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Multivariate Evolutionary and Tweedie GLM Methods for Estimating Motor Vehicle Insurance Claims Reserves

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ABSTRACT

An insurance policy is a contract agreement between the policyholder and the insurance company. For the contract agreement to run, policyholders need to pay premiums to insurance companies. On the other hand, the insurance company must underwrite the risk if the policyholder does submission of claims. It is necessary to estimate the reserves of claims for the company insurance accurately to prepare several funds for settlement of claim. Generalized Linear Model (GLM) can be used to estimate the claim values in a univariate form which only consists of 1 LoB (Line of Business). In practice, almost every insurance company has various types of LoB which depends on one another. Therefore, the GLM can be expanded to a multivariate GLM which can be used to estimate the claim data with more than one LoB. The researcher also wants to compare between an estimated reserve calculations of Swiss Re Group's claims using the Multivariate Evolutionary GLM Adaptive Simple Method and GLM with the Tweedie Family Distribution Approach to find a more accurate method of finding claim reserves for each line of Swiss Re Group's business data.

Keywords: multivariate evolutionary distribution GLM with the adaptive model, simple GLM with the tweedie family distribution approach

1 INTRODUCTION

Insurance policy is a contract agreement between the policyholder (the insured) and the insurance company (the insurer). Policyholders need to pay premiums to the insurance company. In return, the insurance company must bear the risk if the policyholder makes a claim, and the insurance company needs to set aside money from premium payments to be used as claim reserves. Claim reserves, as a liability, certainly affects the level of the company's solvency. Therefore, it is necessary to estimate the exact claim reserves so that the insurance company can prepare a certain amount of funds for settlement of claims.

Estimation of claims reserves can be done using GLM (Generalized Linear Model). GLM can be used to estimate claim with univariate data which only consists of 1 LoB (Line of Business). In practice, almost every insurance company has various types of LoB which are dependent on one another [1]. Therefore, GLM was extended to multivariate GLM which can be used to estimate claim data with more than one LoB. The random factors on multivariate GLM will change recursively and the dependence between these factors will be calculated using the common shock approach [1].

Furthermore, the Tweedie family distribution method will be used to estimate the claim reserves with univariate data. Meanwhile, multivariate evolutionary GLM method will be used to estimate multivariate data. Then which method can produce the highest level of accuracy? This is the topic that we will discuss in this research.

2 MATERIAL AND METHODS

In calculating estimated claim reserves, it is necessary to pay attention to the form of claim data used. There are 2 forms of claim data model, namely univariate and multivariate. Many journals have discussed claim reserving for univariate data, including the Simple GLM method with the Tweedie family distribution approach [2], the Bornhuetter-Ferguson method [3], Benktander [4] and the Chain Ladder [5].

The Chain Ladder and Bornhuetter-Ferguson methods are traditional methods for predicting outstanding claims for a long tail business [6]. Chain ladder is the method most often used to predict the size of future claims using internal company information in the past with a run-off triangle and an estimator of age-to-age factor same as the Bornhuetter-Ferguson method [5]. However, this Chain Ladder method has several weaknesses, namely the model uses past data so that it is no longer in accordance with the latest data, which results in a fundamental discontinuity of the estimation sequence [7]. This method also cannot differentiate between the claim reserves for IBNR and RBNS [8]. In addition, the Benktander method only produces accurate estimates for stable claims data only, this can be seen from the MSE value [4].

Meanwhile, for multivariate claim data, the reserve can be calculated using the Tweedie family distribution and the multivariate evolutionary GLM using the Adaptive Estimation approach [9]. Evolutionary models are grouped into 2 types based on the distribution assumptions used, namely the Gaussian model and the non-Gaussian model. The Gaussian model relies on the assumptions of Gaussian observations and model factors. The recursive estimator for the model factor used in the Gaussian model is the dual Kalman filter [10], which is the best linear estimator based on the mean square error. Then, for the non-Gaussian model, the dual Kalman filter is no longer the best linear estimator for the factors, because the distribution assumption used in this model has deviated from the Gaussian assumption. Then instead [11] created a particle filter [12] that is used as a simulation-based solution [13, 14] for the univariate evolutionary GLM framework using the second-order Bayesian revision [15] estimation procedure. So, in the multivariate evolutionary GLM there are two Adaptive estimation approaches that will be used, namely a particle filter with learning parameters for the general framework and a dual Kalman filter for the special cases of the Gaussian model. The two filters mentioned above place a greater emphasis on recent data thereby increasing the possibility of producing a more accurate projection of future claims.

