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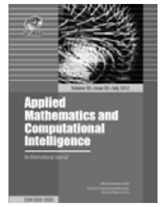

MHD free convection and mass transfer flow of a dissipative fluid over a vertical plate with time-dependent plate velocity and constant heat flux

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Abstract: *This paper presents a numerical analysis of the MHD free convection and mass transfer flow of a dissipative fluid along a vertical plate with time–dependent plate velocity, the surface of which is exposed to a constant heat flux. The non-linear system of partial differential equations is numerically solved by the implicit finite difference scheme of Crank–Nicolson's type. The velocity, temperature and concentration profiles for different values of plate velocity are plotted. The influence of the buoyancy ratio parameter, dissipation number, Grashof number, Prandtl number, Schmidt number and magnetic parameter on heat and mass transfer are discussed. It is found that, the velocity as well as temperature increases with effects dissipation number. the velocity and temperature decreases with increases in the magnetic parameter, while concentration profiles increases with increases in magnetic parameter. The velocity increases as well as the temperature and the concentration decreases with increases buoyancy ratio parameter. the velocity increases as well as the temperature and the concentration decreases with increases Grashof number.*

Keywords: *Time-dependent plate velocity, MHD, free convection, dissipative fluid, vertical plate*

PACS: *44.05.+e , 44.20.+b , 44.25.+f , 47.11.Bc , 47.15.Cb , 47.65.-d*

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1 Introduction

Industrial manufacturing processes such as plasma studies, petroleum industries Magneto-hydrodynamics power generator cooling of clear reactors, boundary layer control in aerodynamics. Many authors have studied the effects of magnetic field on mixed, natural and force convection

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heat and mass transfer problems. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. Magnetohydrodynamic has attracted the attention of a large number of scholars due to its diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering it finds its application in MHD pumps, MHD bearings etc. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. The phenomena of mass transfer are also very common in theory of stellar structure and observable effects are detectable, at least on the solar surface. The study of effects of magnetic field on free convection flow is important in liquid-metals, electrolytes and ionized gases. The thermal physics of hydromagnetic problems with mass transfer is of interest in power engineering and metallurgy.

Numerous works have studied this problem, the first of which, Pohlhausen [1], did not consider viscous dissipation but obtained a solution employing the integral method. Harris et al. [2] investigated the transient free convection from a vertical plate when the plate temperature is suddenly changed, obtaining an analytical solution (for small time values) and a numerical solution until the steady-state is reached. Polidori et al. [3] proposed a theoretical approach to the transient dynamic behaviour of a natural convection boundary-layer flow when a step variation of the uniform heat flux is applied, using the Karman–Pohlhausen integral method. Other authors studied the effect of the surface temperature oscillation [4,5]. Kassem [6] solved the problem for unsteady free-convection flow from a vertical moving plate subjected to constant heat flux. Gebhart [7] was the first to study who studied the problem taking viscous dissipation into account and this author defined the non-dimensional dissipation parameter. Takhar and Soundalgekar [8] studied the effect of a harmonic oscillation in the plate temperature in the form of a travelling wave convected in the direction of the free-stream of viscous incompressible fluids. Pantokratoras [9] solved the problem in a stationary situation using the finite-difference method, with isothermal and uniform flux boundary conditions in the wall, taking into account viscous dissipation. Soundalgekar et al. [10] solved the transient problem with an isothermal vertical wall. When heat and mass transfer occurs simultaneously, it leads to a complex fluid motion (the combination of temperature and concentration gradients in the fluid will lead to buoyancy-driven flows). This problem arises in numerous engineering processes, for example, biology and chemical processes, nuclear waste repositories and the extraction of geothermal energy. Soundalgekar and Ganesan [11] solved the problem of transient free convection with mass transfer on an isothermal vertical flat plate. Gokhale and Samman [12] studied the effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. They obtained many conclusions concerning the effect of the variations of the different non-dimensional parameters that defined the problem on the time required to reach the steady-state. When the presence of a uniform magnetic field is considered, a new problem can be studied, "Unsteady free convection MHD with coupled heat and mass transfer". This problem has attracted the interest of many researchers in view of its application in astrophysics, geophysics fluid dynamics and engineering. Shanker and Kishan [13] studied the effects of mass transfer on the MHD flow past an impulsively started infinite vertical plate with variable temperature or constant heat flux. Ganesan and Rani [14] solved the unsteady free convection flow over a vertical cylinder under the influence of a magnetic field problem, without taking into account the viscous dissipation. Hossain et al. [15] considered surface temperature oscillations, using three different methods, including perturbation and asymptotic methods, the local non-similarity method and an implicit finite-difference method. Aboeldahad and Elbarbary [16] employed a numerical solution using a fourth-order Runge–Kutta scheme to study the Hall effects on the heat and mass transfer. The natural convection flow of a conducting visco-elastic liquid between two heated vertical plates under the influence of transverse magnetic field has been studied by Sreehari Reddy et al [17]. Suneetha et al.[18] have analyzed the thermal radiation effects on hydromagnetic free convection flow past an impulsively started vertical plate with variable surface temperature and concentration is analyzed by taking into account of the heat due to viscous dissipation. Recently Suneetha et al. [19] studied the effects of thermal radiation on the natural conductive heat and mass transfer of a viscous incompressible gray absorbing-emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation. Very recently Hiteesh [20] studied the boundary layer steady flow and heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field.

