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Transport Equation for the Joint Distribution Functions of Certain Variables in Convective Turbulent Flow in Presence of Coriolis Force undergoing a First Order Reaction

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Abstract: *In this paper, the joint distribution functions for simultaneous velocity, temperature, concentration fields in turbulent flow undergoing a first order reaction in presence of Coriolis force have been studied. The various properties of the constructed joint distribution functions have been discussed. The transport equations for one and two point joint distribution functions of velocity, temperature, concentration in convective turbulent flow due to first order reaction in presence of Coriolis force have been derived.*

Keywords: *Coriolis force*, *Concentration, Distribution functions, Turbulent flow, First order reaction.*

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1 Introduction

In molecular kinetic theory in physics a particle's distribution function is a function of seven variables, $f(x, y, z, v_x, v_y, v_z)$ which gives the number of particles per unit volume in phase space. It is the number of particles per unit volume having approximately the velocity (v_x, v_y, v_z) near the place (x, y, z) and time t. The distributions function as used in physics. Particle distribution functions are often used in plasma physics to describe wave-particle interactions and velocity-space instabilities. Distribution functions are also used in fluid mechanics, statistical mechanics, fluid and nuclear physics. In the past, several researchers discussed the distribution functions in the statistical theory of turbulence. G. K. Batchelor [1] studied the theory

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of homogeneous turbulence. Lundgren [2] derived the transport equation for the distribution of velocity in turbulent flow. Bigler [3] gave the hypothesis that in turbulent flames, the thermo chemical quantities can be related locally to few scalars and considered the probability density function of these scalars. Kishore [4] studied the distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. S. B. Pope [5] studied the statistical theory of turbulence flames. Also, Pope [6] derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow. Kollman and Janica [7] derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient flux model. Kishore and Singh [8] derived the transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Also Kishore and Singh [9] have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. The Coriolis force helps to clarify the relation between angular momentum and rotational kinetic energy and how an inertial force can have a significant affect on the movement of a body and still without doing any work. On a rotating earth the Coriolis force acts to change the direction of a moving body to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. This deflection is not only instrumental in the large-scale atmospheric circulation, the development of storms, and the sea-breeze circulation Atkinson [10], it can even affect the outcome of baseball tournaments. Also a first-order reaction is defined a reaction that proceeds at a rate that depends linearly only on one reactant concentration. Later, some researchers extended their works including Coriolis force. In the continuation, Azad and Sarker [11] studied the Statistical theory of certain distribution functions in MHD turbulence in a rotating system in presence of dust particles. Sarker and Azad [12] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system. Sarker and Azad [13], Azad and Sarker [14] deliberated the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi- time in a rotating system and dust particles. Azad and Sarker [15] discussed the decay of temperature fluctuations in MHD turbulence before the final period in a rotating system. Also, Azad et al. [16], Sarker et al. [17], Azad et al. [18], Aziz et al. [19], Azad et al. [20] discussed the First Order Reactant in MHD turbulence before the final period of decay for the case of multi-point multi-time and multi -point single time considering rotating system and dust particles. Following the above researchers, Aziz et al. [21, 22**],** Azad et al. [23] had further studied the statistical theory of certain distribution functions in MHD turbulent flow for velocity and concentration considering first order reaction with a rotating system and dust particles. Aziz et al. [24] extended their study for the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system in presence of dust particle. Sarker, Bkar Pk and Azad[25] studied the hhomogeneous dusty fluid turbulence in a first order reactant for the case of multi -point and multi -time prior to the final period of decay. Azad, Molla and Z. Rahman, [26] studied the transport equatoin for the joint distribution function of velocity, temperature and concentration in convective tubulent flow in presence of dust particles. Molla, Azad and Z. Rahman [27] discussed the decay of temperature fluctuations in homogeneous turbulenc before the finaln period in a rotating system. Bkar et al. [28], Bkar et al. [29, 30] premeditated the first-order reactant in homogeneous dusty fluid turbulence prior to the ultimate phase of decay for four-point correlation considering rotating system. Bkar P.K., et al. [31, 32] had studied the decay of MHD turbulence before the final period for four- point correlation among dust particle and rotating system. M. H. U. Molla et al. [33] studied the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow in presence of Coriolis force.

