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A new method for solving fully fuzzy linear fractional programming with a triangular fuzzy numbers

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Abstract: *In this paper, we propose a method of solving the fully fuzzy linear fractional programming (FFLFP) problems, where all the parameters and variables are triangular fuzzy numbers. In the proposed method, the given FFLFP problem is decomposed into three crisp linear fractional programming (CLFP) problems with bounded variables constraints, three CLFP problems are solved separately and by using its optimal solutions, the fuzzy optimal solution to the given FFLFP problem is obtained. Fuzzy ranking functions and addition of nonnegative variables were not used and there is no restriction on the elements of coefficient matrix in the proposed method. Finally, numerical example are used in order to show the efficiency and superiority of the proposed method.*

Keywords: *fully fuzzy linear fractional programming problem; Fractional programming; Triangular fuzzy number.*

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1 Introduction

There have been significant developments in the theory and applications of fractional programming in the last decades. In most real-world situations, the possible values of coefficients of a linear fractional programming problem are often only imprecisely or ambiguously known to the expert [1]. With this observation in mind, it would be certainly more appropriate to interpret the coefficients as fuzzy numerical data.

Generally, two types of problems implying fuzzy uncertainity are studied in the literature. Fuzzy approaches to solve deterministic problems could be developped and also fuzzy models, implying fuzzy goals and fuzzy coefficients, could be defined and solved.

Bellman and Zadeh [2] proposed the concept of decision making in fuzzy environment. Tanaka et al.[3] adopted this concept for solving mathematical programming problems. Zim-

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merman [4] proposed the first formulation of fuzzy linear programming. Inuiguchi et al. [5] used the concept of continuous piecewise linear membership function for fuzzy linear programming problems. Fang et al. [6] developed a method for solving LP problems with fuzzy coefficients in constraints. Buckley and Feuring [7] proposed a method to find the solution for a fully fuzzified linear programming problem by changing the objective function into a multiobjective LP problem. Maleki et al. [8] solved the LP problems by the comparison of fuzzy numbers in which all decision parameters are fuzzy numbers. Maleki [9] proposed a method for solving LP problems with vagueness in constraints by using ranking function. Zhang et al. [10] introduced a method for solving FLP problems in which coefficients of objective function are fuzzy numbers. Nehi et al. [11] developed the concept of optimality for LP problems with fuzzy parameters by transforming FLP problems into multiobjective LP problems. Ramik [12] proposed the FLP problems based on fuzzy relations. Ganesan and Veeramani [13] proposed an approach for solving FLP problem involving symmetric trapezoidal fuzzy numbers without converting it into crisp LP problems. Hashemi et al. [14] introduced a two phase approach for solving fully fuzzified linear programming. Jimenez et al. [15] developed a method using fuzzy ranking method for solving LP problems where all the coefficients are fuzzy numbers .

Saraj and Safaei[16] suggests the use of a taylor series for fuzzy linear fractional bi level multi objective programming problems. Lottfi et al. [17] proposed a method to obtain the approximate solution of FFLP problems. pointed out that there is no method in literature for finding the fuzzy optimal solution of fully fuzzy linear programming problems and proposed a new method to find the fuzzy optimal solution of FFLP problems with equality constraints .

Kumar et al. [18] proposed a new method is proposed to find the fuzzy optimal solution of same type of fuzzy linear programming problems. Pop et al. [19] proposed a method to solve the fully fuzzified linear fractional programming problem , where all the variables and parameters are represented by triangular fuzzy numbers. The proposed approach is first to transform the original fuzzy problem into a determin-istic multiple objective linear fractional programming problem with quadratic constraints. This transformation is obtained by using the extension principle of Zadeh and Kerees method for the evaluation of the fuzzy constraints. In this paper, we introduce a new approach which is easy to handle and has a natural interpretation. The rest of this paper is organized as follows: Section 2 some basic definitions and arithmetics between two triangular fuzzy numbers are reviewed. In Section 3 formulation of FFLFP problems are discussed. In Section 4 a new method is proposed for solving FFLFP problems. The paper ends with conclusions in Section 5.