The object of the present paper is to study the transient free convection and mass transfer flow of an incompressible dissipative viscous fluid past a vertical plate, under the influence of a uniform transverse magnetic field in the presence of constant heat flux and time-dependent plate velocity. The dimensionless governing equations are solved by using implicit finite difference scheme of Crank–Nicolson's type.

2 Mathematical model

Consider the free convection flow of a dissipative fluid through an infinite vertical plate with constant heat flux and time-dependent plate velocity (Fig. 1) under the action of a transverse magnetic field. Under these assumptions and Boussinesq' s approximation, the flow is governed by the following system of equations:

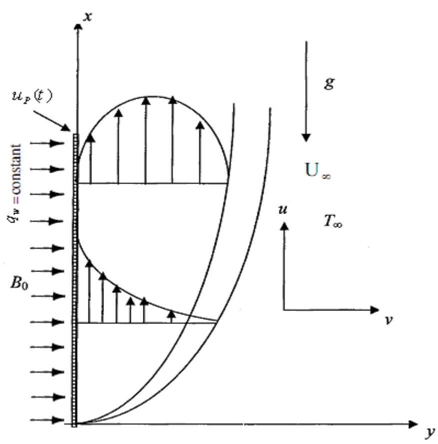

Figure. 1. Sketch of the physical model

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\beta g\left(T-T_\infty\right)+\beta^* g\left(c-c_\infty\right)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

Energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Mass equation:

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2} \tag{4}$$

where u and v are components of the velocity in x and y directions, respectively, υ is the kinematic viscosity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric expansion coefficient for mass transfer, g is the acceleration due to gravity, ρ is the density, σ fluid electrical conductivity, B_0 is magnetic induction, α is fluid thermal diffusivity, c_p is specific heat at constant pressure, T is the temperature, T_∞ is the temperature of the fluid far away from the plate, C is the concentration, C_∞ is the concentration far away from the plate and D is the molecular diffusivity.

The necessary initial and boundary conditions are:

$$: \ u=0\ ,v=0\ ,\ T=T_\infty\ ,\ c=c_\infty\quad t\le 0$$

$$: \ u=0\ ,v=0\ ,\ T=T_\infty\ ,\ c=c_\infty\ \text{ at }\ x=0\ t\rangle 0$$

$$: \ u=u_P(t)\ ,v=0\ ,\ -k\frac{\partial T}{\partial y}=q\ ,\ c=c_\infty\ \text{ at }\ y=0\ \ t\rangle 0 \tag{5}$$

$$: \ u=0\ ,T\to T_\infty\ ,\ c\to c_\infty\ ,\ c=c_\infty\ \text{ at }\ y\to\infty\ t\rangle 0$$

Now introduce the following non dimensional quantities:

$$\tau=\frac{tu_p^2}{\upsilon}\ ,\ U=\frac{u}{u_p}\ ,\ V=\frac{v}{u_p}\ ,\ X=\frac{xu_p}{\upsilon}\ ,\ Y=\frac{yu_p}{\upsilon}$$

$$\theta = \frac{T - T_\infty}{q_w L / k} \; , \; C = \frac{c - c_\infty}{c_w - c_\infty} \; , \; Gr = \frac{g\,\beta \upsilon q_W L}{k u_p^3} \; , \; Sc = \frac{\upsilon}{D} \tag{6}$$

$$N = \frac{g \upsilon \beta^* \left(c_w - c_\infty\right)}{u_p^3} \; , \; M = \frac{\sigma B_0^2 \upsilon}{\rho u_p^2} \; , \; \varepsilon = \frac{k u_p^2}{c_p q_W L} \; , \; \Pr = \frac{\upsilon}{\alpha}$$

where L is the wall height, X is the dimensionless axial coordinate, Y is the dimensionless axial coordinate perpendicular to X , U, V is the dimensionless velocities, θ is the dimensionless temperature, C is the non-dimensional species concentration, q_w is the heat flux at the plate, Sc is the Schmidt number, N is the buoyancy ratio parameter, Gr is the Grashof number, M is the magnetic parameter, $\Pr$ is the prandtl number and ε is the dissipation number.