But at this stage, one is met with the difficulty that the N-point distribution function depends upon the N+1-point distribution function and thus result is an unclosed system. This so-called closer problem is encountered in turbulence, Kinetic theory and other non-linear system.

In this paper, we have studied the joint distribution function for simultaneous velocity, temperature, concentration fields in turbulent flow in presence of Coriolis force undergoing a first order reaction. Finally, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have been discussed.

2 Methodology

2.1 Basic equations

The equation of motion and field equations of temperature and concentration in presence of Coriolis force are shown by

$$\frac{\partial u_\alpha}{\partial t}+u_\alpha\frac{\partial u_\alpha}{\partial x_\beta}=-\frac{\partial}{\partial x_\beta}\int_0^\infty\frac{1}{4\pi}\frac{\partial}{\partial x'_\beta}\left\{u_\alpha(x',t)\frac{\partial}{\partial x'_\beta}\cdot u_\alpha(x',t)\right\}\frac{dx'_\beta}{\left|x_\beta-x'_\beta\right|}+\nu\frac{\partial}{\partial x_\beta}\frac{\partial}{\partial x_\beta}u_\alpha-2\in_{m\alpha\beta}\Omega_m u_\alpha\,, \tag{1}$$

$$\frac{\partial\theta}{\partial t}+u_\alpha\frac{\partial\theta}{\partial x_\beta}=f\frac{\partial}{\partial x_\beta}\frac{\partial}{\partial x_\beta}\theta\,, \tag{2}$$

$$\frac{\partial c}{\partial t}+u_\alpha\frac{\partial c}{\partial x_\beta}=\mathrm{D}\frac{\partial}{\partial x_\beta}\frac{\partial}{\partial x_\beta}c-Rc\,, \tag{3}$$

where u and x are vector quantities in the whole process. u_α (x, t) = Fluctuating velocity component, θ (x, t) =Temperature fluctuation, c = Concentration of contaminants, ν = Kinematics viscosity, f= Coefficient of thermal conductivity, D= Diffusive coefficient for contaminants, $\in_{m\alpha\beta}$ = Alternating tensor, Ω_m= Angular velocity of a uniform rotation, R=constant reaction rate.

Formulation of the problem

We consider the turbulence and the concentration fields are homogeneous, also consider a large ensemble of mixture of miscible fluids in which each member is an infinite incompressible heat conducting fluid in turbulent state. The fluid velocity u, temperature θ and concentration c are randomly distributed functions of position and time and satisfy their field equations. Different members of ensemble are subjected to different initial conditions and the aim is to find out a way by which we can determine the ensemble averages at the initial time. The present aim is to construct a joint distribution functions, study its properties and derive an equation for its evolution of this joint distribution functions in presence of Coriolis force undergoing a first order reaction.

Joint distribution function in convective turbulence and their properties

It may be considered that the fluid velocity u, temperature θ, concentration c at each point of the flow field in turbulence. Lundgren (1967) and Sarker and Kishore (1991, 1999) has studied the flow field on the basis of one variable character only (namely the fluid u) but we can study it for two or more variable characters as well. The corresponding to each point of the flow field, we have three measurable characteristics. We represent the three variables by v, ϕ and ψ and denote the pairs of these variables at the points $x^{(1)},x^{(2)}--------,x^{(n)}$ as $\left(v^{(1)},\phi^{(1)},\psi^{(1)}\right)$, $\left(v^{(2)},\phi^{(2)},\psi^{(2)}\right)$,

……, $\left(v^{(n)},\phi^{(n)},\psi^{(n)}\right)$ at a fixed instant of time. It is possible that the same pair may be occurring more than once; therefore, we simplify the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Instead of considering discrete points in the flow field if we consider the continuous distribution of the variables and ψ over the entire flow field, statistically behaviour of the fluid may be described by the distribution function $F(v,\phi,\psi)$ which is normalized so that

$$\int F(v,\phi,\psi)dv\,d\phi\,d\psi = 1\,,$$

where the integration ranges over all the possible values of v, ϕ and ψ. We shall make use of the same normalization condition for the discrete distributions also. The joint distribution functions of the above quantities can be defined in terms of Dirac Delta-functions.