2 Preliminaries

In this section, some necessary backgrounds and notions of fuzzy set theory are reviewed.

2.1 Basic definitions

Definition 2.1 [20]. The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\widetilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\widetilde{A}} : X \longrightarrow [0, 1]$. The assigned value indicate the membership grade of the element in the set A.
The function $\mu_{\widetilde{A}}$ is called the membership function and the set $\widetilde{A} = \{(x, \mu_{\widetilde{A}}); x \in X\}$ defined by $\mu_{\widetilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2 [20]. A fuzzy number $\widetilde{A} = (a, b, c)$is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & a \leqslant x \leqslant b, \\ \dfrac{(x-c)}{(b-c)} & b \leqslant x \leqslant c, \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.3[20]. A triangular fuzzy number (a , b, c) i s said to be non-negative fuzzy number iff $a \geqslant 0$.
Definition 2.4[20]. Two triangular fuzzy numbers $\widetilde{A} = (a, b, c)$ and $\widetilde{B} = (e, f, g)$ are said to be equal if and only if $a = e, b = f, c = g$
Definition 2.5 Let $\widetilde{A} = (a, b, c)$ and $\widetilde{B} = (e, f, g)$ be two triangular fuzzy numbers, then
$\widetilde{A} \precsim \widetilde{B} \quad iff \quad a \leqslant e, b \leqslant f, c \leqslant g$

2.2 Arithmetic operations

In this subsection, arithmetic operations between two triangular fuzzy numbers, defined on universal set of real numbers R, are reviewed [20]. Let $\widetilde{A} = (a, b, c)$ and $\widetilde{B} = (e, f, g)$ be two triangular fuzzy numbers then
$(i)\widetilde{A} \oplus \widetilde{B} = (a, b, c) \oplus (e, f, g) = (a + e, b + f, c + g)$
$(ii) - \widetilde{A} = -(a, b, c) = (-c, -b, -a)$
$(iii)\widetilde{A} \ominus \widetilde{B} = (a, b, c) \ominus (e, f, g) = (a - g, b - f, c - e)$
(iv) Let $\widetilde{A} = (a, b, c)$ be any triangular fuzzy number and $\widetilde{B} = (x, y, z)$ be a non-negative triangular fuzzy number then

$$\widetilde{A} \otimes \widetilde{B} = \begin{cases} (ax, by, cz) & a \geqslant 0, \\ (az, by, cz) & a < 0, c \geqslant 0, \\ (az, by, cx) & c < 0. \end{cases}$$

3 Fully fuzzy linear fractional programming Problem

Consider the following fully fuzzy linear fractional programming problems with m fuzzy inequality constraints and n fuzzy variables may be formulated as follows:

$$\begin{array}{ll} & \text{Maximize (or Minimize)} \quad \widetilde{L}(\widetilde{X}) \approx \dfrac{(\widetilde{C^T} \otimes \widetilde{X}) \oplus \widetilde{\alpha}}{(\widetilde{D^T} \otimes \widetilde{X}) \oplus \widetilde{\beta}} \\ \text{(P)} & \text{subject to} \qquad \widetilde{A} \otimes \widetilde{X} \precsim \widetilde{b} \\ & \widetilde{X} \text{ is a non-negative triangular fuzzy number,} \end{array}$$

where $\widetilde{C^T} = [\widetilde{c_j}]_{1\times n}, \widetilde{D^T} = [\widetilde{d_j}]_{1\times n}, \widetilde{X} = [\widetilde{x_j}]_{n\times 1}, \widetilde{A} = [\widetilde{a_{ij}}]_{m\times n}, \widetilde{b} = [\widetilde{b_i}]_{m\times 1}$ and $\widetilde{c_j}, \widetilde{d_j}, \widetilde{x_j}, \widetilde{a_{ij}}, \widetilde{b_i}, \widetilde{\alpha}, \widetilde{\beta} \in F(R)$.
For some values of $\widetilde{X}$, $(\widetilde{D^T} \otimes \widetilde{X}) \oplus \widetilde{\beta}$ may be equal to fuzzy zero. to avoid such cases, $(\widetilde{D^T} \otimes \widetilde{X}) \oplus \widetilde{\beta}$ is generally set to be greater than fuzzy zero.
$\widetilde{X} = [\widetilde{x_j}]_{n\times 1}$ is said to be exact fuzzy optimal solution of FFLFP problem (P) if it satisfies the following characteristics:
(i) $\widetilde{X}$ is a non-negative fuzzy number,
(ii)$\widetilde{A} \otimes \widetilde{X} \precsim \widetilde{b}$

