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Comparison on the rigid and flexible model of attitude maneuvering of a simple multi-body satellite

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Abstract: *Rigid body assumption of a satellite model has been a common practice in spacecraft attitude manoeuvring. However, with the increasing demand for greater functionality of space activities, requires bigger and wider solar panels to cater the power needs. In this paper, simple rigid and flexible multi-body satellite model is derived using basic Newton's second law and Assumed Mode Method. The response from both model are then simulated using MATLAB software while comparison is done to illustrate the significance of the flexible behaviour that inherited in the satellite system. With the negligence of flexible interference in the rigid model, it is likely to execute an exact attitude motion while the flexible model would yield a harmonic motion that is due to the vibratory motion of the solar panels.*

Keywords: *Satellite, Flexible, Rigid, Model, Comparison*

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1 Introduction

With the increasing demand satellite functionality to meet mission demand, complicated design of satellite has become a common design specification. Bounded on contains such as mass and power, modern satellite often carry lightweight deployable appendages such as solar panels, booms, antennas which are inherently flexible in nature. In addition to that, in the terms of mass constrains, the solar panels have thin thickness which amplifies the flexibility of the panels. These flexibilities

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is capable of causing instability of the satellite which in threatens to destabilize and attitude inaccuracy may occurs [1]. The modelling of the rotating multi-body systems has developed mainly to cater such uncertainties of satellite attitude control and for numerical simulation of attitude behaviour [2]. The coupling effect from the rigid hub and the flexible member present in the multi-body system pose great difficulties in developing the model. Pointed out in publication [3] increasing complexity of the satellite design with greater size, flexibility and demanding precision and autonomy makes the modelling becomes more inadequate due to the presence of complex coupling effect between each structure. Thus, obtaining the basic model of the satellite structure becomes crucial, while slowly incorporate the unmodelled parameters into the basic model in order to obtain a much reliable dynamics.

Modelling methodology of the satellite was published by Tyc George in [3]. The assumed mode method is applied in derivation of the dynamics of flexible satellite. However, thorough understanding of the flexible behaviour, demonstrated by H.H.Yoo and S.H.Shin on their research on Vibration analysis on rotating cantilever beam in year 1997[4]. Articles [5-6] have shown that the flexible elements of spacecraft can have a destabilizing influence. They have established a method of designing a linear, proportional control system employing root-locus plots and eigenvalue analyses. The control loop gains were based on a dynamic model, using hybrid coordinates, of a spacecraft containing long flexible beams. An essentially similar approach was made by DiLorenzo and Santinelli.[7] A linear, proportional, control system was also designed by considering the equations of motion of the spacecraft and those of the flexible elements.

To obtain the flexible model of a single panelled satellite multi-body as shown in Figure 1, Rayleigh-Ritz Assumed Mode Method, a classical method for the calculation of the natural vibration frequency of a structure in the second is applied. The mode shape function of the model is obtained from the Euler-Bernoulli beam theory where the frequency equation of the cantilever has been well established. The Euler-Bernoulli beam is assumed to provide an appropriate solution in three-dimensional motion due to the thin design of the panels and that gyroscopic effect does not significantly affect the panels. As for the mode frequencies, it is obtained by the resonance state of the cantilever [8-9].

Researchers [10-13] has established the dynamics of satellites based on a point mass assumption of the flexible member of the multi-body. In addition to that, their work is limited to the rotation about a single principle axis of the body. In this paper, the mass distribution of the flexible member is based on the actual rectangular shaped panel while the responses based on the modelling on all 3 axis of rotation are also included.

2 Flexible Dynamics

Composed of a multi-body configuration of a rigid body and flexible structured appendage, the satellite executes flexible behaviour contributed by the vibratory motion of the appendage. The fix-free configuration of the joint of the appendage arrangement may be idealized with a cantilever beams, and subjects to rotational and vibrational motion sourced from the rigid frame. The need to include the flexible motion into the dynamics makes the basic Newton Law undesirable. An alternative method named “Rayleigh-Ritz Assumed Mode Method”, is applied instead to obtain the model that describes the behaviour of the satellite. Unlike the rigid model derived in section 2, it initiates with the expression of the total energy of the satellite system which comprised of both kinetics and potential energy. Figure 1 shows the schematic diagram of the multi-body satellite’s rigid hub and the elastic appendage. The three global Cartesian axes of the satellite are the axes that the torque is applied along and is termed Roll (φ), Pitch (θ) and Yaw (ψ) while the local axes of the solar panel are denominated as x, y and z.