Then the governing equations reduce to the following non-dimensional boundary-layer equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{7}$$

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + \theta Gr + NC - Mu \tag{8}$$

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial Y^2} + \varepsilon \left(\frac{\partial U}{\partial Y} \right)^2 \tag{9}$$

$$\frac{\partial C}{\partial \tau} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \tag{10}$$

The dimensionless boundary conditions become:

$: \; U = 0 \, , V = 0 \; , \; \theta = 0 \; , \; C = 0 \, , \quad \tau \le 0,$

$: \; U = 0 \; , V = 0 \; , \; \theta = 0 \; , \; C = 0$ at $X = 0, \quad \tau \rangle 0,$

$$: \; U = U_P(\tau) \;, V = 0 \;, \; \frac{\partial \theta}{\partial Y} = -1 \,, C = 1 \quad \text{at} \quad Y = 0, \quad \tau \rangle 0, \tag{11}$$

$$: \; U = 0 \;\; , \theta = 0 \;\; , \; C = 0 \quad \text{at} \quad Y \to \infty, \; \tau \rangle 0.$$

3 Numerical Solution of the problem

The governing equations (7-10) are steady, coupled and non-linear with boundary conditions. An implicit finite - difference technique of Crank–Nicolson has been employed to solve the nonlinear coupled equations, as described (Thomas algorithm) in Carnahan et al [21]. The finite difference equations corresponding to equations (7–10) are as follows:

$$\frac{U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^{n} - U_{i-1,j}^{n}}{2\Delta X} + \frac{V_{i,j+1}^{n+1} - V_{i,j-1}^{n+1} + V_{i,j+1}^{n} - V_{i,j-1}^{n}}{4\Delta Y} = 0, \tag{12}$$

$$\frac{U_{i,j}^{n+1} - U_{i,j}^{n}}{\Delta \tau} + U_{i,j}^{n} \frac{U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^{n} - U_{i-1,j}^{n}}{2\Delta X} + v_{i,j}^{n} \frac{U_{i,j+1}^{n+1} - U_{i,j-1}^{n+1} + U_{i,j+1}^{n} - U_{i,j-1}^{n}}{4\Delta Y}$$

$$= \frac{U_{i,j+1}^{n+1} - 2U_{i,j}^{n+1} + U_{i,j-1}^{n+1} + U_{i,j+1}^{n} - 2U_{i,j}^{n} + U_{i,j-1}^{n}}{2\Delta Y^2} + Gr \frac{\theta_{i,j}^{n+1} + \theta_{i,j}^{n}}{2}$$

$$+ N \frac{C_{i,j}^{n+1} + C_{i,j}^{n}}{2} - M \frac{U_{i,j}^{n+1} + U_{i,j}^{n}}{2}, \tag{13}$$

$$\frac{\theta_{i,j}^{n+1} - \theta_{i,j}^{n}}{\Delta \tau} + U_{i,j}^{n} \frac{\theta_{i,j}^{n+1} - \theta_{i-1,j}^{n+1} + \theta_{i,j}^{n} - \theta_{i-1,j}^{n}}{2\Delta X} + v_{i,j}^{n} \frac{\theta_{i,j+1}^{n+1} - \theta_{i,j-1}^{n+1} + \theta_{i,j+1}^{n} - \theta_{i,j-1}^{n}}{4\Delta Y}$$

$$= \frac{1}{\Pr} \frac{\theta_{i,j+1}^{n+1} - 2\theta_{i,j}^{n+1} + \theta_{i,j-1}^{n+1} + \theta_{i,j+1}^{n} - 2\theta_{i,j}^{n} + \theta_{i,j-1}^{n}}{2\Delta Y^2}$$

$$+ \varepsilon \left(\frac{U_{i,j+1}^{n+1} - U_{i,j-1}^{n+1} + U_{i,j+1}^{n} - U_{i,j-1}^{n}}{4\Delta Y} \right)^2 \tag{14}$$

$$\frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta \tau} + U_{i,j}^{n} \frac{C_{i,j}^{n+1} - C_{i-1,j}^{n+1} + C_{i,j}^{n} - C_{i-1,j}^{n}}{2\Delta X} + v_{i,j}^{n} \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1} + C_{i,j+1}^{n} - C_{i,j-1}^{n}}{4\Delta Y}$$

$$= \frac{1}{Sc} \frac{C_{i,j+1}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j-1}^{n+1} + C_{i,j+1}^{n} - 2C_{i,j}^{n} + C_{i,j-1}^{n}}{2\Delta Y^2} \tag{15}$$