2.1 Observation Component

Observations and explanatory factors are linked by the structure of the mean. In this framework, the modified Hoerl curve (discrete version of the Gamma curve) allows the use of the calendar year effect. This curve is determined using the functions j and $log(j)$ to estimate the development pattern of the claims. The advantages of the Hoerl curve are an efficient model, resistant to fluctuations in observations, and extrapolation outside the range of the observed development year. In the context of evolutionary reserves, this curve permits systematic changes in claims activity over time. The structure of the mean can be written using the log-link function on the Hoerl curve as follows:

$$\log\left(\mu_{i,j}^{(n)}\right) = a_i^{(n)} + r_i^{(n)}.\log(j) + s_i^{(n)}.j + h_t^{(n)} \tag{1}$$

where $a_i^{(n)}$ is accident year factor, $r_i^{(n)}$ and $s_i^{(n)}$ are factors of Hoerl curve that determine the pattern of development of the ith accident year, and $h_t^{(n)}$ is a factor of the calendar year. Note that $a_i^{(n)}$, $r_i^{(n)}$ and $s_i^{(n)}$ are accident-year-specific, while $h_t^{(n)}$ is calendar-year-specific. The structure of the mean can be modified according to the case. One special case of the multivariate evolutionary GLM framework is the multivariate Gaussian model assuming Gaussian observations (where log transformations can be applied if the distribution of observations is log normal). The Multivariate Gaussian model allows dependency between calendar years between LoBs. The observation components for this particular case are:

$$Y_{i,j}^{(n)} = a_i^{(n)} + r_i^{(n)}.\log(j) + s_i^{(n)}.j + h_t^{(n)} + \varsigma_{i,j}^{(n)} \qquad \varsigma_{i,j}^{(n)} \sim Normal(0, \sigma_{\varsigma^{(n)}}^2) \tag{2}$$

2.2 State Component

The recursive evolution of random factors is determined through state components. The observation component is the structure for the standard GLM in reserve. In the standard structure, the same fixed parameter values are used to capture the development year effect (e.g. $r_i^{(n)} = r^{(n)}, s_i^{(n)} = s^{(n)}$in terms Hoerl curve) for all accident years. This means that one average development pattern is assumed for all accident years. The difference in computation of reserves using a multivariate evolutionary GLM framework with simple GLM lies in the provisions of the factors $a_i^{(n)}, r_i^{(n)}, s_i^{(n)}$ and $h_t^{(n)}$ which exists in the mean structure. These factors are random and keep growing over time. So that every accident year has its own development pattern. The evolution of each accident year can be determined using a time series process such as the ARMA process. In simple terms, the evolution of a state uses a random process. The evolution of $a_i^{(n)}$, $r_i^{(n)}$ and $s_i^{(n)}$ is as follows:

$$a_i^{(n)} = a_{i-1}^{(n)} + {}_a\epsilon_i^{(n)} \qquad {}_a\epsilon_i^{(n)} \sim Normal\left(0, \sigma_{{}_a\epsilon^{(n)}}^2\right) \tag{3}$$

$$r_i^{(n)} = r_{i-1}^{(n)} + {}_r\epsilon_i^{(n)} \qquad {}_r\epsilon_i^{(n)} \sim Normal\left(0, \sigma_{{}_r\epsilon^{(n)}}^2\right) \tag{4}$$

$$s_i^{(n)} = s_{i-1}^{(n)} + {}_s\epsilon_i^{(n)} \qquad {}_s\epsilon_i^{(n)} \sim Normal\left(0, \sigma_{{}_s\epsilon^{(n)}}^2\right) \tag{5}$$

where $\sigma^2_{{}_a\epsilon_i^{(n)}}, \sigma^2_{{}_r\epsilon_i^{(n)}}, \sigma^2_{{}_s\epsilon_i^{(n)}}$ is the variance of the respective errors ${}_a\epsilon_i^{(n)}$, ${}_r\epsilon_i^{(n)}$, ${}_s\epsilon_i^{(n)}$ in evolution. These variances are model parameters that must be estimated.

The evolution of the calendar factor $h_t^{(n)}$ is determined by a random process that has been modified to involve interdependencies between calendar years via a common shock approach.

$$h_t^{(n)} = h_{t-1}^{(n)} + {}_h\epsilon_t^{(n)} + \lambda^{(n)}.\ {}_h\tilde{\epsilon}_t \qquad {}_h\epsilon_t^{(n)} \sim Normal\left(0, \sigma^2_{{}_h\epsilon^{(n)}}\right) \tag{6}$$

There are two sources of disturbance in this evolution, namely line specific disturbance ${}_h\epsilon_t^{(n)}$ and common shock disturbance ${}_h\tilde{\epsilon}_t$. And the variance is $\sigma^2_{{}_h\epsilon^{(n)}}$, $\sigma^2_{{}_h\tilde{\epsilon}}$ and $\lambda^{(n)}$ are model parameters. The dependence between calendar years is affected by ${}_h\tilde{\epsilon}_t$. This common shock disturbance can represent changes in calendar year t that affect all lines simultaneously. The effect of the common shock on each row is often different due to the difference in the effect received on several rows, therefore it is better to use the scale factor $\lambda^{(n)}$ to adjust for the effect of the common shock on each row.

2.3 State Space Matrix Representation

This section describes a matrix representation of the framework for the multivariate evolutionary GLM. This matrix representation is referred to as a state space representation. The purpose of the matrix representation for the evolutionary model is to represent the relationship between observations and random factors and the development of random factors over time. So that it can describe the model estimation well.