(iii) If there exist $\widetilde{\mathring{X}} = [\widetilde{\mathring{x}_j}]_{n\times 1}$ such that $\widetilde{A} \otimes \widetilde{\mathring{X}} \precsim \widetilde{b}$, then $\widetilde{L}(\widetilde{X}) \succ \widetilde{L}(\widetilde{\mathring{X}})$ (in case of maximization problem) and$\widetilde{L}(\widetilde{X}) \prec \widetilde{L}(\widetilde{\mathring{X}})$ (in case of minimization problem).

Let $\widetilde{X} = [\widetilde{x_j}]_{n\times 1}$ be the fuzzy optimal solution of FFLFP problem (P) . If there exist $\widetilde{Y} = [\widetilde{y_j}]_{n\times 1}$ such that

(i) $\widetilde{Y}$ is a non-negative fuzzy number,

(ii)$\widetilde{A} \otimes \widetilde{Y} \precsim \widetilde{b}$

(iii) $\widetilde{L}(\widetilde{X}) \approx \widetilde{L}(\widetilde{Y})$ then $\widetilde{Y}$ is said to be an alternative fuzzy optimal solution of (P).

4 Proposed method to find the fuzzy optimal solution of FFLFP problems

In this section, a new method is proposed to find the fuzzy optimal solution of FFLFP problems, The steps of the proposed method are as follows:

Step 1 Substituting $\widetilde{C^T} = [\widetilde{c_j}]_{1\times n}, \widetilde{D^T} = [\widetilde{d_j}]_{1\times n}, \widetilde{X^T} = [\widetilde{x_j}]_{n\times 1}, \widetilde{A} = [\widetilde{a_{ij}}]_{m\times n}, \widetilde{b} = [\widetilde{b_i}]_{m\times 1}$ the problem (P) may be written as:

$$\text{Maximize (or Minimize)} \quad \widetilde{L} \approx \sum_{j=1}^{n} \Big(\frac{(\widetilde{c_j^T} \otimes \widetilde{x_j}) \oplus \widetilde{\alpha}}{(\widetilde{d_j^T} \otimes \widetilde{x_j}) \oplus \widetilde{\beta}} \Big)$$

$$\text{subject to} \quad \sum_{j=1}^{n} \widetilde{a_{ij}} \otimes \widetilde{x_j} \precsim \widetilde{b_i} \qquad \forall i = 1 \dots m.$$

$\widetilde{x_j}$ is a non-negative triangular fuzzy number.

Step 2 If all the parameters $\widetilde{c_j}, \widetilde{d_j}, \widetilde{x_j}, \widetilde{a_{ij}}, \widetilde{b_i}, \widetilde{\alpha}$and $\widetilde{\beta}$ are represented by triangular fuzzy numbers (p_j^1, p_j^2, p_j^3), (q_j^1, q_j^2, q_j^3), (x_j, y_j, z_j), $(a_{ij}^1, a_{ij}^2, a_{ij}^3)$, (b_i, g_i, h_i, k_i), $(\alpha^1, \alpha^2, \alpha^3)$ and $(\beta^1, \beta^2, \beta^3)$respectively then the FFLFP problem, obtained in Step 1, may be written as:

$$\text{Maximize (or Minimize)} \ (L_1, L_2, L_3) \approx \sum_{j=1}^{n} \Big(\frac{\big[(p_j^1, p_j^2, p_j^3) \otimes (x_j, y_j, z_j)\big] \oplus (\alpha^1, \alpha^2, \alpha^3)}{\big[(q_j^1, q_j^2, q_j^3) \otimes (x_j, y_j, z_j)\big] \oplus (\beta^1, \beta^2, \beta^3)} \Big)$$

$$\text{subject to} \qquad \sum_{j=1}^{n} (a_{ij}^1, a_{ij}^2, a_{ij}^3) \otimes (x_j, y_j, z_j) \precsim (b_i, g_i, h_i) \qquad \forall i = 1 \dots m.$$

$(x_j, y_j, z_j)$is a non-negative triangular fuzzy number.

