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Magnetohydrodynamic Stagnation-point Flow towards a Permeable Stretching/Shrinking Sheet with Slip and Heat Generation/Absorption Effects

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ABSTRACT

This study of the magnetohydrodynamic (MHD) stagnation-point flow towards a permeable stretching/shrinking sheet in the presence of slip and heat generation/absorption effects is considered. The governing equations in the form of partial differential equations are transformed into a system of ordinary differential equations by using similarity transformation, and then solved numerically using bvp4c function in Matlab software. The variations of the numerical solutions for the skin friction coefficient and the local Nusselt number as well as velocity and temperature profiles are obtained for several values of the governing parameters. It is found that the solution is unique for the stretching case whereas dual (first and second) solutions exist for the shrinking case in certain range of parameters.

Keywords: Heat Generation/Absorption, Magnetohydrodynamic, Slip effect, Stretching/Shrinking Sheet.

1 INTRODUCTION

For the past few years, the study of suction/injection effect has attracted the attention of many researchers under various conditions. Hamid and Arifin [1] have studied the effect of suction/injection on MHD Marangoni convection boundary layer flow in nanofluid. They found that the suction/injection has the significant effect on velocity and temperature profiles. Samyuktha and Ravindran [2] analyzed the effects of suction/injection and thermal radiation on mixed convection flow over a vertical stretching sheet embedded in a porous medium. RamReddy and Pradeepa [3] have considered the influence of suction/injection on free convection flow over a vertical plate in porous medium saturated with a micropolar fluid. Other than that, Ganapathirao and Ravindran [4] have investigated the effect of non-uniform slot suction/injection into mixed convection MHD flow over a vertical wedge with chemical reaction. The results obtained indicate the local skin friction coefficient, the local Nusselt and Sherwood numbers increase with suction while decrease by an increasing of injection. Since then, many researchers have been working on the suction or injection effect with various physical conditions such as Ahmad [5], Ganapathirao and Ravindran [6], Lin et al. [7], Mohamed et al. [8], Rizwan-ul-Haq [9], Rosali et al. [10], Zaidi and Mohyud-Din [11], and Zeeshan and Majeed [12].

The study of heat generation/absorption also has attracted many researchers. Freidoonimehr et al. [13] have investigated the MHD stagnation-point flow past a stretching/shrinking sheet in the presence of heat generation/absorption and chemical reaction effects. The effect of the nanoparticle on magnetohydrodynamic boundary layer flow over a stretching sheet in the presence of heat generation/absorption with heat and mass fluxes is studied by Venkataramanaiah et al [14]. Khan et al. [15] explored the effects of melting and heat generation/absorption on unsteady Falkner-Skan flow of Carreau nanofluid over a wedge. Recently, Soomro et al. [16] studied the heat generation/absorption effects in the presence of nonlinear thermal radiation along a moving slip surface. Plenty of investigators offered study on heat generation/absorption effect by considering different physical effects (see Eid and Mahny [17], Ganga et al. [18], Hayat et al. [19][20][21][22], Hussain et al. [23], Mohamed [24], Qayyum et al. [25]).

Motivated by the aforementioned works, we analyze, in this paper, the behavior of the magnetohydrodynamic stagnation-point flow towards a permeable stretching/shrinking sheet with slip and heat generation/absorption effects. In the present study, we extend the work of Aman et al. [26]. This study is different from that investigated by Aman et al. [26], where we consider the heat generation/absorption and suction/injection effects. The governing partial differential equations are converted into ordinary differential equations by similarity transformations, before being solved numerically using the bvp4c function in Matlab software. The results on suction/injection and heat generation/absorption effects are explored and discussed in detail. The dual solutions are expected to exist for the shrinking case.