The one-point joint distribution function $F_1^{(1)}\left(v^{(1)},\phi^{(1)},\psi^{(1)}\right)$ is defined in such a way that $F_1^{(1)}\left(v^{(1)},\phi^{(1)},\psi^{(1)}\right)dv^{(1)}d\phi^{(1)}d\psi^{(1)}$ is the probability that the fluid velocity, temperature and concentration field at a time t are in the element $dv^{(1)}$ about $v^{(1)}$, $d\phi^{(1)}$ about $\phi^{(1)}$ and $d\psi^{(1)}$ about $\psi^{(1)}$ respectively and is given as

$$F_1^{(1)}\left(v^{(1)},\phi^{(1)},\psi^{(1)}\right)=\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle\,, \tag{4}$$

where, δ is the Dirac delta-function defined as:

$$\int\delta(u-v)\,dv=\begin{cases}1, & \text{at the point } u=v,\\ 0, & \text{otherwise}.\end{cases}$$

Two-point joint distribution function is given by

$$F_2^{(1,2)}=\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\,\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\varphi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle\,, \tag{5}$$

and three point distribution functions is shown by

$$\begin{aligned}F_3^{(1,2,3)}=\langle&\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\,\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\varphi^{(2)}\right)\\&\times\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\varphi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle.\end{aligned} \tag{6}$$

Similarly, we can define an infinite numbers of multi-point joint distribution functions $F_4^{(1,2,3,4)}$, $F_5^{(1,2,3,4,5)}$ and so on. The joint distribution functions so constructed have the following properties:

(A) Reduction properties

Integration with respect to pair of variables at one-point, lowers the order of distribution function by one. For example:

$$\int F_1^{(1)} dv^{(1)}\, d\varphi^{(1)}\, d\psi^{(1)} = 1,$$

$$\int F_2^{(1,2)} dv^{(2)}\, d\varphi^{(2)}\, d\psi^{(2)} = F_1^{(1)},$$

$$\int F_3^{(1,2,3)} dv^{(3)}\, d\varphi^{(3)}\, d\psi^{(3)} = F_2^{(1,2)},$$

and so on.

Also the integration with respect to any one of the variables reduces the number of Delta-functions from the distribution function by one as:

$$\int F_1^{(1)} dv^{(1)} = \langle \delta\left(\theta^{(1)} - \varphi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right)\rangle,$$

$$\int F_1^{(1)} d\varphi^{(1)} = \langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right)\rangle,$$

and

$$\int F_2^{(1,2)} dv^{(2)} = \langle \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right)\rangle,$$

and so on.

(B) Separation properties

The pairs of variables at the two points are statistically independent of each other if these points are far apart from each other in the flow field i.e.,

$$\lim_{\left|x^{(2)} - x^{(1)}\right| \to \infty} F_2^{(1,2)} = F_1^{(1)} F_1^{(2)},$$

and similarly,

$$\lim_{\left|x^{(3)} - x^{(2)}\right| \to \infty} F_3^{(1,2,3)} = F_2^{(1,2)} F_1^{(3)} \quad etc.$$

(C) Coincidence property

When two points coincide in the flow field, the components at these points should be obviously the same that is $F_2^{(1,2)}$ must be zero. Thus:

$$v^{(2)} = v^{(1)}, \phi^{(2)} = \phi^{(1)} \quad and \quad \psi^{(2)} = \psi^{(1)}.$$

But also $F_2^{(1,2)}$ must have the property

$$\int F_2^{(1,2)} dv^{(2)}\, d\phi^{(2)}\, d\psi^{(2)} = F_1^{(1)}.$$

And hence it follows that:

$$\lim_{\left|x^{(2)} - x^{(1)}\right| \to \infty} F_2^{(1,2)} = F_1^{(1)}\, \delta\left(v^{(2)} - v^{(1)}\right) \delta\left(\phi^{(2)} - \phi^{(1)}\right) \delta\left(\psi^{(2)} - \psi^{(1)}\right).$$

Similarly

$$\lim_{\left|x^{(3)} - x^{(2)}\right| \to \infty} F_3^{(1,2,3)} = F_2^{(1,2)}\, \delta\left(v^{(3)} - v^{(1)}\right) \delta\left(\phi^{(3)} - \phi^{(1)}\right) \delta\left(\psi^{(3)} - \psi^{(1)}\right) \quad etc.$$