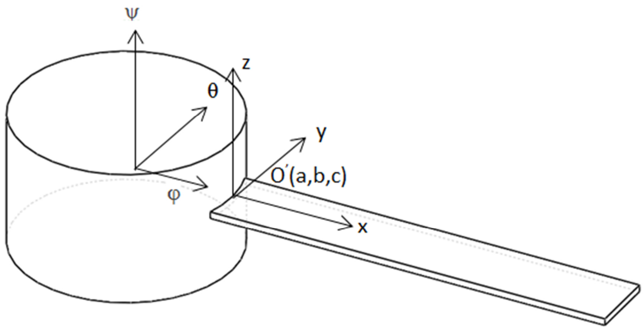

Figure 1: diagram

To obtain the total energy of the system, the Kinetic Energy, T first obtained and is described in Eq. (1); Variable I is the moment of inertia of the rigid hub of the satellite, $\dot{\theta}$ is the angular rate of motion of the satellite body, while, ρ, $\bar{V}_x$, $\bar{V}_y$ and $\bar{V}_z$ is the mass per unit volume of solar panel, and velocity of the appendage in x, y and z axis respectively.

$$T = \begin{bmatrix} T_\varphi \\ T_\theta \\ T_\psi \end{bmatrix} = \frac{1}{2} I\dot{\theta}^2 + \frac{1}{2}\sum_{i=1}^{3} \rho \left[\int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L \bar{V} \cdot \bar{V}\, dx\, dy\, dz \right] \tag{1}$$

The tangential velocity $\bar{V}$ is equivalent to the cross product of the rate of angular rotation to the orthogonal distance to the point of interest. $\bar{V}$ may be obtained in Eq. (2).

$$\bar{\mathrm{V}}_\mathrm{i} = \begin{bmatrix} \bar{\mathrm{V}}_\mathrm{x} \\ \bar{\mathrm{V}}_\mathrm{y} \\ \bar{\mathrm{V}}_\mathrm{z} \end{bmatrix} = \begin{bmatrix} \bar{\dot{\varphi}} \times \mathrm{R_a} \\ \bar{\dot{\theta}} \times \mathrm{R_b} \\ \bar{\dot{\psi}} \times \mathrm{R_c} \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\mathrm{u}} \\ 0 \end{bmatrix} \tag{2}$$

Where $\dot{\mathrm{u}}$ denote is the elastic deformation (the lateral displacement) of the appendage at time t and distance x. When small deflection is considered, notation u may be assumed in Eq. (3).

$$u = z_u k \tag{3}$$

The total potential energy of the system under Euler-Bemouli assumption is:

$$U = \sum_{i=1}^{3} \int_0^L EI\ddot{z}_u{}^2 dx \tag{4}$$

Where, EI is the uniform flexural rigidity of the appendage, and $\ddot{z}_u$ is the second partial derivative of z_u with respect to x.

2.1 Local to Global Axis Conversion

The transformation matrix for translation is to provide the definition of the relation between the local and the coordinate in the linear translational manner. Referring to Figure 1, the local coordinate matrix, **M** is related to the global reference, **G** by the transformation matrix **H** and is defined in Eq. (5).

$$G = M + H = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{5}$$

The orthogonal position vector, **R** to the axis rotation is describe in Eq. (6)

$$R = \begin{bmatrix} \mathrm{R_a} \\ \mathrm{R_b} \\ \mathrm{R_c} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} [G] \tag{6}$$

2.2 Energy Equations

Obtaining the total energy equation with respect to θ torque application involves the kinetic and the potential energy. To obtain the kinetic energy due to the torque, substitute Eq. (2) – (4) into Eq. (1), and leads to Eq (7).