Now, since (x_j, y_j, z_j) is a triangular fuzzy number, then

$$x_j \leqslant y_j \leqslant z_j \qquad j = 1 \dots n$$

The above relation is called bounded constraints.

Step 3 Assuming $\big[(p_j^1, p_j^2, p_j^3) \otimes (x_j, y_j, z_j)\big] \oplus (\alpha^1, \alpha^2, \alpha^3) = (r_j^1, r_j^2, r_j^3)$, $\big[(q_j^1, q_j^2, q_j^3) \otimes (x_j, y_j, z_j)\big] \oplus (\beta^1, \beta^2, \beta^3) = (s_j^1, s_j^2, s_j^3)$ $(a_{ij}^1, a_{ij}^2, a_{ij}^3) \otimes (x_j, y_j, z_j) = (m_i, n_i, t_i)$ the FFLFP problem, obtained in Step 2, may be written as:

$$\text{Maximize (or Minimize)} \quad (L_1, L_2, L_3) \approx \sum_{j=1}^{n} \left(\frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)} \right)$$

$$\text{subject to} \qquad \sum_{j=1}^{n} (m_i, n_i, t_i) \precsim (b_i, g_i, h_i) \qquad \forall i = 1 \ldots m.$$

and all decision variables are non-negative.

Step 4 Using arithmetic operations and partial ordering relations, the fuzzy linear fractional programming problem, obtained in Step 3, is converted into the following CLFP problem:

$$\text{Maximize (or Minimize)} \quad L_1 = \sum_{j=1}^{n} \left(\frac{r_j^1}{s_j^1} \right) \quad \left(\text{lower value of } \sum_{j=1}^{n} \left(\frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)} \right) \right)$$

$$\text{Maximize (or Minimize)} \quad L_2 = \sum_{j=1}^{n} \left(\frac{r_j^2}{s_j^2} \right) \quad \left(\text{middle value of } \sum_{j=1}^{n} \left(\frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)} \right) \right)$$

$$\text{Maximize (or Minimize)} \quad L_3 = \sum_{j=1}^{n} \left(\frac{r_j^3}{s_j^3} \right) \quad \left(\text{upper value of } \sum_{j=1}^{n} \left(\frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)} \right) \right)$$

$$\text{subject to} \ \sum_{j=1}^{n} m_i \leqslant b_i, \ \forall i = 1 \ldots m. \left(\text{lower value of } \sum_{j=1}^{n} (m_i, n_i, t_i) \precsim (b_i, g_i, h_i) \right)$$

$$\sum_{j=1}^{n} n_i \leqslant g_i, \ \forall i = 1 \ldots m. \left(\text{middle value of } \sum_{j=1}^{n} (m_i, n_i, t_i) \precsim (b_i, g_i, h_i) \right)$$

$$\sum_{j=1}^{n} t_i \leqslant h_i, \ \forall i = 1 \ldots m. \left(\text{upper value of } \sum_{j=1}^{n} (m_i, n_i, t_i) \precsim (b_i, g_i, h_i) \right)$$

and all decision variables are non-negative.