Continuity equation in terms of distribution functions

An infinite number of continuity equations can be derived for the convective turbulent flow and the continuity equations can be easily expressed in terms of distribution functions and are obtained directly by div u = 0.

$$\begin{aligned}
\left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle &= \left\langle \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} d\varphi^{(1)} d\psi^{(1)} \right\rangle = \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} d\varphi^{(1)} d\psi^{(1)} \right\rangle \\
&= \frac{\partial}{\partial x_\alpha^{(1)}} \int \left\langle u_\alpha^{(1)} \right\rangle \left\langle F_1^{(1)} \right\rangle dv^{(1)} d\varphi^{(1)} d\psi^{(1)} = \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_1^{(1)} dv^{(1)} d\varphi^{(1)} d\psi^{(1)}, \qquad (7) \\
&= \int \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} v_\alpha^{(1)} dv^{(1)} d\varphi^{(1)} d\psi^{(1)} = 0,
\end{aligned}$$

and similarly

$$\int \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} \phi_\alpha^{(1)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} = 0, \qquad (8)$$

which are the first order continuity equations in which only one point distribution function is involved. For second-order continuity equations, if we multiply the continuity equation by

$$\delta\left(u^{(2)} - v^{(2)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right).$$

And if we take the ensemble average, we obtain:

$$0 = \left\langle \delta\left(u^{(2)} - v^{(2)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right) \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)u_\alpha^{(1)} \right\rangle,$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int \left\langle u_\alpha^{(1)}\, \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\right\rangle\left\langle \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)\right\rangle,$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_2^{(1,2)} dv^{(1)} d\phi^{(1)} d\psi^{(1)}, \tag{9}$$

and similarly, $0 = \frac{\partial}{\partial x_\alpha^{(1)}} \int \phi_\alpha^{(1)} F_2^{(1,2)} dv^{(1)} d\phi^{(1)} d\psi^{(1)}.$ (10)

The Nth-order continuity equations are

$$0 = \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_N^{(1,2,---N)} dv^{(1)} d\phi^{(1)} d\psi^{(1)}, \tag{11}$$

and

$$0 = \frac{\partial}{\partial x_\alpha^{(1)}} \int \phi_\alpha^{(1)} F_N^{(1,2,---N)} dv^{(1)} d\phi^{(1)} d\psi^{(1)}. \tag{12}$$

The continuity equations are symmetric in their arguments i.e.

$$\begin{aligned} &\frac{\partial}{\partial x_\alpha^{(r)}} \int \left(v_\alpha^{(r)} F_N^{(1,2,---s---r---N)} dv^{(r)} d\varphi^{(r)} d\psi^{(r)} \right) \\ &= \frac{\partial}{\partial x_\alpha^{(s)}} \int \left(v_\alpha^{(s)} F_N^{(1,2,---r---s---N)} dv^{(s)} d\varphi^{(s)} d\psi^{(s)} \right). \end{aligned} \tag{13}$$

Since, the divergence property is an important property and it is easily verified by the use of the property of distribution function as:

$$\frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_1^{(1)} dv^{(1)} d\phi^{(1)} d\psi^{(1)} = \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle u_\alpha^{(1)} \right\rangle = \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = 0, \tag{14}$$

and all the properties of the distribution function obtained in section (4) can also be easily verified.

Equations for the evolution of joint distribution functions

This, in fact is done by making use of the definitions of the constructed distribution functions, the transport equation for $F(v,\phi,\psi,x,t)$ is obtained from the definition of F and from the transport equations (1), (2), (3). Differentiating equation (4) we get,

$$\frac{\partial}{\partial t}F_1^{(1)}=\frac{\partial}{\partial t}\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle=\left\langle \delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$
$$+\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\right\rangle+\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$
$$=\left\langle -\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}}{\partial t}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle \tag{15}$$
$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial \theta^{(1)}}{\partial t}\frac{\partial}{\partial \varphi^{(1)}}\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\right\rangle$$
$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\frac{\partial c^{(1)}}{\partial t}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle.$$