The region of integration is considered as a rectangle with sides $X_{max}(=1)$ and $Y_{max}(=10)$, where corresponding to $Y \to \infty$ which lies far from the momentum , energy and concentration boundary layers. An appropriate mesh sizes considered for the calculation are $\Delta X = 0.01$, $\Delta Y = 0.05$ and $\Delta\tau = 0.005$.

4 Results and discussion

The velocity, temperature and concentration profiles have been computed by using implicit finite difference scheme of Crank–Nicolson's type. The numerical calculations are carried out for the effect of the flow parameters such as plate velocity ($U_P(\tau)$), Prandtl number (Pr), Schmidth number (Sc), Grashof number (Gr), magnetic parameter (M), dissipation number (ε), buoyancy ratio parameter (N), on the velocity, temperature and concentration distribution of the flow fields are presented graphically in figure 2-14.

The effects of dissipation number (ε) on the transient velocity and temperature profiles for different values of plate velocity are shown in Figs. 2-3. It is observed that the velocity and temperature increases with dissipation number for different values of plate velocity. In fact dissipation number increases the thickness of the boundary layer and depth of diffusion of the thermal boundary layer increases. Figs 4-5 display the influence of buoyancy ratio parameter (N) on the transient velocity and temperature profiles for different values of plate velocity. It is clear that increasing the buoyancy ratio parameter tends to increases the velocity as well as the species temperature. The hydrodynamics and the thermal boundary layer become thick as the buoyancy ratio parameter increases.

Figs 6-8 illustrate the dimensionless velocity, temperature and concentration profiles for different values of plate velocity and for magnetic parameter (M). It is obvious that, the velocity and temperature decreases with increases in magnetic parameter, while concentration profiles increases with increases in magnetic parameter. The presence of the transverse magnetic field produces a resistive force the fluid flow. This force is called the Lorentz force, which leads to slow down the motion of electrically conducting fluid, which tends to increase the temperature and concentration. However, the influence of magnetic parameter (M) on the concentration is insignificant.

The effects of Grashof number (Gr) on the velocity, temperature, and concentration profiles for different values of plate velocity are shown in Figs. 9-11. It is observed that the velocity and temperature increases with increase in Grashof number while concentration decreases with increase in the Grashof number.

Figs. 12-13 depict the velocity and temperature profiles for different values of plate velocity and for different values of Prandtl number (Pr). It is observed that the velocity and temperature decreases with increase in the Prandtl number.

The influences of Schmidth number (Sc) on the concentration profiles for different values of plate velocity are shown in Fig. 14. It is observed that the concentration decreases with increase in Schmidth number.