From the above decomposition problem, we construct the following CLFP problems namely, middle level fractional problem (MLFP), upper level fractional problem (ULFP) and lower level fractional problem (LLFP) as follows:

$$\text{(MLFP)} \quad \text{Maximize (or Minimize)} \quad L_2 = \sum_{j=1}^{n} \left(\frac{r_j^2}{s_j^2} \right)$$

$$\text{subject to} \qquad \sum_{j=1}^{n} n_{ij} \leqslant g_i \qquad \forall i = 1 \ldots m.$$

all decision variables are non-negative.

$$\text{(ULFP)} \quad \text{Maximize (or Minimize)} \quad L_3 = \sum_{j=1}^{n} \left(\frac{r_j^3}{s_j^3} \right)$$

$$\text{subject to} \qquad \sum_{j=1}^{n} t_{ij} \leqslant h_i \qquad \forall i = 1 \ldots m.$$

$$\sum_{j=1}^{n} \left(\frac{r_j^3}{s_j^3} \right) \geqslant L_2^*$$

all variables in the constraints and objective function in ULFP must satisfy the bounded constraints, replacing all values of the decision variables which are obtained in MLFP and all decision variables are non-negative.

$$\text{(LLFP)} \quad \text{Maximize (or Minimize)} \quad L_1 = \sum_{j=1}^{n} \left(\frac{r_j^1}{s_j^1}\right)$$

$$\text{subject to} \quad \sum_{j=1}^{n} m_{ij} \leqslant b_i \qquad \forall i = 1 \dots m.$$

$$\sum_{j=1}^{n} \left(\frac{r_j^1}{s_j^1}\right) \leqslant L_2^*$$

all variables in the constraints and objective function in LLFP must satisfy the bounded constraints, replacing all values of the decision variables which are obtained in the MLFP and ULFP and all decision variables are non-negative.

where L_2^* is the optimal objective value of MLFP.

Step 5 Find the optimal solution x_j, y_j and z_j by solving the MLFP, ULFP and LLFP problem obtained in step 4.

Step 6 Find the fuzzy optimal solution by putting the values of x_j, y_j and z_j in $\widetilde{x} = (x_j, y_j, z_j)$.

5 Numerical examples

Let us consider the following FFLFP problem and solve it by the proposed method.

$$\text{Maximize } \widetilde{L} \approx \frac{(2,4,7) \otimes \widetilde{x_1} \oplus (1,3,4) \otimes \widetilde{x_2} \oplus (1,2,4)}{(1,2,3) \otimes \widetilde{x_1} \oplus (3,5,8) \otimes \widetilde{x_2} \oplus (0,1,2)}$$

$$\text{subject to } (0,1,2)\widetilde{x_1} \oplus (1,2,3)\widetilde{x_2} \precsim (1,10,27)$$

$$(1,2,3)\widetilde{x_1} \oplus (0,1,2)\widetilde{x_2} \precsim (2,11,28)$$

$\widetilde{x_1}, \widetilde{x_2}$ is a non-negative triangular fuzzy number.

Solution: Let $\widetilde{x_1} = (x_1, y_1, z_1)$ and $\widetilde{x_2} = (x_2, y_2, z_2)$then given FFLFP problem may be written as:

$$\text{Maximize } (L_1, L_2, L_3) \approx \frac{(2,4,7) \otimes (x_1, y_1, z_1) \oplus (0,3,4) \otimes (x_2, y_2, z_2) \oplus (1,2,4)}{(1,2,3) \otimes (x_1, y_1, z_1) \oplus (3,5,8) \otimes (x_2, y_2, z_2) \oplus (1,1,2)}$$

$$\text{subject to } (0,1,2) \otimes (x_1, y_1, z_1) \oplus (1,2,3) \otimes (x_2, y_2, z_2) \precsim (1,10,27)$$

$$(1,2,3) \otimes (x_1, y_1, z_1) \oplus (0,1,2) \otimes (x_2, y_2, z_2) \precsim (2,11,28)$$

$(x_1, y_1, z_1)$and(x_2, y_2, z_2) is a non-negative triangular fuzzy number.

Using Step 3, the above FFLFP problem may be written as:

$$\text{Maximize } (L_1, L_2, L_3) \approx \frac{(2x_1 + 1x_2, 4y_1 + 3y_2 + 2, 7z_1 + 4z_2 + 4)}{(1x_1 + 3x_2 + 1, 2y_1 + 5y_2 + 1, 3z_1 + 8z_2 + 2)}$$

$$\text{subject to } (0x_1 + 1x_2, 1y_1 + 2y_2, 2z_1 + 3z_2) \precsim (1,10,27)$$

$$(1x_1 + 0x_2, 2y_1 + 1y_2, 3z_1 + 2z_2) \precsim (2,11,28)$$

$(x_1, y_1, z_1)$and(x_2, y_2, z_2) is a non-negative triangular fuzzy number.

