App. Math. and Comp. Intel., Vol. 4 (1) (2015) 361–367

http://amci.unimap.edu.my

A note on "A new method for solving fully fuzzy linear fractional programming with a triangular fuzzy numbers"

Sapan Kumar Das[a], T. Mandal [a], S. A. Edalatpanah[b]

[a]*Department of Mathematics, National Institute of Technology Jamshedpur, India, 831014*
[b]*Young Researchers Club, Lahijan Branch, Islamic Azad University, Iran,*

Received: 19 June 2015; Accepted: 9 September 2015

Abstract: *The objective of this paper is to deal with a kind of fuzzy fractional programming problem where all the parameters and variables are triangular fuzzy numbers. We point out an error in the recently published article (Safaei, Appl. Math. Comp. Intel., 3 (2014) 273-281.) and then correct it. An example is also presented to demonstrate the new form.*

Keywords: *Triangular fuzzy number; Fractional programming; Fully fuzzy mathematical programming.*

PACS: *02.20.-a, 02.70.-c, 02.60.-x*

1 Introduction

There have been significant developments in the theory and applications of fractional programming in the last decades. In most real world environment, the values of coefficients of a linear fractional programming problem are often only imprecisely known to the expert. So the researchers are introduced fuzzy programming techniques; see [1-2] and references therein.

Recently, Safaei in [2] proposed a new method for solving fully fuzzy linear fractional programming (FFLFP) with a triangular fuzzy numbers. In Section 4 of the mentioned paper, the

* Corresponding Author: cool.sapankumar@gmail.com (S. Kumar Das)

author assumed that fuzzy optimal solution of fully fuzzy linear fractional programming problem can be obtained by transforming it into crisp linear fractional programming problem:

$$\text{Max (or Min) } (L_1, L_2, L_3) \approx \sum_{j=1}^{n} \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}, \tag{1}$$

$$\text{subject to } \sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \leq \tilde{b}_i \ \forall i = 1,2,3.......m.$$

$\tilde{x}_j$ is a non-negative triangular fuzzy number, and,

$$\text{Max (or Min) } (L_1, L_2, L_3) \approx \sum_{j=1}^{n} \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}, \tag{2}$$

$$\text{subject to } \sum_{j=1}^{n} (m_i, n_i, t_i) \leq (b_i, g_i, h_i) \ \forall i = 1,2,3....m,$$

and all decision variables are non-negative.

We observe that the model is not correct. In this paper, we demonstrate the shortcomings of the proposed method and explain a correct model for the solution of FFLFP.

2 Preliminaries

Here, we first state some definitions which we refer to later.

Definition 2.1 [1]: A fuzzy number $\tilde{A} = (b,c,a)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(X) = \begin{cases} \frac{(x-b)}{(c-b)}, & b \leq x \leq c, \\ \frac{(x-a)}{(c-a)}, & c \leq x \leq a, \\ 0, & else. \end{cases}$$

Definition 2.2 [1]:

A triangular fuzzy number (b,c,a) is said to be non-negative (non-positive) triangular fuzzy number if and only if $b \geq 0$ $(a \leq 0)$.

Definition 2.3 [1]:

Two triangular fuzzy number $\widetilde{A}=(b,c,a)$ and $\widetilde{B}=(e,f,d)$ are said to be equal if and only if $b=e, c=f, a=d$.

Definition 2.4:

Let $\widetilde{A}=(b,c,a)$, $\widetilde{B}=(e,f,d)$ be two triangular fuzzy number then the fuzzy arithmetic is defined as follows:

(i) $\widetilde{A}+\widetilde{B}=(b,c,a)+(e,f,d)=(b+e,c+f,a+d)$,

(ii) $-\widetilde{A}=(-a,-c,-b)$,

(iii) $\widetilde{A}-\widetilde{B}=(b,c,a)-(e,f,d)=(b-d,c-f,a-e)$,

(iv) Let $\widetilde{A}=(b,c,a)$ be any triangular fuzzy number and $\widetilde{B}=(e,f,d)$ be a non-negative triangular fuzzy number then one may have,

$$\widetilde{A}\otimes\widetilde{B}=\widetilde{A}\widetilde{B}=\begin{cases}(be,cf,ad) & if\ b\geq 0,\\ (bd,cf,ad) & if\ b<0, a\geq 0,\\ (bd,cf,cd) & if\ c<0,\end{cases}$$

(v) $\dfrac{\widetilde{A}}{\widetilde{B}}=(\dfrac{b}{d},\dfrac{c}{f},\dfrac{a}{e})$

3 Main Results

Here, we show that the model of [2] is based on a false interpretation. From [2] and Esq.(1-2), it is easy to see that the author construct his model based on the following assumptions :

Maximize (or Minimize) L₁= $\sum_{j=1}^{n}(\frac{r_j^1}{s_j^1})$ (Lower value of $\sum_{j=1}^{n}\frac{(r_j^1,r_j^2,r_j^3)}{(s_j^1,s_j^2,s_j^3)}$) (3)

Maximize (or Minimize) $L_2 = \sum_{j=1}^{n} (\frac{r_j^2}{s_j^2})$ (middle value of $\sum_{j=1}^{n} \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}$)

Maximize (or Minimize) $L_3 = \sum_{j=1}^{n} (\frac{r_j^3}{s_j^3})$ (upper level of $\sum_{j=1}^{n} \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}$)

Subject to $\sum_{j=1}^{n} m_i \le b_i, \quad \forall i = 1, 2, 3......m.$

$$\sum_{j=1}^{n} n_i \le g_i, \quad \forall i = 1, 2, 3......m.$$

$$\sum_{j=1}^{n} t_i \le h_i, \quad \forall i = 1, 2, 3......m.$$

and all decision variables are non-negative.