2 MATHEMATICAL FORMULATION

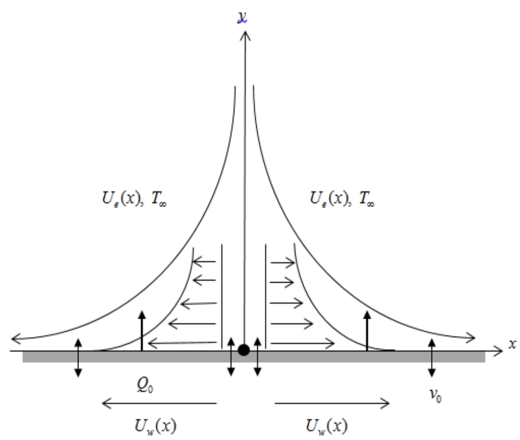
Let us consider a steady stagnation-point flow over a stretching/shrinking sheet in a viscous fluid with suction/injection, slip and heat generation/absorption effects as shown in Figure 1, where x and y are the Cartesian coordinates measured along the surface and normal to it, respectively. It is assumed that the velocity of stretching/shrinking sheet is $u_w(x) = bx$, where $b > 0$ is the stretching sheet and $b < 0$ is the shrinking sheet, while the free stream velocity is $u_e(x) = ax$ where a is a positive constant. B_o is a uniform magnetic field of strength which is assumed to be applied in the positive y-direction normal to the plate. The induced magnetic field is assumed to be small compared to the applied magnetic field and it is neglected. Under these assumptions, the boundary layer equations are (see Ishak et al. [27], Bhattacharyya et al. [28] and Aman et al. [26])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

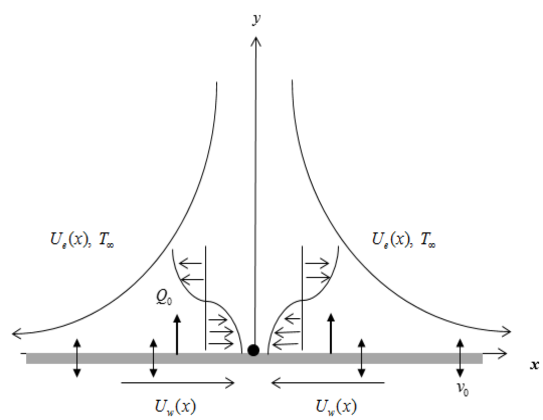
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}\left(u_e-u\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}\left(T-T_\infty\right) \qquad (3)$$

where u and v are velocity components corresponding to the along x and y axes, respectively, T denotes the fluid temperature, ν denotes the kinematic viscosity, σ denotes the electrical conductivity, α denotes the thermal diffusivity of the fluid, C_p denotes the specific heat capacity at constant pressure, Q_0 denotes the temperature dependent heat generation/absorption coefficient and ρ denotes the density of a fluid.



(a) Stretching sheet



(b) Shrinking sheet

Figure 1 : Physical model and coordinate system for the proposed problem.

We assume that the equations (1)-(3) are subjected to the boundary conditions

$$u\left(x,0\right)=u_w+\mathrm{L}\left(\frac{\partial u}{\partial y}\right),\quad v\left(x,0\right)=v_0,\quad \mathrm{T}\left(x,0\right)=T_w+S\left(\frac{\partial T}{\partial y}\right)$$

$$u\left(x,y\right)\to u_e\left(x\right),\quad T\left(x,y\right)\to T_\infty,\quad \text{as } y\to\infty. \qquad (4)$$

Here, v_0 is the mass velocity with $v_0>0$ for suction and $v_0<0$ for injection, L is the slip length, S is a proportionality constant and T_∞ is the ambient temperature. Further the similarity variables are introduced as follows:

$$\psi = \left(\nu x u_e\right) f\left(\eta\right),\ \theta\left(\eta\right) = \frac{T - T_\infty}{T_w - T_\infty},\ \eta = \left(\frac{u_e}{\nu x}\right) y, \tag{5}$$

where $\psi\left(x, y\right)$ is the stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. $f\left(\eta\right)$ is the dimensionless stream function and $\theta\left(\eta\right)$ is the dimensionless temperature. Equation (1) is identically satisfied. By substituting (5) into Equations (2) and (3), the following ordinary differential equations will be obtained

$$f''' + ff'' + 1 - f'^2 + M\left(1 - f'\right) = 0 \tag{6}$$

$$\frac{1}{\Pr}\theta'' + f\theta' + Q\theta = 0 \tag{7}$$

and the boundary conditions becomes

$$f\left(0\right) = \gamma,\ f'\left(0\right) = \varepsilon + \delta f''\left(0\right),\ \theta\left(0\right) = 1 + \beta\theta'\left(0\right),$$
$$f'\left(\infty\right) \to 1,\ \theta\left(\infty\right) \to 0 \tag{8}$$