$$T_\theta = \frac{1}{2} I\dot{\theta}^2 + \frac{1}{2}\sum_{i=1}^{3} \rho \begin{bmatrix} \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L \dot{\theta}^2 [(a+x)^2 + (c+z)^2]\, dx\, dy\, dz \\ + \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L \dot{\theta}^2 {z_u}^2\, dx\, dy\, dz \\ + \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L \dot{\theta}\dot{z}(a+x)\, dx\, dy\, dz + \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L {\dot{z}_u}^2\, dx\, dy\, dz \end{bmatrix} \tag{7}$$

2.2 Equation Discretization

Discretization of the equation may be done via introduction of the assumed equation as in Eq.(8). The lateral displacement of any point on the appendage can be described by the product of a spatial function (the mode shape), and a harmonic time function:

$$z_{i,u}(x,t) = \sum_{i=1}^{n} q_i(t) \emptyset_i(x) \tag{8}$$

Variable $\emptyset_i(x)$ is called the mode shape, while $q_i(t)$ describes modal generalized coordinate for the i- th mode, and n denotes the number of modes retained in the approximation. The mode shape function for the appendage is obtained as, see [8].

$$\emptyset_i(x) = C_n \left\{ (\cos\beta_n x - \cosh\beta_n x) + \frac{(\sin\beta_n L - \sinh\beta_n L)}{(\cos\beta_n L + \cosh\beta_n L)} (\sin\beta_n x - \sinh\beta_n x) \right\} \tag{9}$$

The C_n is the arbitrary constant and β_n is determined via the assumed boundary condition. In this case, the boundary condition is a fixed-free configured beam and may be obtained through the following Eq.(10).

$$\cos\beta_n L + \cosh\beta_n L + 1 = 0 \tag{10}$$

C_n is obtained through normalization of the mode shape function whereby it may only ranges between 0 to 1. To achieve such condition, C_n is chosen must gives

$$\int_0^1 \left(\emptyset_i(x)\right)^2 dx = 1 \tag{11}$$

However, the mode shape function is orthogonal in the following condition

$$\int_0^1 \emptyset_i(x)\emptyset_j(x)dx = 0 \qquad when\ i \neq j \tag{12}$$

Incorporate the assumed equation, Eq. (8) into total kinetic energy and potential energy would yield Eq. (13) – (14).

$$T_\theta = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}\sum_{i=1}^{3}\rho\begin{bmatrix} \int_0^z\int_{-\frac{w}{2}}^{\frac{w}{2}}\int_0^L \dot{\theta}^2[(a+x)^2+(c+z)^2]\,dx\,dy\,dz \\ +\int_0^z\int_{-\frac{w}{2}}^{\frac{w}{2}}\int_0^L \dot{\theta}^2(q_i\emptyset_i)^2\,dx\,dy\,dz \\ +\int_0^z\int_{-\frac{w}{2}}^{\frac{w}{2}}\int_0^L \dot{\theta}\dot{q}_i\emptyset_i(a+x)\,dx\,dy\,dz + \int_0^z\int_{-\frac{w}{2}}^{\frac{w}{2}}\int_0^L (\dot{q}_i\emptyset_i)^2\,dx\,dy\,dz \end{bmatrix} \tag{13}$$

$$U_\theta = \sum_{i=1}^{n}\int_0^{L_i} EI\ddot{\emptyset}_i^{\,2} {q_i}^2 dx \tag{14}$$

2.3 Lagrange Equation

From here, the final dynamics may be obtained by using the Lagrange Equation for four generalized coordinates represented by A_j (j=1,2), which are the angular rotation φ and elastic motion q.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{A}_j}\right) - \frac{\partial T}{\partial A_j} + \frac{\partial U}{\partial A_j} = Q \tag{15}$$

Q represents the generalized forces of the system. Substitute Eq. (13) and Eq. (14) into Eq. (15) and the final mathematical modal equation for the satellite for θ axis torque application is obtained.