However, based on Definition 2.4, above extension is not true. Therefore, the fully fuzzy linear fractional programming problem of Esq.(1-2)cannot be transformed into three crisp objective problem Eq.(3) . Hence, all results proposed in [2] are wrong.

In the following, we present a correction version of Eq.(3). Based on Definitions 2.4, we can transform the fuzzy objective function into three crisp objective functions. So we can rewritten the Eq.(3) as follows:

Maximize (or Minimize) $L_1 = \sum_{j=1}^{n} (\frac{r_j^1}{s_j^3})$ (Lower value of $\sum_{j=1}^{n} \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}$). (4)

Maximize (or Minimize) $L_2 = \sum_{j=1}^{n} (\frac{r_j^2}{s_j^2})$ (Middle value of $\sum_{j=1}^{n} \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}$).

Maximize (or Minimize) $L_3 = \sum_{j=1}^{n} (\frac{r_j^3}{s_j^1})$ (upper level of $\sum_{j=1}^{n} \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}$).

Subject to $\sum_{j=1}^{n} m_i \le b_i, \quad \forall i = 1,2,3......m.$

$$\sum_{j=1}^{n} n_i \le g_i, \quad \forall i = 1,2,3......m.$$

$$\sum_{j=1}^{n} t_i \le h_i, \quad \forall i = 1,2,3......m.$$

and all decision variables are non-negative.

Next, we consider an example to show that our demonstrations.

Consider the following problem:

Maximize $\tilde{L} \approx \dfrac{(2,4,7) \otimes \tilde{x}_1 + (1,3,4) \otimes \tilde{x}_2 + (1,2,4)}{(1,2,3) \otimes \tilde{x}_1 + (3,5,8) \otimes \tilde{x}_2 + (0,1,2)}$.

Subject to $(0,1,2)\tilde{x}_1 + (1,2,3)\tilde{x}_2 \le (1,10,27)$,

$(1,2,3)\tilde{x}_1 + (0,1,2)\tilde{x}_2 \le (2,11,28)$,

$\tilde{x}_1, \tilde{x}_2$ is a non-negative triangular fuzzy number.

Solution. Let $\tilde{x}_1 = (x_1, y_1, z_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2)$. The given fully fuzzy linear fractional programming (FFLFP) problem may be written as:

Maximize $(L_1, L_2, L_3) \approx \dfrac{(2,4,7) \otimes (x_1, y_1, z_1) + (0,3,4) \otimes (x_2, y_2, z_2) + (1,2,4)}{(1,2,3) \otimes (x_1, y_1, z_1) + (3,5,8) \otimes (x_2, y_2, z_2) + (1,1,2)}$

Subject to $(0,1,2) \otimes (x_1, y_1, z_1) + (1,2,3) \otimes (x_2, y_2, z_2) \le (1,10,27)$,

$(1,2,3) \otimes (x_1, y_1, z_1) + (0,1,2) \otimes (x_2, y_2, z_2) \le (2,11,28)$,

(x_1, y_1, z_1) and (x_2, y_2, z_2) is non-negative triangular fuzzy number.

By the method of [2], we have get the fuzzy optimal solution of problem is $\widetilde{L}(\widetilde{x}) = (1.34, 2, 2.31)$. However, we know that these three crisp linear fractional programming problem objective functions are not correct. Therefore, in the following, we solve the problem based on the correct version of the method. We consider the following crisp linear fractional programming problem as follows:

Maximize $L_1 = \dfrac{2x_1 + x_2 + 1}{3z_1 + 8z_2 + 2}$.

Maximize $L_2 = \dfrac{4y_1 + 3y_2 + 2}{2y_1 + 5y_2 + 1}$.

Maximize $L_3 = \dfrac{7z_1 + 4z_2 + 4}{x_1 + 3x_2}$.

Subject to $0x_1 + x_2 \le 1$,

$y_1 + 2y_2 \le 10$,

$2z_1 + 3z_2 \le 27$,

$x_1 + 0x_2 \le 2$,

$2y_1 + y_2 \le 11$,

$3z_1 + 2z_2 \le 28$,

and all variables are no-negative triangular fuzzy numbers.

After solving this problem using same algorithm used by [2], we get the fuzzy optimal solution of FFLFP problem is $\widetilde{L}(\widetilde{x}) = (1.5, 2, 2.8)$.

4 Conclusions

In this paper, we studied a new model for solving fully fuzzy linear fractional programming problems proposed in [2]. We have shown that this model is not correct. Furthermore, a correct version is given in this paper.

References

[1] H. S. Najafi and S. A. Edalatpanah. A note on "A new method for solving fully fuzzy linear programming problem". *Applied Mathematical Modelling*, 37(14):7865-7867, 2013.

[2] N. Safaei, A new method for solving fully fuzzy linear fractional programming with a triangular fuzzy numbers, *Applied Mathematics and Computational Intelligence*, 3(1):273-281, 2014.