Using equation (1), (2) and (3) in the equation (15) we get

$$\frac{\partial}{\partial t}F_1^{(1)}=\left\langle -\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\begin{bmatrix}-u_\alpha^{(1)}\dfrac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}-\dfrac{\partial}{\partial x_\beta^{(1)}}\displaystyle\int \frac{1}{4\pi}\frac{\partial}{\partial x_\beta^{(2)}}\left\{u_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\cdot u_\alpha^{(2)}\right\}\\ \times\dfrac{dx_\beta^{(2)}}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|}+\nu\dfrac{\partial}{\partial x_\beta^{(1)}}\dfrac{\partial}{\partial x_\beta^{(1)}}u_\alpha^{(1)}-2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\end{bmatrix}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$
$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\left\{-u_\alpha^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}+f\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\theta^{(1)}\right\}\frac{\partial}{\partial\varphi^{(1)}}\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\right\rangle$$
$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\left\{-u_\alpha^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}+\mathrm{D}\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}c^{(1)}\right\}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)-Rc^{(1)}\right\rangle,$$
$$\Rightarrow\frac{\partial F_1^{(1)}}{\partial t}+\left\langle -\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_\alpha^{(1)}\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$
$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_\alpha^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle$$
$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_\alpha^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$
$$+\left\langle \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\left[-\frac{\partial}{\partial x_\beta^{(1)}}\left\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}u_\alpha^{(2)}u_\alpha^{(2)}\frac{dx_\beta^{(2)}}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|}\right\}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right]\right\rangle$$

$$+\left\langle \delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\nu\left(\frac{\partial}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}u_{\alpha}^{(1)}\right)\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$
$$+\left\langle -\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)2\in_{m\alpha\beta}\Omega_{m}u_{\alpha}^{(1)}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$
$$+\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)f\left(\frac{\partial}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}\right)\theta^{(1)}\frac{\partial}{\partial \varphi^{(1)}}\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\right\rangle \tag{16}$$
$$+\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\mathrm{D}\left(\frac{\partial}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}\right)c^{(1)}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$
$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)Rc^{(1)}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle=0.$$

Various terms in the above equation can be simplified as that they may be expressed in terms of one point and two point distribution functions. The 2^{nd} , 3^{rd} and 4^{th} terms on the left hand side of the above equation are simplified in a similar fashion and take the forms as follows

$$\left\langle -\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\alpha}^{(1)}\frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$
$$=\left\langle \delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\alpha}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle,\qquad\left[\because\frac{\partial u_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}}=-1\right], \tag{17}$$

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\alpha}^{(1)}\frac{\partial \theta^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle$$
$$=\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\alpha}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle, \tag{18}$$

and

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_{\alpha}^{(1)}\frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$
$$=\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_{\alpha}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle \tag{19}$$

Adding equation (17), (18) and (19) we get,

$$\left\langle \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\alpha}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle+\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)u_{\alpha}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle$$
$$+\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_{\alpha}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$\begin{aligned}
&=\frac{\partial}{\partial x_{\beta}^{(1)}}\left\langle u_{\alpha}^{(1)}\left(\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right)\right\rangle,\\
&=\frac{\partial}{\partial x_{\beta}^{(1)}}v_{\alpha}^{(1)}F_{1}^{(1)}, \qquad \left[\textit{Applying the properties of distribution function}\right]\\
&=v_{\alpha}^{(1)}\frac{\partial F_{1}^{(1)}}{\partial x_{\beta}^{(1)}}.
\end{aligned}\tag{20}$$

We reduce the 5th and 6th terms on left hand side of equation (16),

$$\begin{aligned}
&\left\langle \delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\left[-\frac{\partial}{\partial x_{\beta}^{(1)}}\left\{\frac{1}{4\pi}\int \frac{\partial}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial x_{\beta}^{(2)}}u_{\alpha}^{(2)}u_{\alpha}^{(2)}\frac{dx_{\beta}^{(2)}}{\left|x_{\beta}^{(1)}-x_{\beta}^{(2)}\right|}\right\}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right]\right\rangle\\
&=\frac{\partial}{\partial v_{\alpha}^{(1)}}\left[-\frac{1}{4\pi}\int\left(\frac{\partial}{\partial x_{\beta}^{(2)}}\frac{\partial}{\left|x_{\beta}^{(1)}-x_{\beta}^{(2)}\right|}\right)\right]\left(v_{\alpha}^{(2)}\frac{\partial}{\partial x_{\beta}^{(2)}}\right)^{2}F_{2}^{(1,2)}dx^{(2)}dv^{(2)}\,d\phi^{(2)}\,d\psi^{(2)},
\end{aligned}\tag{21}$$