For $\dot{\varphi}$:

$$\left\{I_\theta + \sum_{i=1}^{n} \rho \left[\begin{array}{l} \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L [(a+x)^2 + (c+z)^2]\, dx\, dy\, dz \\ \quad + \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^{L_i} q^2 \emptyset_{ni} \emptyset_{mi}\, dx\, dy\, dz \end{array}\right]\right\} \ddot{\theta} + \frac{1}{2}\sum_{i=1}^{n} \left\{\left[\rho \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L \emptyset_i (a+x)\, dx\, dy\, dz\right] \ddot{q}\right\} = \tau_\theta \tag{16}$$

For q:

$$\frac{1}{2}\sum_{i=1}^{n} \left\{\left[\rho \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L \emptyset_i (a+x)\, dx\, dy\, dz\right] \ddot{\theta} + \left[\rho \int_0^z \int_{-\frac{w_i}{2}}^{\frac{w}{2}} \int_0^L \emptyset_{ni} \emptyset_{mi}\, dx\, dy\, dz\right] \ddot{q} + \left[2\int_0^L EI \ddot{\emptyset}_{ni} \ddot{\emptyset}_{mi}\, dx - \dot{\theta}^2 \rho \int_0^Z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L \emptyset_{ni} \emptyset_{mi}\, dx\, dy\, dz\right] q\right\} = 0 \tag{17}$$

2.4 φ and ψ Torque Application

The Derivation for both dynamics due to φ and ψ torque application is similar to that of the section above, Hence, the equation of motion for model for θ torque is shown in Eq. (18), while ψ torque application yields Eq. (19).

For $\dot{\varphi}$:

$$\left\{I_\varphi + \rho\left[\int_0^Z \int_{-\frac{W}{2}}^{\frac{W}{2}} \int_0^L [(b+y)^2 + (c+z)^2] dx\, dy\, dz\right]\right\} \ddot{\theta} = \tau_\varphi \tag{18}$$

For $\dot{\psi}$:

$$\left\{I_\psi + \rho\left[\int_0^Z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L [(a+x)^2 + (b+y)^2] dx\, dy\, dz\right]\right\} \ddot{\psi} = \tau_\psi \tag{19}$$

3 Simplification

Further reduce the model is done by assuming that the transformation matrix **H** is defined as in Eq. (20)

$$H = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} \tag{20}$$

Hence, new equation of motion the satellite dynamics is reviewed as in Eq. (21)-(24)

For $\dot{\theta}$:

$$\left\{ I_\theta + \sum_{i=1}^{n} \rho \left[\begin{array}{c} \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L [(a+x)^2 + z^2]\, dx\, dy\, dz \\ + \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^{L_i} q^2 \emptyset_{ni} \emptyset_{mi}\, dx\, dy\, dz \end{array} \right] \right\} \ddot{\theta} + \frac{1}{2} \sum_{i=1}^{n} \left\{ \left[\rho \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L \emptyset_i (a+x)\, dx\, dy\, dz \right] \ddot{q} \right\} = \tau_\theta \tag{21}$$

For q:

$$\frac{1}{2} \sum_{i=1}^{n} \left\{ \left[\rho \int_0^z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L \emptyset_i (a+x)\, dx\, dy\, dz \right] \ddot{\theta} + \left[\rho \int_0^z \int_{-\frac{w_i}{2}}^{\frac{w}{2}} \int_0^L \emptyset_{ni} \emptyset_{mi}\, dx\, dy\, dz \right] \ddot{q} + \left[2 \int_0^L EI \ddot{\emptyset}_{ni} \ddot{\emptyset}_{mi}\, dx - \dot{\theta}^2 \rho \int_0^Z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L \emptyset_{ni} \emptyset_{mi}\, dx\, dy\, dz \right] q \right\} = 0 \tag{22}$$

For $\dot{\varphi}$:

$$\left\{ I_\varphi + \rho \left[\int_0^Z \int_{-\frac{W}{2}}^{\frac{W}{2}} \int_0^L [y^2 + z^2] dx\, dy\, dz \right] \right\} \ddot{\varphi} = \tau_\varphi \tag{23}$$

For $\dot{\psi}$:

$$\left\{ I_\psi + \rho \left[\int_0^Z \int_{-\frac{w}{2}}^{\frac{w}{2}} \int_0^L [(a+x)^2 + y^2] dx\, dy\, dz \right] \right\} \ddot{\psi} = \tau_\psi \tag{24}$$

4 Rigid Model

In the case where the satellite body is further generalized as a lump object, the flexible motion is neglected and is assumed to execute non-deformable behaviour. In this case, the basic Newton's second law of rotational motion is used to obtain the dynamic equation, which is as shown in Eq. (25).

$$\tau = I\ddot{\theta} \tag{25}$$

Parameter $\ddot{\theta}$ is the angular acceleration of the satellite due to the torque application. Since the satellite is assumed rigid, Eq. (25) is applicable to all the 3 axes of rotation of the satellite.

