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Satellite Attitude Estimation in Simulated Non-Gaussian White Noise using Particle Filter and Extended Kalman Filter

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ABSTRACT

Extended Kalman filter (EKF) has been found as most widely used algorithm for state estimation due to its simplicity for implementation and theoretically attractive in the sense that minimizes the variance of the estimation error. Nevertheless it is known that EKF algorithm strictly assumed that the nature of the noise or errors in the system is Gaussian white noise. Yet, in real world this is not always true, which will lead to less accurate estimation. However, there is an estimation approach that does not require the assumption of a specific noise as EKF which is particle filter (PF), which hypothetically can provide more accurate estimation under non-Gaussian noise condition. Hence, this work will study and compare accuracy performance of both estimation algorithms in simulated non-Gaussian white noise for satellite attitude application.

Keywords: Extended Kalman filter; Particle filter; Satellite attitude estimation; Non-Gaussian white noise

1 INTRODUCTION

Satellite attitude determination is one of the important aspects in Attitude Determination and Control System (ADCS) of a satellite. Satellite attitude is important to be determined in a satellite system to be fed back to controller in accomplishing a specific satellite mission such as Earth observation, communication, scientific research and many other missions. However, not all states are directly available may be due to malfunction sensor or as a way to obtain a substantial reduction of sensors which represents a cost and hardware complexity reduction. Hence, state estimation is required to provide the current state of the satellite attitude.

Since decades, a great number of research works have been devoted to the problem of estimating the attitude of a spacecraft based on a sequence of noisy vector observations such as [1][2][3][4]. Different algorithms have been designed and implemented in satellite attitude estimation problem. Early applications relied mostly on the Kalman filter for attitude estimation. Kalman filter was the first applied algorithm for attitude estimation for the Apollo space program in 1960s. Due to limitation of Kalman filter which work optimal for linear system only, several famous new approaches have been implemented to deal with the nonlinearity in satellite attitude system including Extended Kalman Filter (EKF) [4][5][6], Unscented Kalman Filter (UKF) [7][8][9], Particle Filter (PF) [10][11] and predictive filtering [12][13]. EKF is an extended version of Kalman filter for nonlinear system whereby the nonlinear equation is approximated by linearized equation through Taylor series expansion. UKF, an alternative to the EKF uses a deterministic sampling technique known as the unscented transform to pick a minimal set of sample points called sigma points to propagate the non-linear functions. All Kalman based approaches assume the noise in the system is Gaussian white noise process. However, in real world this is not always true, which will lead to less accurate estimation. While, PF is a nonlinear estimation algorithm that approximates the nonlinear function using a set of random samples without restricted to a specific noise distribution as Kalman based approaches, which hypothetically it will provide more accurate estimation in non-Gaussian white noise condition. Therefore, this work will study the performance of EKF and PF algorithms' accuracy using estimation of the satellite attitude under simulated Gaussian white noise and non-Gaussian white noise.

In signal processing system, noise is a general term for unwanted uncertainty signal that disturb and affect quality of the true signal [14]. Noise may arise in signals of interest to various scientific and technical fields. Through literature, the most well-known noise classification in various scientific and technical fields is Gaussian white noise [15]. Gaussian white noise is a noise that has white properties and distributed with Gaussian form. However in the real world, there are many other types of noise. A type of noise can be classified based on its statistical properties and named after the color and its probability distribution function (PDF) [14]. There are three famous types of colour for random noise in a system which are white, pink/flicker and brown/brownian noise [16]. White noise is perhaps the most familiar noise in signal processing [17]. It has been used widely as a noise model in many physical and engineering applications [18]. A type of noise can also be characterized based on its probability distribution. Amongst frequently used noise distribution model in many engineering fields are Gaussian, exponential, lognormal, inverse power-law, and uniform distribution [16]. In satellite attitude field practice, the non-Gaussianity of the noise could be due to geomagnetic field measurement as been reported in TechSAT real data, where double-peaked distribution of the geomagnetic field measurement by three-axis magnetometer data was observed [19]. Errors due to multipath effects [20] and gravitational field fluctuations generated during warm inflation also may lead to the non-Gaussian distributed noise [21].