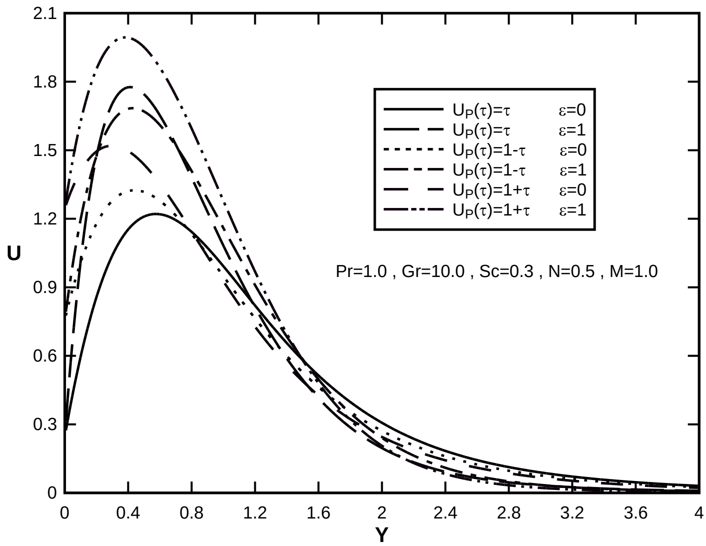

Figure 2. Effect of dissipation number (ε) on dimensionless velocity Profiles

for different values of plate velocity

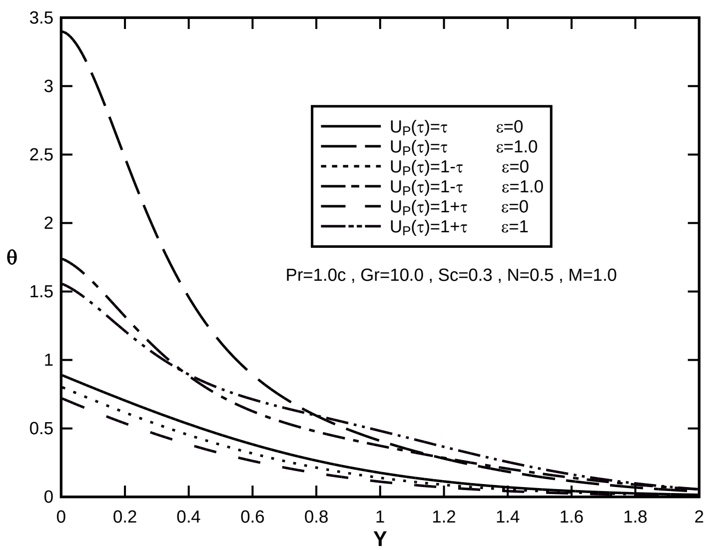

Figure 3. Effect of dissipation number (ε) on dimensionless temperature distributions