4 Dynamic Simulation

Simulation on both the rigid and flexible dynamic model is done using the SIMULINK tool box of MATLAB. The flexible dynamic behaviour of the satellite is demonstrated via the simulation and compared with the rigid model simulation. The simulation environment assumed that there is no external disturbance existed, and that the attitude manoeuvring is done on 1 axis at a time, while the other 2 axis remain constant. The initial condition for the satellite is assumed to be constant. The input of the torque application is shown in the Figure 2. The specification of the dummy satellite is derived based on RazakSAT satellite as shown in Table 1.

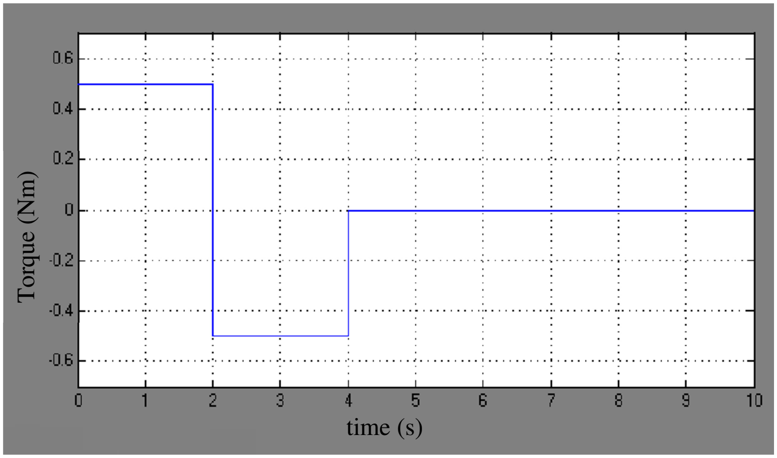

Figure 2: Input Torque

Table 1: Satellite Specification

Item	Value
z	0.02 m
w	0.567 m
a	0.456 m
ρ	291.07 Kg/m^3
E	150 GPa
M	158 kg
L	0.818 m

4 Results

The output of the attitude manoeuvring of the satellite is as shown in Figure 3 to Figure 7 where the response of the satellite on each axis of rotation will be illustrated. Figure 3 illustrates the comparison of the overall attitude manoeuvring of the satellite for rigid and flexible model.

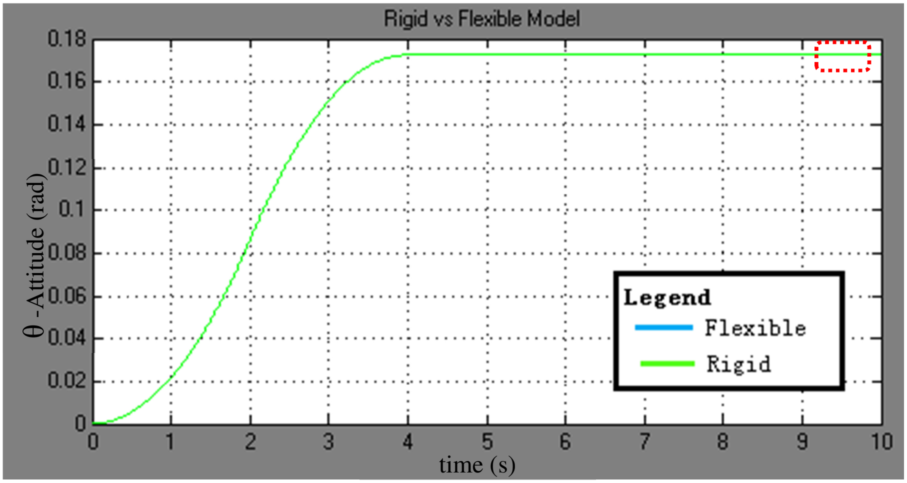

Figure 3: Comparison of attitude manoeuvring of the satellite (θ Axis)

In Figure 3, the red dotted section is enlarged in Figure 4 to show the deviation between the rigid and flexible model.