The organization of this paper proceeds as follows. Section 2 presents the nonlinear mathematical model of the observer. Section 3 describes briefly the nonlinear estimation algorithms used in this work which are Extended Kalman Filter and Particle Filter. Section 4 presents and discusses the results of the estimation performance. Lastly, Section 5 provides the paper's conclusions.

2 NONLINEAR MATHEMATICAL MODEL OF THE OBSERVER

A nonlinear observer is a nonlinear dynamic system that is used to estimate the unknown states from one or more noisy measurements. Mathematically, the nonlinear observer design is described as follows. Given the actual nonlinear system dynamics and measurement described by continuous-time model [22].

$$\dot{x} = f(x) + w \tag{1}$$

$$y = h(x) + v \tag{2}$$

Then, the observer is modelled as

$$\dot{\hat{x}} = f(\hat{x}) \tag{3}$$

$$\hat{y} = h(\hat{x}) \tag{4}$$

In Eqs. (1)-(4), $x \in R^n$ is the state vector and $y \in R^p$ is the output vector, w and v denote the noise or uncertainty vector in the process and measurement respectively. While $\hat{x}$ and $\hat{y}$ denote the corresponding estimates.

In this work, the system is designed to estimate the satellite's angular velocity $(\omega_x, \omega_y, \omega_z)$ by using Euler angles attitude $(\emptyset, \theta, \varphi)$ measurement only. Hence the state vector is $x = [\omega_x, \omega_y, \omega_z, \emptyset, \theta, \varphi]^T$, while the state equation is

$$\dot{x} = \left[\dot{\omega}_x, \dot{\omega}_y\ , \dot{\omega}_z, \dot{\emptyset}, \dot{\theta}, \dot{\varphi}\right]^T \tag{5}$$

with

$$\dot{\omega}_x = -\left(\frac{I_z - I_y}{I_x}\right)\omega_y\omega_z + 3{\omega_0}^2\frac{(I_z - I_y)}{I_x}s\emptyset c\emptyset c^2\theta \tag{6}$$

$$\dot{\omega}_y = -\left(\frac{I_x - I_z}{I_y}\right)\omega_x\omega_z + 3{\omega_0}^2\frac{(I_z - I_x)}{I_y}s\theta c\theta c\emptyset \tag{7}$$

$$\dot{\omega}_z = -\left(\frac{I_y - I_x}{I_z}\right)\omega_x\omega_y + 3{\omega_0}^2\frac{(I_x - I_y)}{I_z}s\emptyset c\theta s\theta \tag{8}$$

$$\dot{\emptyset} = [\omega_x + \omega_0 c\theta s\varphi\ ] + s\emptyset t\theta\left[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)\right] + c\emptyset t\theta[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \tag{9}$$

$$\dot{\theta} = c\emptyset\left[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)\right] - s\emptyset[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \tag{10}$$

$$\dot{\varphi} = \frac{s\emptyset}{c\theta}\left[\omega_y + \omega_0(c\emptyset c\varphi + s\emptyset s\theta s\varphi)\right] + \frac{c\emptyset}{c\theta}[\omega_z + \omega_0(-s\emptyset c\varphi + c\emptyset s\theta s\varphi)] \tag{11}$$

where $I = diag\left[I_x, I_y, I_z\right]$, $\dot{\omega} = \left[\dot{\omega}_x,\ \dot{\omega}_y,\ \dot{\omega}_z\right]$, $\omega = \left[\omega_x,\ \omega_y,\ \omega_z\right]$ represent satellite's moment of inertia, angular acceleration and angular velocity vectors respectively. While $\emptyset$ is rotational angle about X-axis (roll); θ is rotational angle about Y-axis (pitch); and φ is rotational angle about Z-axis (yaw). In the above equation *c, s* and *t* denote cosine, sine, and tangent functions, respectively. While, ω_0 is the orbital rate of the spacecraft.