for different values of plate velocity

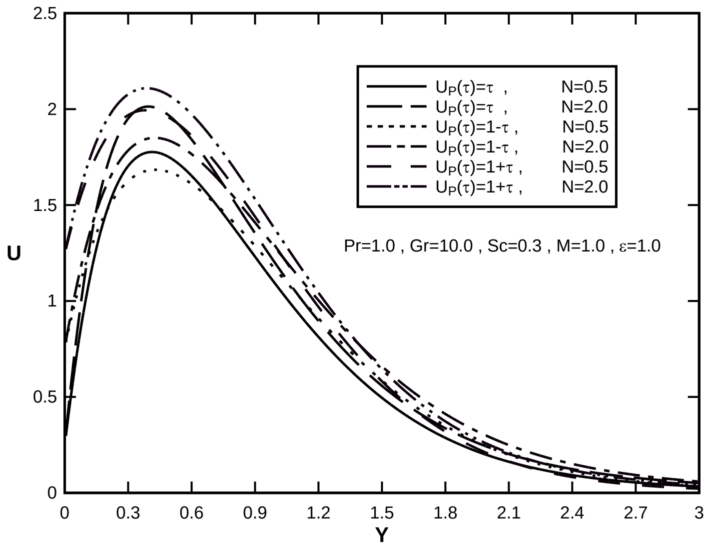

Figure 4. Effect of buoyancy ratio parameter (N) on dimensionless velocity

Profiles for different values of plate velocity

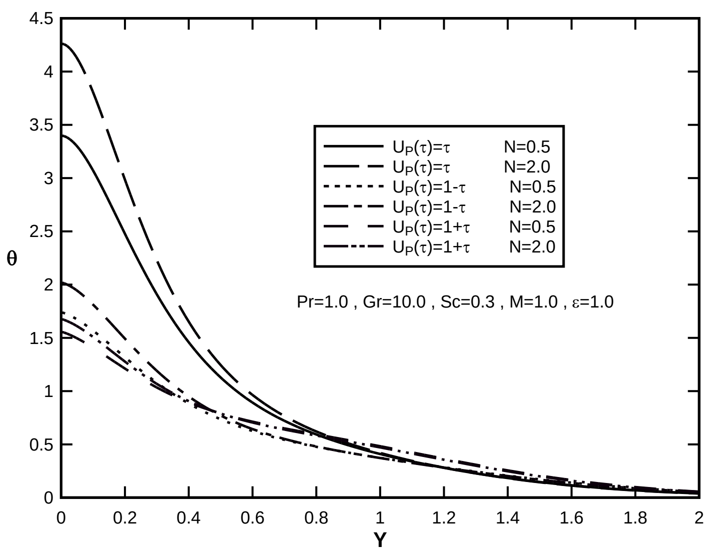

Figure 5. Effect of buoyancy ratio parameter (N) on dimensionless

temperature distributions for different values of plate velocity

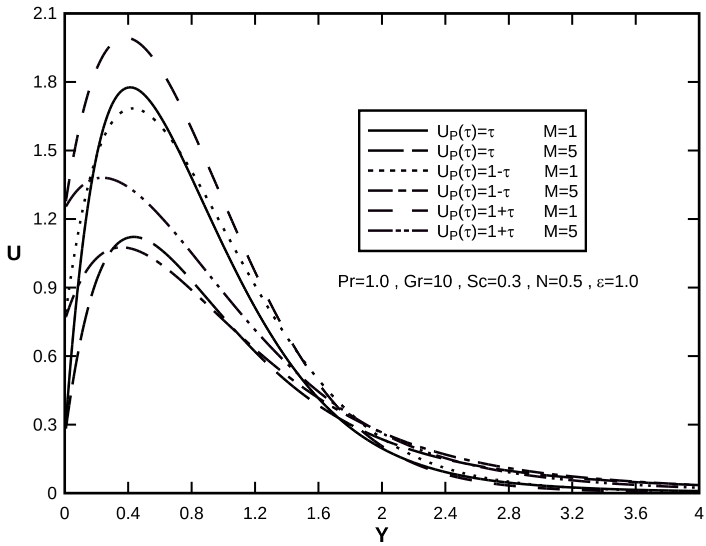

Figure 6. Effect of magnetic parameter (M) on dimensionless velocity Profiles for different values of plate velocity

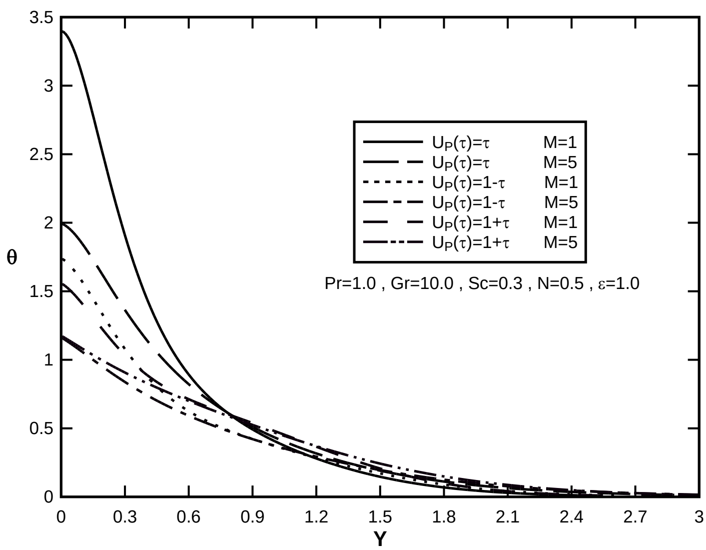

Figure 7. Effect of magnetic parameter (M) on dimensionless temperature distributions for different values of plate velocity