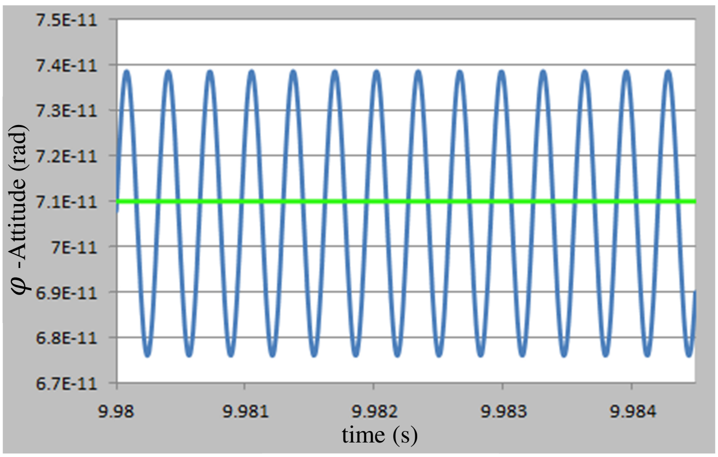

Figure 4: Enlarged Red-dotted section of Figure 3

From Figure 4, it is observable that the attitude of the satellite is undergoing harmonic motion on post-actuation. This however, is not visible on the rigid model due to the negligence in terms of the flexible behaviour inherited by the satellite. The vibratory motion of the solar panel is shown in Figure 8.

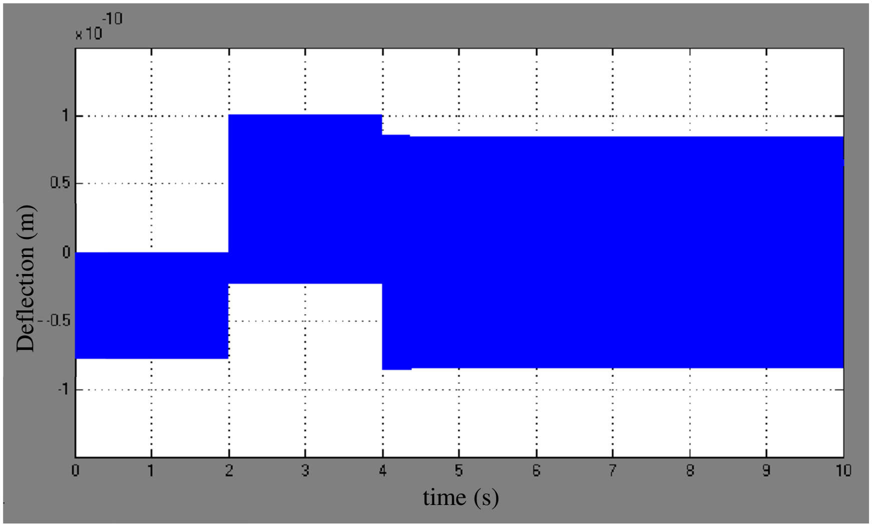

Figure 5: Vibratory motion of Panel

From Figure 5, it shows that there are 3 phase of vibration of the solar panel. At 0s<t<2s, the vibration of the solar panels is biased towards the negative region. However, at 2s<t<4s, the direction of the vibration suddenly switched to the positive region. This is due to the sudden change in the actuation torque applied on the satellite as shown in Figure 2. On post-actuation, the vibration stabilized and oscillates through the neutral condition of the panel. For this flexible model, the panel vibrational amplitude remains the same on post-actuation due to the negligence of damping effect on the vibration. Figure 5 and 6 shown below is the response for the φ and ψ axis of rotation.