Meanwhile, the measurement equation of the observer is

$$y = h(x) = \begin{bmatrix} \emptyset \\ \theta \\ \varphi \end{bmatrix} \tag{12}$$

3 NONLINEAR ESTIMATION ALGORITHMS

3.1 Extended Kalman Filter

In this work, Extended Kalman Filter (EKF) is used as one of the methods since it is widely used estimation algorithm in real practice of spacecraft community and theoretically attractive in the sense that it minimizes the variance of the estimation error. EKF algorithm is described as below. [23]

Let the continuous model in Eqs. (1) and (2) are transformed into the discrete-time model such that

$$x_k = f(x_{k-1}) + w_{k-1} \tag{13}$$
$$y_k = h(x_k) + v_k \tag{14}$$

Here the subscript of the variables denotes the time step, while w_{k-1} and v_k are restricted assumed as Gaussian distributed noises with mean zero and covariance R_w and R_v respectively such that $w_{k-1} \sim N(0, R_w)$ and $v_k \sim N(0, R_v)$. Then, the estimated state is obtained through the following step:

Step 1: Set the initial state estimate $\hat{x}_0 = \hat{x}_{0|0}$ and variance $P_0 = P_{0|0}$. (15)

Step 2: Repeat

(i) Prediction step (priori estimate)

- Jacobian of $f(x_{k-1})$: $F_{k-1} = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1|k-1}}$ (16)
- Predicted state estimate: $\hat{x}_{k|k-1} = f\left(\hat{x}_{k-1|k-1}\right)$ (17)
- Predicted covariance estimate: $P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^T + R_w$ (18)

(ii) Update step (posteriori estimate)

- Jacobian of $h(x_k)$: $H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k|k-1}}$ (19)
- Kalman gain: $K_k = P_{k|k-1} H_k^T \left[H_k P_{k|k-1} H_k^T + R_v\right]^{-1}$ (20)
- Updated state estimate: $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left[y_k - h\left(\hat{x}_{k|k-1}\right)\right]$ (21)
- Updated covariance estimate: $P_{k|k} = [I - K_k H_k] P_{k|k-1}$ (22)

3.2 Particle Filter

In this work, Particle Filter (PF) is used as one of the methods since it does not require any assumption about the state-space or the noise of the system to be Gaussian as restricted in conventional method EKF. The method approximates the posterior density using a set of particles. PF algorithm is described as below. [24]

Let the continuous-time model in Eqs. (1) and (2) are transformed into the discrete-time model as described in Eqs. (13) and (14) written again for convenience

$$x_k = f(x_{k-1}) + w_{k-1} \tag{23}$$
$$y_k = h(x_k) + v_k \tag{24}$$

Here the subscript of the variables denotes the time step, while w_{k-1} and v_k are process and measurement noises respectively with variance R_w and R_v. Then, the estimated state is obtained through the following step:

Step 1: Set the number of particles N_s and set the initialization

- Initial state estimate: $\hat{x}_0 = \hat{x}_{0|0}$ (25)
- Initial particles: $\chi_0^i = \hat{x}_{0|0}$ for $i = 1,2,\cdots,N_s$ (26)
- Initial weight: $w_0^i = \frac{1}{N_s}$ for $i = 1,2,\cdots,N_s$ (27)

Step 2: Repeat

(i) Sequential importance sampling

- Draw particles: $\chi_k^i \sim N\big(x_k; f_{k-1}(\chi_{k-1}^i), R_w\big)$ for $i = 1,2,\cdots,N_s$ (28)
- Compute the weight for each particle:

$$w_k^i = w_{k-1}^i N\big(z_k; h_k(\chi_k^i), R_v\big) \quad \text{for } i = 1,2,\cdots,N_s \tag{29}$$

- Calculate the total weight: $T = \sum_{i=1}^{N_s} w_k^i$ (30)
- Normalize the weight: $w_k^i = \frac{w_k^i}{T}$ for $i = 1,2,\cdots,N_s$ (31)

(ii) Resampling (To eliminate samples with low importance weights)

- Initialize cumulative sum of weight (CSW) : $c_1 = 0$ (32)
- Construct CSW: $c_i = c_{i-1} + w_k^i$ for $i = 2,3,\cdots,N_s$ (33)
- Start at the bottom of the CSW: $i = 1$ (34)
- Draw a starting point: $u_1 \sim U\left[0, \frac{1}{N_s}\right]$ (35)
- For $j = 1,2,\cdots,N_s$
  - Move along the CSW: $u_j = u_1 + \frac{1}{N_s}(j-1)$ (36)
  - Set if $u_j > c_i$, then update $i = i + 1$ (37)
  - Assign particles: $\chi_k^j = \chi_k^i$ (38)
  - Assign weight: $w_k^j = \frac{1}{N_s}$ (39)