and

$$\begin{aligned}
&\left\langle \delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\nu\left(\frac{\partial}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}u_{\alpha}^{(1)}\right)\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle\\
&=\left\langle \nu\frac{\partial}{\partial v_{\alpha}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}u_{\alpha}^{(1)}\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle,\\
&=\nu\frac{\partial}{\partial v_{\alpha}^{(1)}}\left\langle \frac{\partial}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}u_{\alpha}^{(1)}\left[\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(1)}-v^{(1)}\right)\right]\right\rangle,\\
&=\nu\frac{\partial}{\partial v_{\alpha}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}\left\langle u_{\alpha}^{(1)}\left[\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(1)}-v^{(1)}\right)\right]\right\rangle,\\
&=\nu\frac{\partial}{\partial v_{\alpha}^{(1)}}\underset{x^{(2)}\to x^{(1)}}{Lim}\frac{\partial}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial x_{\beta}^{(2)}}\left\langle \begin{array}{l}\int u_{\alpha}^{(2)}\delta\left(\theta^{(2)}-\varphi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\\ \delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(1)}-v^{(1)}\right)dv^{(2)}d\varphi^{(2)}d\psi^{(2)}\end{array}\right\rangle,\\
&=\frac{\partial}{\partial v_{\alpha}^{(1)}}\underset{x^{(2)}\to x^{(1)}}{Lim}\,\nu\frac{\partial}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial x_{\beta}^{(2)}}\int v^{(2)}F_{2}^{(1,2)}dv^{(2)}d\varphi^{(2)}d\psi^{(2)}.
\end{aligned}\tag{22}$$

We reduce the 7th term on left hand side of equation (16),

$$\left\langle -\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$
$$=\left\langle 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\left[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right]\right\rangle,$$
$$=2\in_{m\alpha\beta}\Omega_m\frac{\partial}{\partial v_\alpha^{(1)}}\left\langle u_\alpha^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle, \tag{23}$$
$$=2\in_{m\alpha\beta}\Omega_m\frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}}\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle=2\in_{m\alpha\beta}\Omega_m F_1^{(1)}.$$

Similarly, 8th, 9th and 10th terms of left hand side of (16) can be simplified as follows:

$$\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)f\left(\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\right)\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle$$
$$=\frac{\partial}{\partial\phi^{(1)}}\lim_{x^{(2)}\to x^{(1)}}f\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int\phi^{(2)}F_2^{(1,2)}dv^{(2)}d\phi^{(2)}d\psi^{(2)}, \tag{24}$$

$$\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\varphi^{(1)}\right)\mathrm{D}\left(\frac{\partial}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\right)c^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$
$$=\frac{\partial}{\partial\psi^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\mathrm{D}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int\psi^{(2)}F_2^{(1,2)}dv^{(2)}d\varphi^{(2)}d\psi^{(2)}, \tag{25}$$

and $\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)Rc^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle=-R\psi^{(1)}\frac{\partial}{\partial\psi^{1}}F_1^{(1)}.$ (26)

Substituting the results (20)-(26) in equation (16), we get the transport equation for one point distribution function $F_1^{(1)}(v,\phi,\psi)$ in turbulent flow in a rotating system undergoing a first order reaction

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\alpha^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+\frac{\partial}{\partial v_\alpha^{(1)}}\left[-\frac{1}{4\pi}\int\left(\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|}\right)\right]\left(v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\right)^2F_2^{(1,2)}dx^{(2)}dv^{(2)}d\varphi^{(2)}d\psi^{(2)}$$
$$+\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\nu\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int v^{(2)}F_2^{(1,2)}dv^{(2)}d\varphi^{(2)}d\psi^{(2)}$$
$$+\frac{\partial}{\partial\varphi^{(1)}}\lim_{x^{(2)}\to x^{(1)}}f\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int\varphi^{(2)}F_2^{(1,2)}dv^{(2)}d\varphi^{(2)}d\psi^{(2)} \tag{27}$$
$$+\frac{\partial}{\partial\psi^{(1)}}\lim_{x^{(2)}\to x^{(1)}}\mathrm{D}\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int\psi^{(2)}F_2^{(1,2)}dv^{(2)}d\varphi^{(2)}d\psi^{(2)}$$
$$+2\in_{m\alpha\beta}\Omega_m F_1^{(1)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{1}}F_1^{(1)}=0.$$