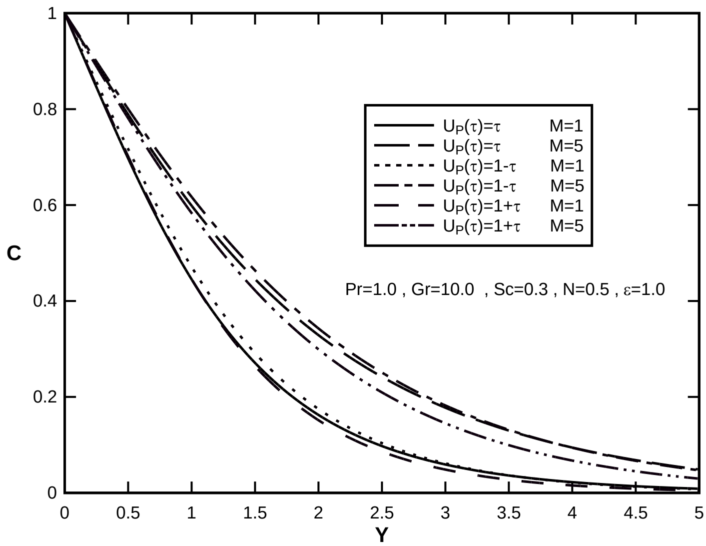

Figure 8. Effect of magnetic parameter (M) on dimensionless

concentration distributions for different values of plate velocity

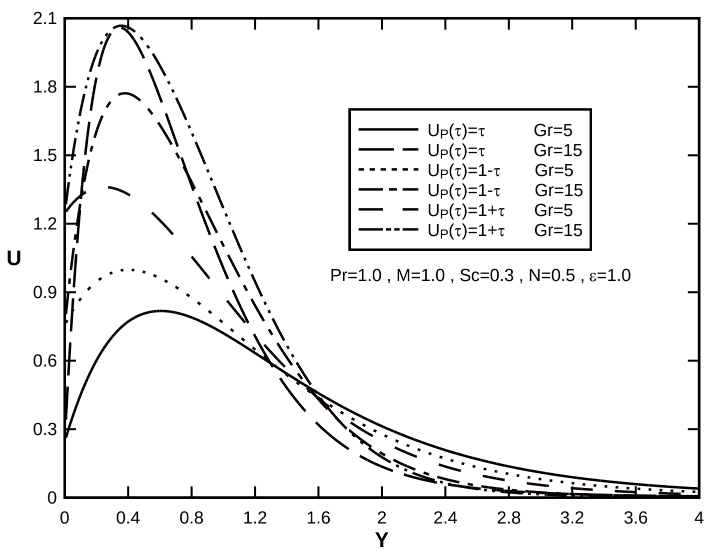

Figure 9. Effect of Grashof number (Gr) on dimensionless velocity Profiles

for different values of plate velocity

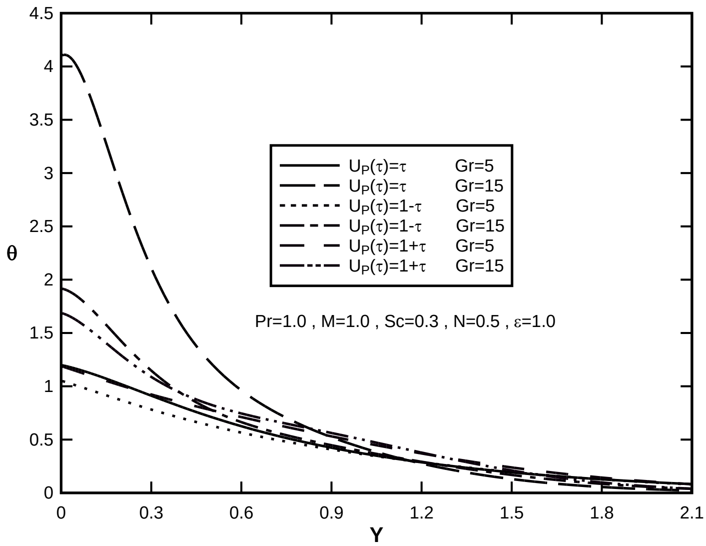

Figure 10. Effect of Grashof number (Gr) on dimensionless temperature distributions for different values of plate velocity

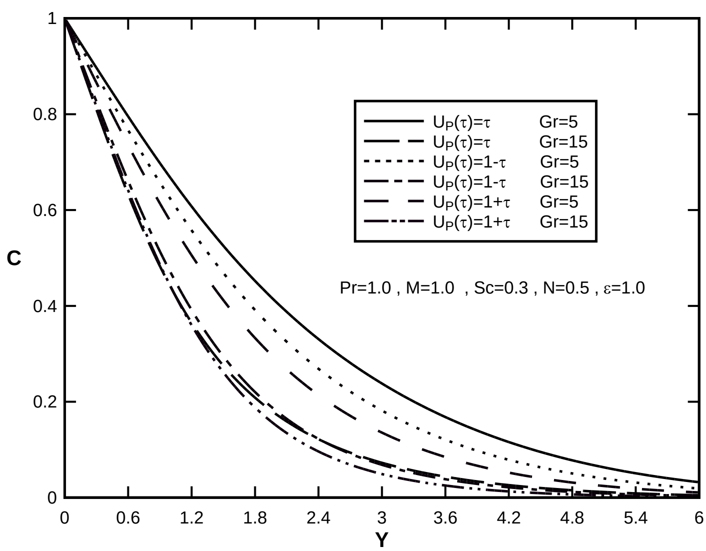

Figure 11. Effect of Grashof number (Gr) on dimensionless concentration distributions for different values of plate velocity