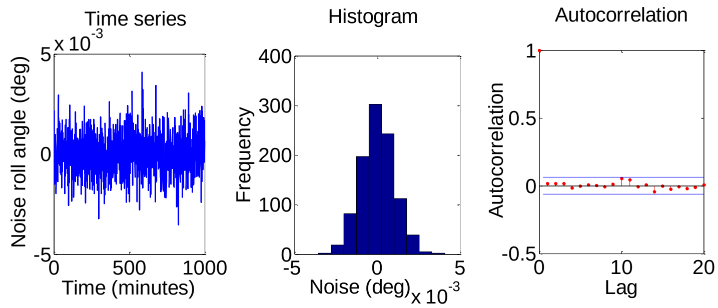
(iii) State estimation

- Compute estimated state: $\hat{x}_{k|k} = \sum_{i=1}^{N_s} \chi_k^i w_k^i$ (40)

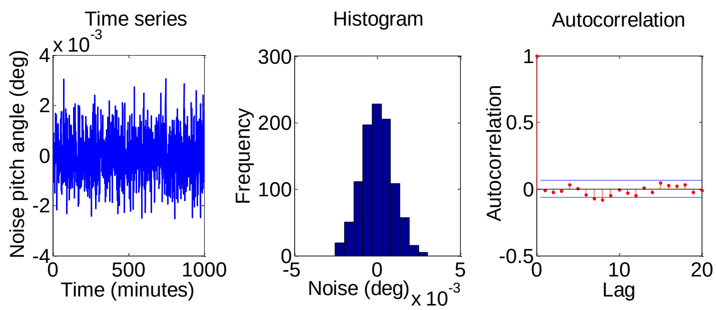
4 RESULTS AND DISCUSSION

In this section, a simulation to study and compare the accuracy performance of EKF and PF under two various type of noise is carried out using MATLAB software.

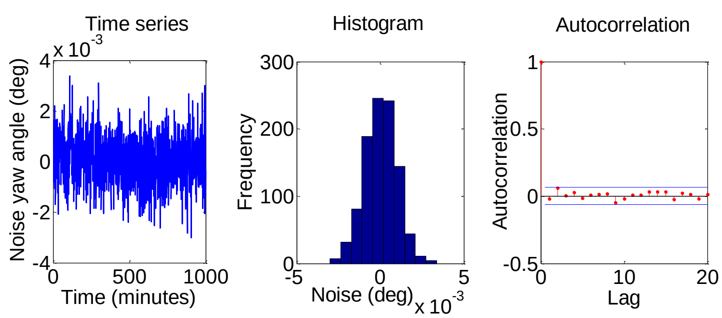
The first study is to compare the performance of EKF and PF under Gaussian white noise circumstances. Figure 1 shows the time series, histogram and autocorrelation plots of measurement corrupted by simulated Gaussian white noise for roll, pitch, and yaw angles. Histogram is plotted in this work to verify whether the time series is generated by Gaussian or non-Gaussian noise. Meanwhile autocorrelation is plotted to verify whether the color of the generated simulated noise is white or non-white colored.



(a)



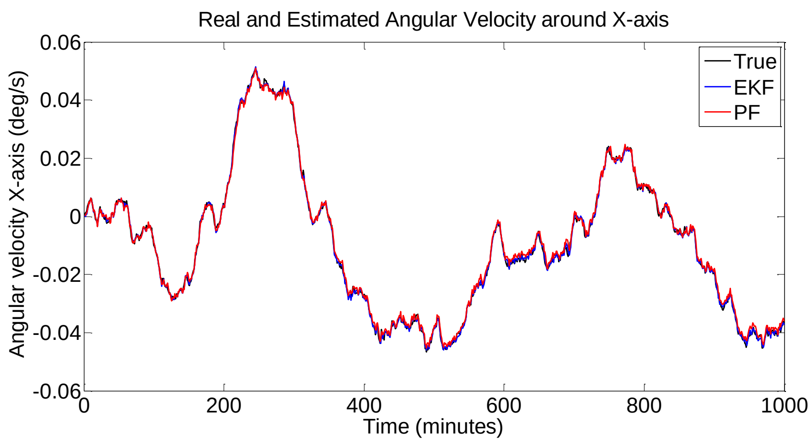
(b)