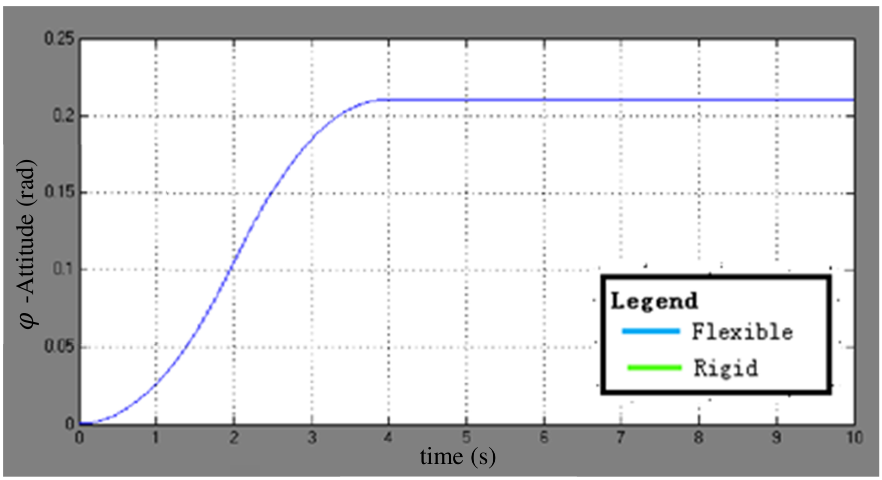

Figure 6: Comparison of attitude manoeuvring of the satellite (φ Axis)

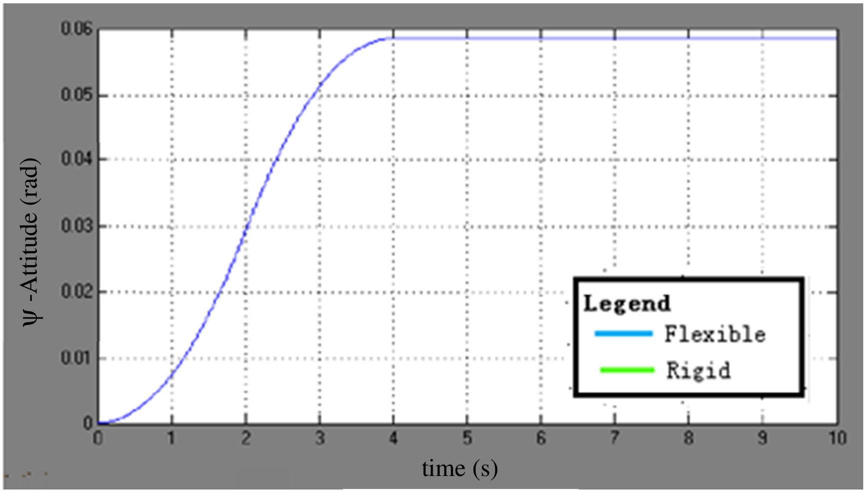

Figure 7: Comparison of attitude manoeuvring of the satellite (ψ Axis)

Based on Figure 6 and 7, there is no observable deviation on the response between the rigid and flexible model. This may be the fact that, on the case on modelling, only the motion on the local z axis is considered.

4 Conclusions

Based on the findings from the comparison on the flexible model and rigid dynamic behaviour, the flexible property executed by the panels via deformation does show a significant effect on the attitude manoeuvring of the satellite. The flexible model provides the alternative yet much more reliable solution to attitude modelling to describe the behaviour of satellite other than the rigid model. The flexible model is capable of describing the vibratory behaviour of the satellite appendages which has the potential to affect the manoeuvring of the satellite and results in undesired outcome. Unlike the rigid dynamics, having the flexible component model is potentially useful in the precision attitude control design while avoiding unwanted reaction. In the meantime, proper action may be taken to reduce the vibration of the appendages and its effect on the overall attitude manoeuvring.

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NOMENCLATURE

φ, θ and ψ	: Axis of rotation of the satellite for X, Y and Z axis
x, y and z	: Axis for the local coordination system for solar panel
a	: Distance of the centres of mass of satellite to the fixed joint of the solar panel
u	: Elastic deformation along y axis
z	: Thickness of the solar panel
w	: Width of the solar panel
x	: The position of a measured mass element along solar panel’s x axis.
$\overline{\omega}$	: Rotation Due to torque application
ρ	: Density of the panel
L	: length of the panel
T	: Kinetic Energy
U	: Potential Energy
$\emptyset_i(t)$	: Mode shape
$q(x)$	: Modal generalized coordinate for the i-th mode
I	: Moment Of Inertia
E	: Young’s Modulus
τ	: Torque
F	: Transformation Matrix of the appendages
$\bar{V}_{xi}$, $\bar{V}_{yi}$ & $\bar{V}_{zi}$	: Velocity of the appendage in x, y and z axis respectively