Similarly, a transport equation for two-point distribution function $F_2^{(1,2)}$ in turbulent flow in rotating system undergoing a first order reaction can be derived by differentiating equation (5) and using equation (1),(2),(3) and simplifying in the same manner which is

$$\frac{\partial F_2^{(1,2)}}{\partial t}+\left(v_\alpha^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\right)F_2^{(1,2)}+\frac{\partial}{\partial v_\alpha^{(1)}}\left[\begin{array}{l}-\frac{1}{4\pi}\int\left(\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\left|x_\beta^{(1)}-x_\beta^{(3)}\right|}\right)\left(v_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\right)^2\\ \times F_3^{(1,2,3)}dx^{(3)}dv^{(3)}\,d\phi^{(3)}\,d\psi^{(3)}\end{array}\right]$$

$$+\frac{\partial}{\partial v_\alpha^{(2)}}\left[-\frac{1}{4\pi}\int\left(\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\left|x_\beta^{(2)}-x_\beta^{(3)}\right|}\right)\left(v_\alpha^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\right)^2F_3^{(1,2,3)}dx^{(3)}dv^{(3)}\,d\varphi^{(3)}\,d\psi^{(3)}\right]$$
$$+v\left(\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(3)}\to x^{(1)}}+\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{x^{(3)}\to x^{(2)}}\right)\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}\int v^{(3)}F_3^{(1,2,3)}dv^{(3)}d\varphi^{(3)}d\psi^{(3)}$$
$$+f\left(\frac{\partial}{\partial\varphi^{(1)}}\lim_{x^{(3)}\to x^{(1)}}+\frac{\partial}{\partial\varphi^{(2)}}\lim_{x^{(3)}\to x^{(2)}}\right)\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}\int\varphi^{(3)}F_3^{(1,2,3)}dv^{(3)}d\varphi^{(3)}d\psi^{(3)} \tag{28}$$
$$+\mathrm{D}\left(\frac{\partial}{\partial\psi^{(1)}}\lim_{x^{(3)}\to x^{(1)}}+\frac{\partial}{\partial\psi^{(2)}}\lim_{x^{(3)}\to x^{(2)}}\right)\frac{\partial}{\partial x_\beta^{(3)}}\frac{\partial}{\partial x_\beta^{(3)}}\int\psi^{(3)}F_3^{(1,2,3)}dv^{(3)}d\varphi^{(3)}d\psi^{(3)}$$
$$+2\in_{m\alpha\beta}\Omega_m F_2^{(1,2)}-R\psi^{(2)}\frac{\partial}{\partial\psi^2}F_2^{(1,2)}=0.$$

Continuing this way, we can derive the equations for evolution of $F_3^{(1,2,3)}, F_4^{(1,2,3,4)}$ and so on. Logically, it is possible to have an equation for every $F_n\ (n\ is\ an\ \mathrm{int}eger)$ but the system of equations so obtained is not closed. It seems that certain approximations will be required thus obtained.

3 Results and Discussion

If the reaction rate R=0, the transport equation for one point joint distribution function $F_1^{(1)}(v,\phi,\psi)$ in turbulent flow undergoing a first order reaction, equation (27) becomes

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\alpha^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+\frac{\partial}{\partial v_\alpha^{(1)}}\left[-\frac{1}{4\pi}\int\left(\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\left|x_\beta^{(1)}-x_\beta^{(2)}\right|}\right)\right]\left(v_\alpha^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\right)^2F_2^{(1,2)}dx^{(2)}dv^{(2)}\,d\phi^{(2)}\,d\psi^{(2)}$$
$$+\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{x^{(2)}\to x^{(1)}}v\frac{\partial}{\partial x_\beta^{(2)}}\frac{\partial}{\partial x_\beta^{(2)}}\int v^{(2)}F_2^{(1,2)}dv^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\frac{\partial}{\partial \varphi^{(1)}} \underset{x^{(2)} \to x^{(1)}}{Lim} f \frac{\partial}{\partial x_\beta^{(2)}} \frac{\partial}{\partial x_\beta^{(2)}} \int \varphi^{(2)} F_2^{(1,2)} dv^{(2)} d\varphi^{(2)} d\psi^{(2)}$$
$$+\frac{\partial}{\partial \psi^{(1)}} \underset{x^{(2)} \to x^{(1)}}{Lim} \mathrm{D} \frac{\partial}{\partial x_\beta^{(2)}} \frac{\partial}{\partial x_\beta^{(2)}} \int \psi^{(2)} F_2^{(1,2)} dv^{(2)} d\varphi^{(2)} d\psi^{(2)} + 2 \in_{m\alpha\beta} \Omega_m F_1^{(1)} = 0, \tag{29}$$