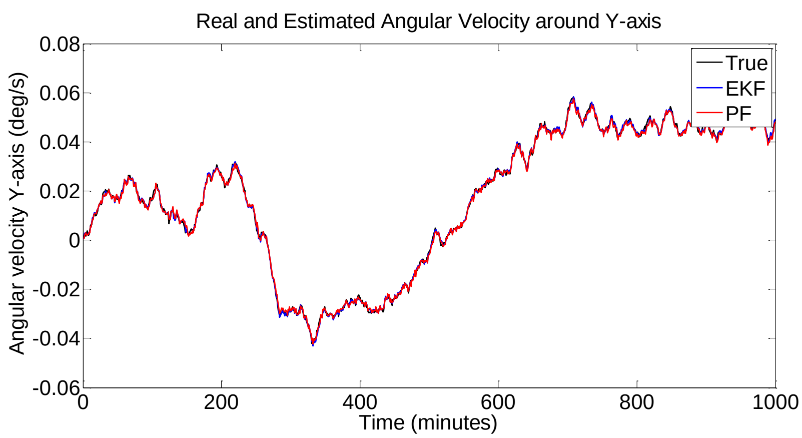
(c)

Figure 1: Time series, histogram and autocorrelation plots of measurement corrupted by Gaussian white noise for (a) roll, (b) pitch, and (c) yaw angles.

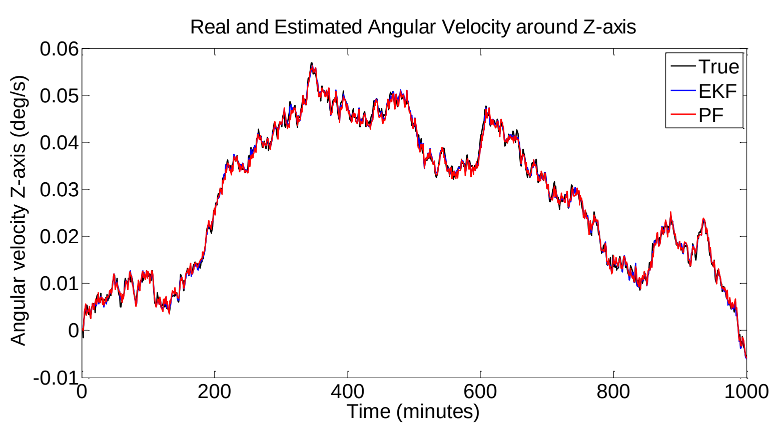
Figure 2 shows the result of estimation by using the measurement of roll, pitch and yaw corrupted by Gaussian white noise. The result shows that both EKF and PF exhibit almost similar performance with insignificant difference and able to track all the three true angular velocities very well. However, by observing the Root Mean Squared Error (RMSE) values of estimated angular velocity as tabulated in Table 1, it is observed that the error norm of EKF is smaller than PF with 0.0014 deg/s and 0.0018 deg/s respectively. Hence, it is concluded that under Gaussian white noise circumstances, EKF provide slightly more accurate estimation than PF. This could be due to capability characteristic of EKF to cater with Gaussian white noise.



(a)



(b)



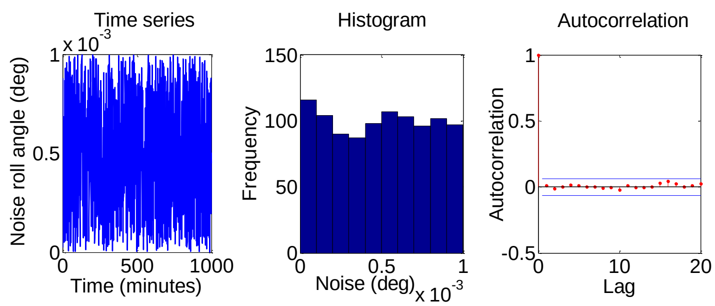
(c)

Figure 2: Comparison between estimated states via EKF and PF versus the true states under measurement corrupted by Gaussian white noise for (a) angular velocity X-axis, (b) angular velocity Y-axis, and (c) angular velocity Z-axis.