where $M = \frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter, $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Q = \frac{Q_0}{\rho C_p a}$ is the heat source parameter, $\gamma = -\frac{v_0}{\sqrt{a\nu}}$, $\gamma > 0$ indicates suction and $\gamma < 0$ corresponds to injection case, ε represents the stretching/shrinking parameter with $\varepsilon > 0$ for a stretching sheet and $\varepsilon < 0$ for a stretching sheet, respectively, $\delta = L\left(\frac{a}{\nu}\right)^{\frac{1}{2}}$ is the velocity slip parameter, $\beta = S\left(\frac{a}{\nu}\right)^{\frac{1}{2}}$ is the thermal slip parameter and prime denotes differentiation with respect to η.

The physical quantities of interest are the skin friction coefficient, C_f and the local Nusselt number, Nu_x which can be defined as Aman et al. [26]:

$$C_f = \frac{\tau_w}{\rho u_e^2 / 2},\ Nu_x = \frac{x q_w}{k\left(T_w - T_\infty\right)}, \tag{9}$$

with μ and k is the dynamic viscosity and the thermal conductivity, respectively. Substituting (5) into (10) and using (9), the following expression can be obtained:

$$\frac{1}{2}\mathrm{Re}_x^{1/2} C_f = f''(0), \quad \mathrm{Re}_x^{-1/2} Nu_x = -\theta'(0), \tag{10}$$

where $\mathrm{Re}_x = \frac{u_e x}{\nu}$ is the local Reynolds number.

3 RESULTS AND DISCUSSION

The ordinary differential equations (6) and (7) with the boundary conditions (8) were solved numerically using the bvp4c function in Matlab software. The numerical results obtained in terms of the skin friction coefficient, $\mathrm{Re}_x^{1/2} C_f$, the local Nusselt number, $\mathrm{Re}_x^{-1/2} Nu_x$, velocity profile, $f'(\eta)$ and temperature profile, $\theta(\eta)$ for different values of suction/injection parameter, γ and heat generation/absorption parameter, Q, while the Prandtl number Pr is fixed at $\mathrm{Pr} = 1$ for the sake of brevity.

To verify the accuracy of the present results, comparison has been made with the previous results of Aman et al. [26]. Table 1 clearly shows the values of skin friction coefficient $f''(0)$ for the case of stretching/shrinking sheet without magnetohydrodynamic effect $M = 0$ (in equation (6)), heat generation/absorption effects $Q = 0$ (in equation (7)), suction/injection effects, $\gamma = 0$ velocity slip effect $\delta = 0$ and thermal slip effect $\beta = 0$ in the boundary conditions (8). The comparisons shows that the present results match well with the previous results obtained by Aman et al. [26], and hence proves that the bvp4c programme is a precise approach in solving the system numerically.

Table 1 : Comparison of the values $f''(0)$ with those of Aman et al. [26] neglecting suction/injection effect $(\gamma = 0)$ in the boundary conditions (9) for both stretching/shrinking sheet.

	Aman et al. [26]		**Present results**	
ε	**First solution**	**Second solution**	**First solution**	**Second solution**
0			1.232588	
-0.1			1.308602	
-0.2			1.373886	
-0.25	1.4022		1.402241	
-0.3	1.4276		1.427576	
-0.35			1.449751	
-0.4	1.4686		1.468613	
-0.45			1.483986	
-0.5	1.4957		1.495670	
-0.6			1.507025	
-0.615	1.5072		1.507241	
-0.7			1.500361	
-0.75	1.4893		1.489298	
-0.8			1.472388	
-0.9			1.418077	
-1	1.3288	0	1.328817	0
-1.05			1.266228	0.012178
-1.1			1.186680	0.049229
-1.13			1.127813	0.085349
-1.15	1.0822	0.1167	1.082200	0.116700
-1.18	1.0004	0.1784	1.000449	0.178361
-1.2			0.932473	0.233650