Using Step 4 o f the proposed method the above FFLFP problem is converted into the following CLFP problem:

$$\text{Maximize } L_1 = \frac{2x_1 + 1x_2}{1x_1 + 3x_2 + 1}$$

$$\text{Maximize } L_2 = \frac{4y_1 + 3y_2 + 2}{2y_1 + 5y_2 + 1}$$

$$\text{Maximize } L_3 = \frac{7z_1 + 4z_2 + 4}{3z_1 + 8z_2 + 2}$$

$$\begin{aligned}\text{subject to } & 0x_1 + 1x_2 \leqslant 1\\ & 1y_1 + 2y_2 \leqslant 10\\ & 2z_1 + 3z_2 \leqslant 27\\ & 1x_1 + 0x_2 \leqslant 2\\ & 2y_1 + 1y_2 \leqslant 11\\ & 3z_1 + 2z_2 \leqslant 28\\ & x_1, y_1, z_1, x_2, y_2, z_2 \geqslant 0\end{aligned}$$

Now, the Middle Level fractional problem is given below:

$$\text{Maximize } L_2 = \frac{4y_1 + 3y_2 + 2}{2y_1 + 5y_2 + 1}$$

$$\begin{aligned}\text{subject to } & 1y_1 + 2y_2 \leqslant 10\\ & 2y_1 + 1y_2 \leqslant 11\\ & y_1, y_2 \geqslant 0\end{aligned}$$

Solving the the above problem using Charnes and Cooper method, [21] we obtain the optimal solution $y_1 = 5.75, y_2 = 0$ and $L_2 = 2$.
Then, the Upper Level fractional problem is given below:

$$\text{Maximize } L_3 = \frac{7z_1 + 4z_2 + 4}{3z_1 + 8z_2 + 2}$$

$$\begin{aligned}\text{subject to} & 1z_1 - 12z_2 \geqslant 0\\ & 2z_1 + 3z_2 \leqslant 27\\ & 3z_1 + 2z_2 \leqslant 28\\ & z_1 \geqslant y_1\\ & z_2 \geqslant y_2\\ & z_1, z_2 \geqslant 0\end{aligned}$$

Solving the above problem with $y_1 = 5.75, y_2 = 0$ using Charnes and Cooper method, [21] we obtain the optimal solution $z_1 = 10.33, z_2 = 0$ and $L_3 = 2.31$.
Finally, the Lower Level fractional problem:

$$\text{Maximize } L_1 = \frac{2x_1 + 1x_2}{1x_1 + 3x_2 + 1}$$

$$\begin{aligned}\text{subject to} & 0x_1 - 5x_2 - 2 \leqslant 0\\ & 0x_1 + 1x_2 \leqslant 1\\ & 1x_1 + 0x_2 \leqslant 2\\ & x_1 \leqslant y_1\\ & x_2 \leqslant y_2\\ & x_1, x_2 \geqslant 0\end{aligned}$$

Solving the above problem with by $y_1 = 5.75, y_2 = 0$ Charnes and Cooper method, [21] we obtain the optimal solution $x_1 = 2.03, y_2 = 0$and $L_1 = 1.34$.
Using Step 6, the fuzzy optimal solution is given by $\widetilde{x_1} = (2.03, 5.75, 10.33), \widetilde{x_2} = (0, 0, 0)$.
The fuzzy optimal value of the given FFLFP problem is $\widetilde{L}(\widetilde{X}) = (1.34, 2, 2.31)$.

6 Conclusion

In this paper a new method is proposed to find the fuzzy optimal solution of FFLFP problems with fuzzy variables and fuzzy constraints. The significant of this paper is solving full fuzzy linear fractional programming problem when the variables are symmetric and non symmetric without using ranking method. Thus the method is very useful in the real world problems where the product is uncertain.