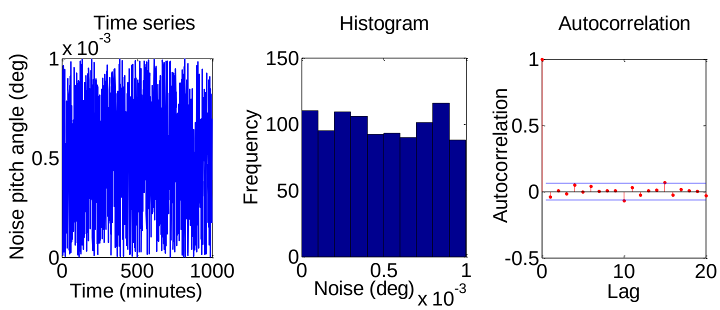
Table 1: RMSE of estimated angular velocity under measurement corrupted by Gaussian white noise.

Algorithm	**RMSE (deg/s)**			
	Angular velocity X-axis, ω_x	**Angular velocity Y-axis, ω_y**	**Angular velocity Z-axis, ω_z**	**Norm**
EKF	8.1167×10^{-4}	8.1875×10^{-4}	7.9481×10^{-4}	**0.0014**
PF	0.0012	9.5102×10^{-4}	9.1561×10^{-4}	**0.0018**

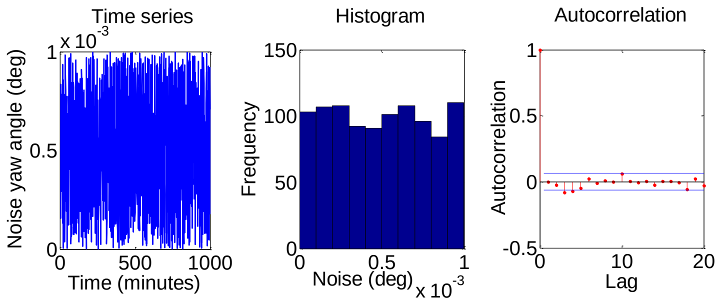
The second study is to analyze the performance of EKF and PF under non-Gaussian white noise circumstances. Figure 3 shows the time series, histogram and autocorrelation plots of measurement corrupted by simulated non-Gaussian white noise for roll, pitch, and yaw angles.



(a)



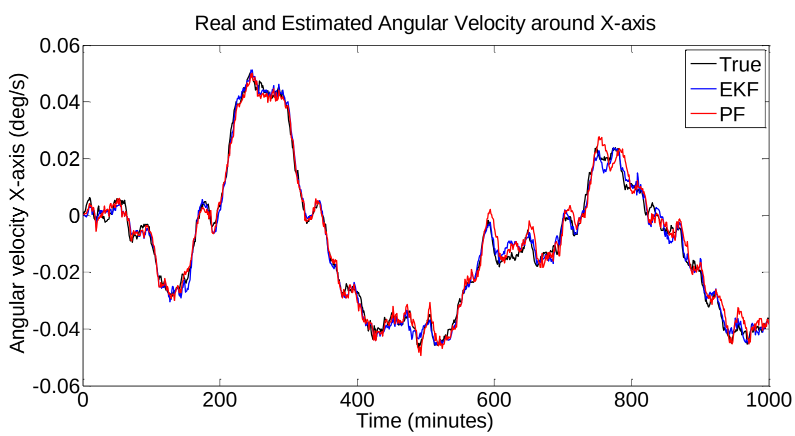
(b)



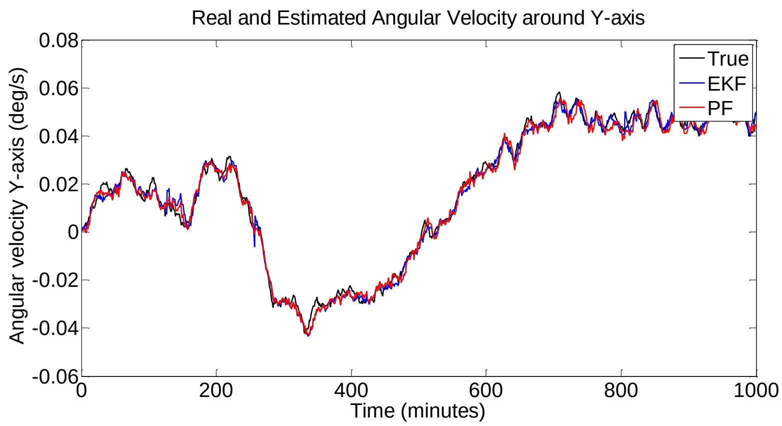
(c)

Figure 3: Time series, histogram and autocorrelation plots of measurement corrupted by non-Gaussian white noise for (a) roll, (b) pitch, and (c) yaw angles.