Figures 2 and 4 show the variations of skin friction coefficient and the local Nusselt number, respectively, for different values of suction/injection parameter γ. These Figures 2 and 4 indicate that there are dual solutions for $\varepsilon_c < \varepsilon < \varepsilon_l$, unique solutions for $\varepsilon > \varepsilon_l$ and no solutions obtained for $\varepsilon < \varepsilon_c$, where ε_c and ε_l are the upper and lower critical values of ε respectively, for which Equations (6) and (7) have no solutions and the full Navier–Stokes and energy equations should be solved. In Figures 2 and 4, the solid lines denote the first solution, while the dash lines denote the second solution. Based on our computation, the critical values ε_c obtained for $\gamma = -0.5, 0$ and 0.1 are $\varepsilon_{c1} = -1.87694$, $\varepsilon_{c2} = -2.48739$ and $\varepsilon_{c3} = -2.63967$ as shown in Figures 2 and 4. The transition from positive (stable) to negative (unstable) values of γ occurs at the turning points, ε_c of the parametric solution curves $(\gamma = -0.5, 0, 0.1)$ which is shown in Figures 2 and 4.

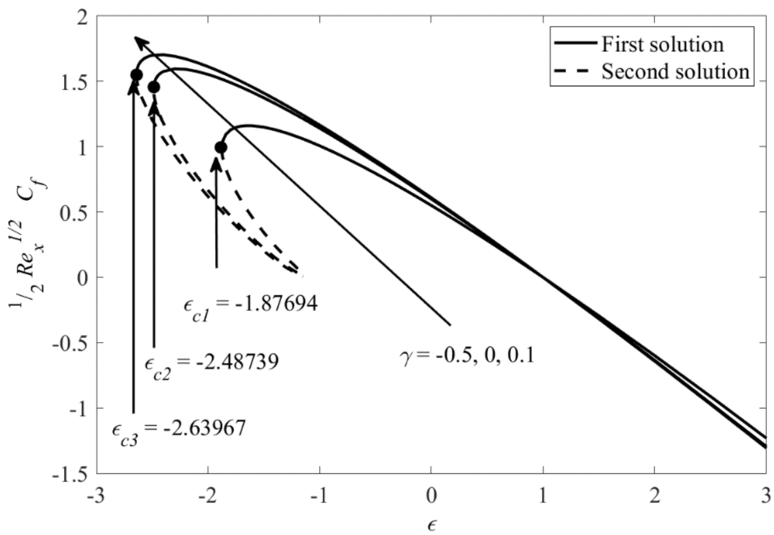


Figure 2 : Variation of the skin friction coefficient $\mathrm{Re}_x^{1/2}\,\mathrm{C}_f$ with ε for different values of γ when $\Pr = 1$, $M = 0.1$, $\delta = 1$, $S = 1$ and $Q = 1$.

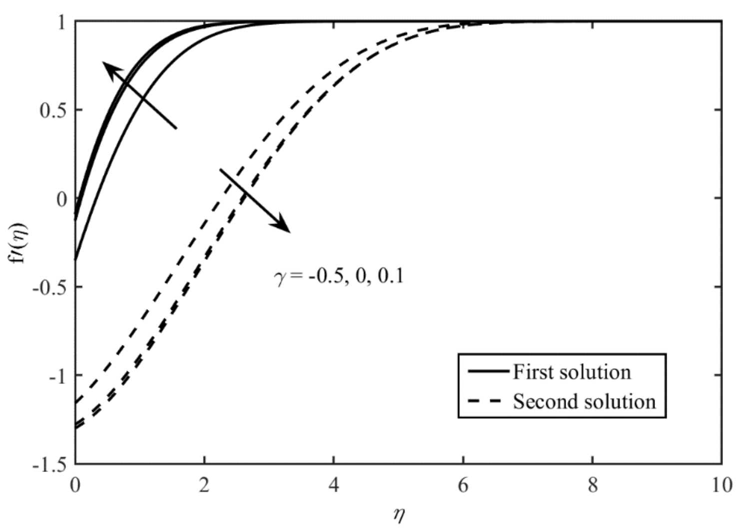


Figure 3 : The velocity profiles $f'(\eta)$ for different values of γ when $\Pr = 1$, $M = 0.1$, $\delta = 1$, $S = 1$, $Q = 1$ and $\varepsilon = -1.5$ (shrinking case).