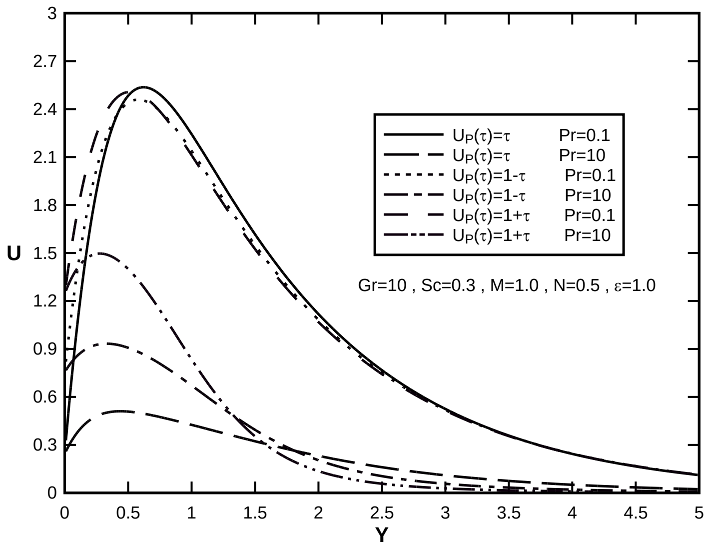

Figure 12. Effect of Prandtl number (Pr) on dimensionless velocity Profiles

for different values of plate velocity

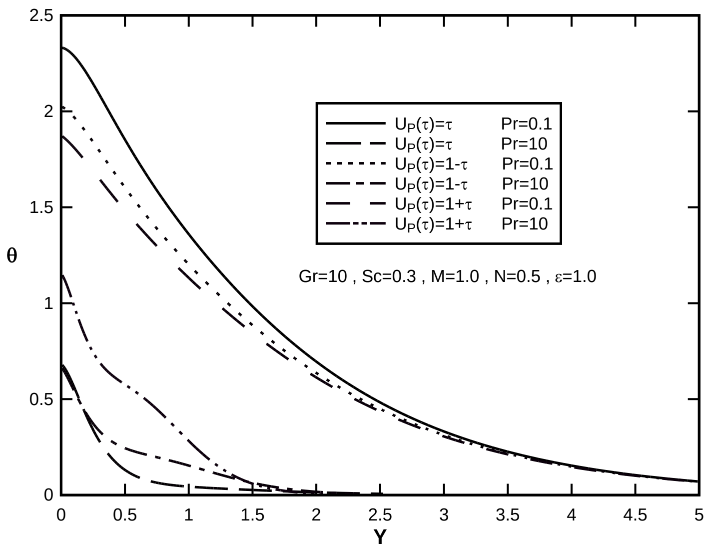

Figure 13. Effect of Prandtl number (Pr) on dimensionless

temperature distributions for different values of plate velocity

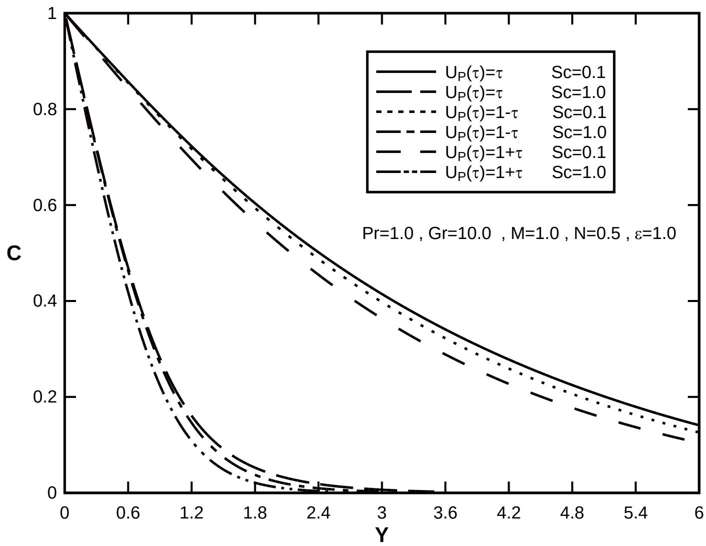

Figure 14. Effect of Schmidth number (Sc) on dimensionless

concentration distributions for different values of plate velocity

5 Conclusions

In this paper the effects of Time-dependent plate velocity on MHD free convection and mass transfer flow of a dissipative fluid over a vertical plate with constant heat flux have been studied numerically. Implicit finite difference scheme of Crank–Nicolson's type is employed to solve the equations.

From the present numerical investigation, following conclusions have been drawn:

Velocity increases, temperature and concentration decreases with an increase in Grashof number (Gr),

Velocity and temperature decreases, concentration increase with an increase in magnetic parameter (M).

With effects of the dissipation number (ε), velocity and temperature increases.

Increase in the buoyancy ratio parameter (N), velocity increases, temperature decreases.

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