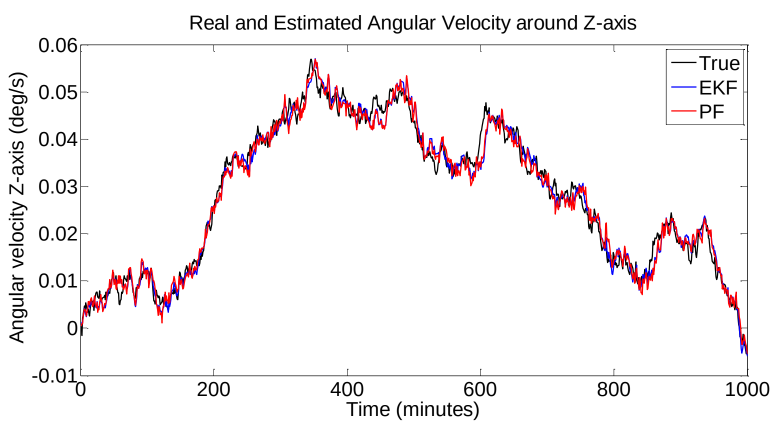
Figure 4 shows the result of estimation by using the measurement of roll, pitch and yaw corrupted by non-Gaussian white noise. The result shows that both EKF and PF also able to track all the three true angular velocities well. However, by observing the RMSE values of estimated angular velocity as tabulated in Table 2, it is observed that the error norm of PF is smaller than EKF with 0.0038 deg/s and 0.0046 deg/s respectively. Hence, it is concluded that under non-Gaussian white noise circumstances, PF provides slightly more accurate than EKF. This could be due to capability characteristic of PF to cater with non-Gaussian densities.



(a)



(b)



(c)

Figure 4**:** Comparison between estimated states by using EKF and PF versus the true states under measurement corrupted by non-Gaussian white noise for (a) angular velocity X-axis, (b) angular velocity Y-axis, and (c) angular velocity Z-axis.

Table 2: RMSE of estimated angular velocity under measurement corrupted by non-Gaussian white noise.

Algorithm	**RMSE (deg/s)**			
	Angular velocity X-axis, $\boldsymbol{\omega_x}$	**Angular velocity Y-axis, $\boldsymbol{\omega_y}$**	**Angular velocity Z-axis, $\boldsymbol{\omega_z}$**	**Norm**
EKF	0.0031	0.0026	0.0022	**0.0046**
PF	0.0024	0.0022	0.0020	**0.0038**

From the studies, it can be seen that EKF provides more accurate estimation compared to PF under Gaussian white noise circumstances. This is due to its main characteristic of EKF which is designed to cater with Gaussian white noise. However, under non-Gaussian white noise circumstances, PF shows its capability to provide better accuracy estimation compared to EKF. Hence it verifies that PF will provide more accurate estimation than EKF under non-Gaussian noise type.

5 CONCLUSION

In this paper, accuracy performance to estimate the satellite attitude under simulated non-Gaussian white noise condition has been studied and compared between EKF and PF algorithms. PF shows its ability to provide more accurate estimation in non-Gaussian white noise circumstances due to its nature that does not restrict the noise to be Gaussian as restricted by EKF. This verifies the assumption that the PF can provide more accurate estimation than EKF under non-Gaussian noise type. Therefore, PF could be a realistic option in practice when one considering statistical performances in terms of accuracy and it is strongly suggested during contingency condition of extremely inaccurate or large uncertainty measurements such as due to unexpected failure of the existing sensor which represent the non-Gaussian noise situation. However, the conventional method EKF is still preferred to be used for less computation on-board implementation for the real-time application. Hence, it is suggested to ADCS designers to design a hybrid tracker that can switch between the EKF and PF depending on the tracking conditions and modes for more efficient on-board implementation.

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