Figure 2 shows the skin friction coefficient increases as γ increases. Physically, this is due to the suction effect increasing the surface shear stress, delay the fluid flow and thus, increase the velocity gradient at the surface which is consistence with the graph in Figure 3. From Figure 2 also, it can be observed that the critical values stretching/shrinking parameter ε_c for which the solution exist increase as γ increases, suggests that suction widens the range of the dual solutions of the similarity Equations (6)–(8). Figure 3 depicts the velocity profile $f'(\eta)$ for different values of suction/injection parameter. For the stable solution, it is clearly indicates that the velocity is increased with an increase in the values of γ. An increase in suction/injection parameter reflects to reduction in momentum boundary layer thickness and thus increases the flow near the surface.

Figure 4 shows the influence of suction parameter on the local Nusselt number which represents the heat transfer rate. The local Nusselt number tends to decreases as γ increases. The effect of suction will lower the thermal boundary layer thickness and it is clearly shown in Figure 5 which represents the temperature profile $\theta(\eta)$.

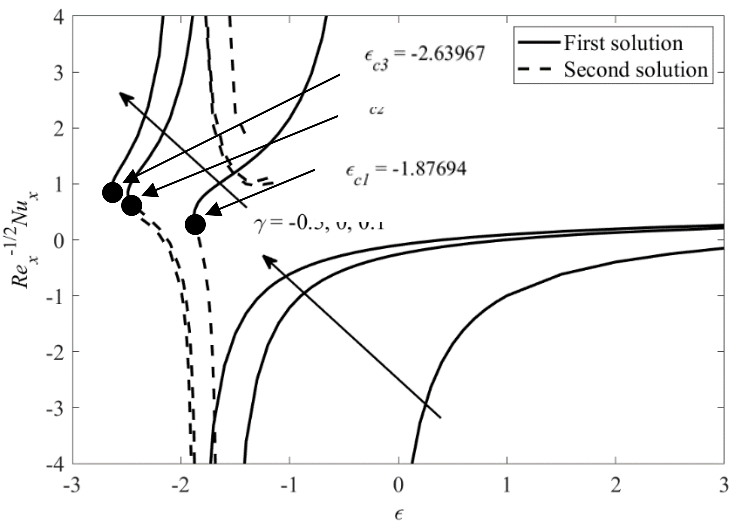


Figure 4 : Variation of the local Nusselt number $\mathrm{Re}_x^{-1/2}\, Nu_x$ with ε for different values of γ when $\Pr = 1$, $M = 0.1$, $\delta = 1$, $S = 1$ and $Q = 1$.

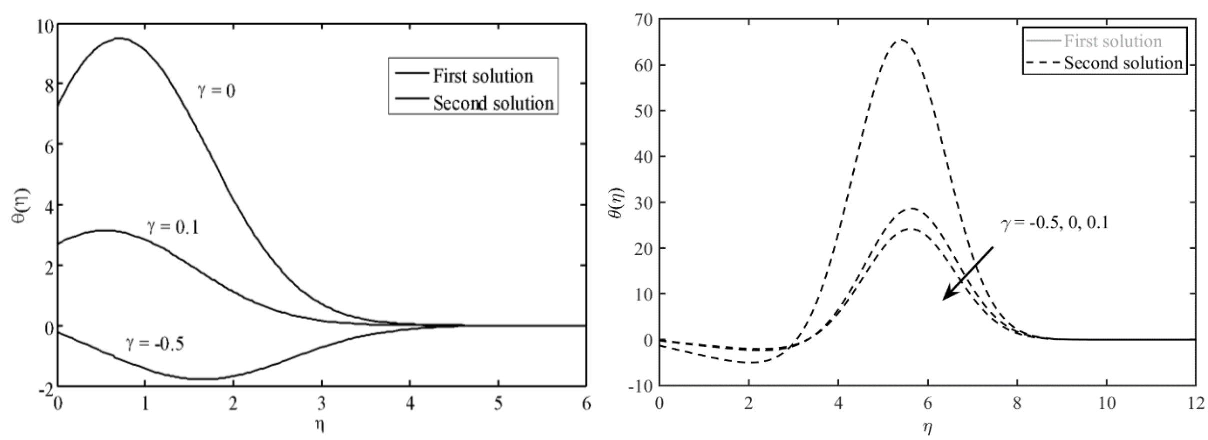


Figure 5 : The temperature profiles $\theta(\eta)$ ((a) first solution and (b) second solution) for different values of γ when $\Pr = 1$, $M = 0.1$, $\delta = 1$, $S = 1$, $Q = 1$ and $\varepsilon = -1.5$ (shrinking case).