References

[1] M. Sakawa, H. Yano, J. Takahashi. Pareto optimality for multi objective linear fractional programming problems with fuzzy parameters.*Inform. Sci.* 63:33-53, 1992.

[2] R.E. Bellman, L.A. Zadeh. Decision making in a fuzzy environment. *Man. Sci.* 17:141-164, 1970.

[3] H. Tanaka, T. Okuda, K. Asai. On fuzzy mathematical programming. *J. Cyb. Syst.* 3:37-46, 1973.

[4] H. J. Zimmerman. Fuzzy programming and linear programming with several objective functions.*Fuzzy Set. Syst.* 1:45-55, 1978 .

[5] M. Inuiguchi, H. Ichihashi, Y. Kume. A Solution Algorithm for Fuzzy Linear Programming with Piecewise Linear Membership Function. *Fuzzy Set. Sys.* 34:15-31, 1990.

[6] S.C. Fang, C.F. Hu, H.F. Wang, S.Y. Wu. Linear Programming with Fuzzy coefficients in Constraints. *Comput. Math. Appli.* 37:63-76, 1999.

[7] J. Buckley, T. Feuring. Evolutionary Algorithm Solution to Fuzzy Problems: Fuzzy Linear Programming. *Fuzzy Set. Syst.* 109:35-53, 2000.

[8] H.R. Maleki M. Tata, M. Mashinchi. Linear Programming with Fuzzy Variables.*Fuzzy Set. Syst.* 109:21-33, 2000.

[9] H.R. Maleki. Ranking Functions and Their Applications to Fuzzy Linear Programming. *Far East J. Math. Sci.* 4:283-301, 2002.

[10] G. Zhang, Y.H. Wu, M. Remias, J. Lu. Formulation of Fuzzy Linear Programming Problems as FourObjective Constrained Optimization Problems. *Appl. Math. Comput.* 139:383-399, 2003.

[11] H.M. Nehi, H.R. Maleki, M. Mashinchi. Solving Fuzzy Number Linear Programming Problem by Lexicographic Ranking Function.*Ital. J. Pur. App. Math.* 15:9-20, 2004.

[12] J. Ramik. Duality in Fuzzy Linear Programming: Some New Concepts and Results.*Fuzzy Optim. Deci. Mak.* 4:25-39, 2005.

[13] K. Ganesan, P. Veeramani. Fuzzy Linear Programs with Trapezoidal Fuzzy Numbers. *Ann. Oper. Res.* 143:305-315, 2006.

[14] S.M. Hashemi , M. Modarres, E. Nasrabadi, M.M. Nasrabadi. Fully Fuzzified Linear Programming, solution and duality.*J. Int. Fuzzy Syst.* 17:253-261, 2006.

[15] M. Jimenez, M. Arenas, A. Bilbao and M. V. Rodrguez, Linear Programming with Fuzzy Parameters: An Interactive Method Resolution. *Euro. J. Oper. Res.* 177:1599-1609, 2007.

[16] M. saraj, N. safaei. fuzzy linear fractional bi-level multi-objective programming problems.*Int. J. Appl. Math. Res.* 1:643-658, 2012.

[17] F.H. Lotfi, T. Allahviranloo, M.A. Jondabeha, L. Alizadeh. Solving a fully fuzzy linear programming using lexicography method and fuzzy approximate solution.*Appl. Math. Modell.* 33:3151-3156, 2009.

[18] A. Kumar, J. Kaur, P. Singh. A new method for solving fully fuzzy linear programming problems.*Appl. Math. Modell.* 35:817-823, 2011.

[19] B. Pop, I.M. Stancu- Minasian. A method of solving fully fuzzified fractional programming problems.*J. Appl. Math. Comput.* 27:227- 242, 2008.

[20] A. Kaufmann, M.M. Gupta.*Introduction to Fuzzy Arithmetic Theory and Applications.* Van Nostrand Reinhold. New York, 1985.

[21] A. Charnes, W.W. Cooper. Programming with linear fractional functions.*Naval. Res. Log. Qua.* 9:181-186, 1962.