which was obtained earlier by M.H.U. Molla [33].

In the absence of the Coriolis force, $\Omega_m = 0$, then the transport equation for one point joint distribution function $F_1^{(1)}(v, \phi, \psi)$ in turbulent flow equation (26) becomes

$$\frac{\partial F_1^{(1)}}{\partial t} + v_\alpha^{(1)} \frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}} + \frac{\partial}{\partial v_\alpha^{(1)}} \left[-\frac{1}{4\pi} \int \left(\frac{\partial}{\partial x_\beta^{(2)}} \frac{\partial}{\left| x_\beta^{(1)} - x_\beta^{(2)} \right|} \right) \right] \left(v_\alpha^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} \right)^2 F_2^{(1,2)} dx^{(2)} dv^{(2)} d\varphi^{(2)} d\psi^{(2)}$$
$$+\frac{\partial}{\partial v_\alpha^{(1)}} \underset{x^{(2)} \to x^{(1)}}{Lim} \nu \frac{\partial}{\partial x_\beta^{(2)}} \frac{\partial}{\partial x_\beta^{(2)}} \int v^{(2)} F_2^{(1,2)} dv^{(2)} d\varphi^{(2)} d\psi^{(2)}$$
$$+\frac{\partial}{\partial \varphi^{(1)}} \underset{x^{(2)} \to x^{(1)}}{Lim} f \frac{\partial}{\partial x_\beta^{(2)}} \frac{\partial}{\partial x_\beta^{(2)}} \int \varphi^{(2)} F_2^{(1,2)} dv^{(2)} d\varphi^{(2)} d\psi^{(2)}$$
$$+\frac{\partial}{\partial \psi^{(1)}} \underset{x^{(2)} \to x^{(1)}}{Lim} \mathrm{D} \frac{\partial}{\partial x_\beta^{(2)}} \frac{\partial}{\partial x_\beta^{(2)}} \int \psi^{(2)} F_2^{(1,2)} dv^{(2)} d\varphi^{(2)} d\psi^{(2)} = 0, \tag{30}$$

which was obtained earlier by N. Kishore and S.R. Singh [9].

To close the system of equations for the joint distribution functions some approximations are required. If we consider the collection of ionized particles i.e., in plasma turbulence case, it can be provided closure form easily by decomposing $F_2^{(1,2)}$ as $F_1^{(1)} F_1^{(2)}$. But such type of approximations can be possible if there is no interaction or correlation between two particles.

If we decompose $F_2^{(1,2)}$ as:

$$F_2^{(1,2)} = (1+\varepsilon) F_1^{(1)} F_1^{(2)}, \tag{31}$$

$$F_3^{(1,2,3)} = (1+\varepsilon)^2 F_1^{(1)} F_1^{(2)} F_1^{(3)}, \tag{32}$$

where ε is the correlation coefficient between the particles. If there is no correlation between the particles, ε will be zero and joint distribution function can be decomposed in usual way. Here, we are considering such type of approximation only to provide closed form of the equation i.e., to approximate two-point equation as one point equation. The transport equation for the joint distribution function of velocity, temperature, and concentration has been shown here to provide an advantageous basis for modeling the turbulent flows in presence of Coriolis force undergoing a first order reaction.

In this study, we have made an attempt for the modeling of various terms such as fluctuating pressure, viscosity and diffusivity in order to close the equation for joint distribution function of velocity, temperature and concentration. Since $F(v, \phi, \psi)$ contains all the statistical information about the velocity at each point, a turbulence model to determine the Reynolds stresses is not needed. However, since $F(v,\phi,\psi)$ is one point statistics, the length scale information is also not needed.

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