It is observed that the increase in heat generation/absorption parameter Q, decrease the local Nusselt number, as shown in Figure 6. This is because heat generation tends to increase the temperature and controlling the heat transfer. The impact of heat generation/absorption on the temperature profile $\theta(\eta)$ is depicted in Figure 7. From Figure 7, the increment in heat generation/absorption increases the temperature at the surface. It is also noticed that the thermal boundary layer thickness increase, so that increase the temperature gradient and in consequences the heat transfer rate at the surface is enhanced, which is consistent with the graph in Figure 6.

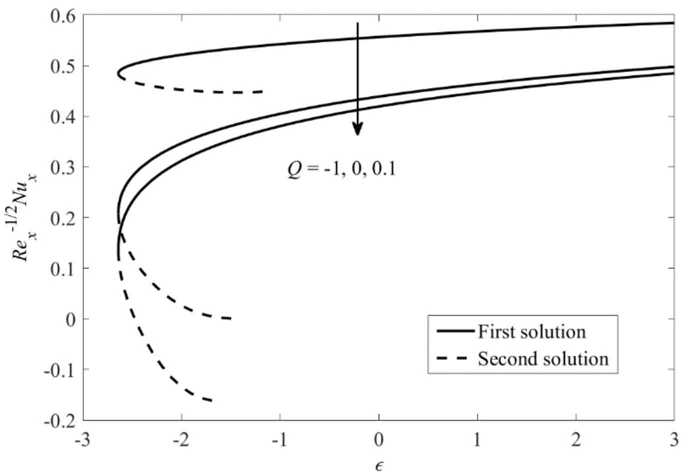


Figure 6 : Variation of the local Nusselt number $\mathrm{Re}_x^{-1/2} Nu_x$ with ε for different values of Q when $\Pr = 1$, $M = 0.1$, $\delta = 1$, $S = 1$ and $\gamma = 0.1$ (suction).

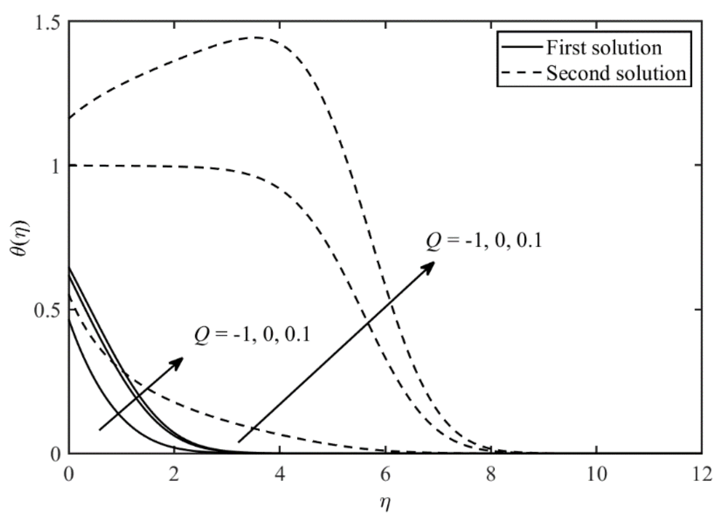


Figure 7 : The temperature profiles $\theta(\eta)$ for different values of Q when $\Pr = 1$, $M = 0.1$, $\delta = 1$, $S = 1$, $\gamma = 0.1$ and $\varepsilon = -1.5$ (shrinking case).

The velocity and temperature profiles which have been shown in Figures 3, 5 and 7 satisfy the far field boundary conditions (8) asymptotically, which support the validity of the numerical results obtained and the existence of the dual solutions.

4 CONCLUSION

This paper considered numerical solutions of the magnetohydrodynamic stagnation-point flow towards a permeable stretching/shrinking sheet with slip and generation/absorption effects and solved numerically using bvp4c function built in Matlab software. The analysis shows that the skin friction coefficient and the local Nusselt number as well as the velocity and temperature were influenced by suction/injection parameter. As the suction suction/injection increases, the skin friction coefficient and the local Nusselt number also increase. Applying the heat generation/absorption increased the local Nusselt number. The present study also has potential to produce dual solutions for the certain range of shrinking case. Summarize your paper and stress the most important points of it.

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