2.3.1 Observation Component

In this framework, claim reserving data can be thought of as a multivariate time series process. Every time there are new observations, each accident year will be the basis for calculating this process. The observation vector in the accident year *i* or Y_i is the vector of all $N.(I - i + 1)$ data claims in the same accident year in the run-off triangle.

2.3.2 State Component

The random evolution of γ_i can be represented in the form of a matrix as;

$$\gamma_i = \gamma_{i-1} + {}_\gamma\epsilon_i, \qquad {}_\gamma\epsilon_i \sim Normal\ (0, Q_{\gamma^\epsilon}) \tag{7}$$

2.4 Adaptive Estimation

Representation of space matrix circumstances from this framework used as developtment from approximation of estimation. that random factor including year non-calender factor $\boldsymbol{y}_i$ (i = 1, ..., I) and year calender factor ψ_I. Parameter that still unknown determined as;

$$\Theta = \left\{\sigma^2_{a\in^{(n)}}, \sigma^2_{r\in^{(n)}}, \sigma^2_{s\in^{(n)}}, \sigma^2_{n^{(n)}}, \sigma^2_{n^{\epsilon}}, \lambda^{(n)}, \phi^{(n)}; n = 1, \ldots, N\right\} \quad (8)$$

where $\phi^{(1)}, \ldots, \phi^{(N)}$ replaced by $\sigma^2_{\zeta^{(1)}}, \ldots, \sigma^2_{\zeta^{(N)}}$ if Gaussian model is used.

Recursive Bayesian Structure will give more weight for new data that used where it can give more response to prediction model where this is more actual to reality and change could be seen gradually from time to time. Recursive Estimation and Framework Structure where random factor determined as recursive using old factor and parameter can increasing random factor calibration when the data is few. Because of that, calibration that used *accident year dimension* as main time dimension for make use of data that availability is more in early of *accident year* [8].

3 RESULTS AND DISCUSSION

3.1 Data Overview

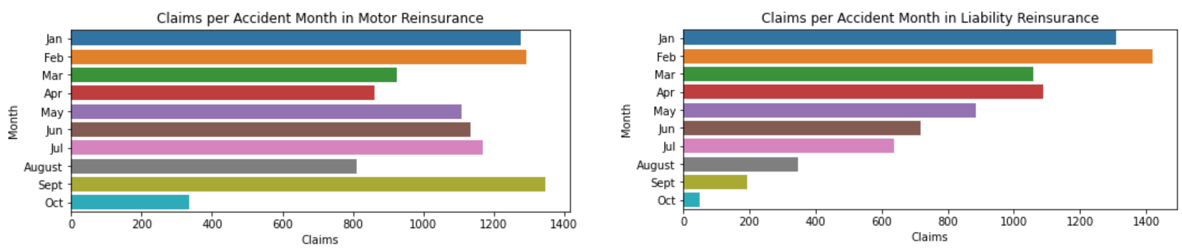
The dataset used in this research are claim values and premium values for line of business liability reinsurance and motor reinsurance services from Swiss Re Group, which is a Comprise P&C Reinsurance and Corporate solution. The used variables are claim values and premium values. There is no missing value in dataset and the range time that is used is January – October 2018. From the claim values that are available in dataset, a runoff triangle is made for each line of business. Tables shown below are premium values and runoff triangles for reported claims per month (cumulative) for each line of business.

Table 1: Reported Claims per Month (cumulative) for Liability Reinsurance

	Premium	**Jan-18**	**Feb-18**	**Mar-18**	**Apr-18**	**May-18**	**Jun-18**	**Jul-18**	**Aug-18**	**Sep-18**	**Oct-18**
01/18	2668.12	199.753	432.583	619.326	839.025	1083.06	1157.37	1191.24	1215.8	1274.03	1309.06
02/18	2579.27	215.194	572.446	825.973	1077.15	1210.44	1301.42	1353.45	1405.11	1421.28	
03/18	2412.58	49.5659	371.774	560.001	711.634	901.323	985.286	1040.27	1059.39		
04/18	2003.16	80.7781	411.143	539.873	783.886	942.896	1089.33	1089.04			
05/18	1727.32	68.09	328.383	547.178	681.812	780.503	883.676				
06/18	1329.15	70.596	335.527	520.456	650.11	716.347					
07/18	1212.89	68.514	256.022	441.441	637.458						
08/18	1112.92	66.9074	215.111	345.75							
09/18	1156.29	47.532	192.312								
10/18	1500.18	48.5309									

Table 2: Reported Claims per Month (cumulative) for Motor Reinsurance

	Premium	Jan-18	Feb-18	Mar-18	Apr-18	May-18	Jun-18	Jul-18	Aug-18	Sep-18	Oct-18
01/18	1555.11	680.333	1061.07	1180.27	1221.81	1247.98	1259.09	1264.65	1257.15	1259.06	1268.84
02/18	1573.92	615.401	1061.63	1177.17	1275.66	1280.48	1288.96	1289.54	1291.86	1293.93	
03/18	1299.77	222.421	787.973	871.215	888.448	919.959	922.329	923.492	923.552		
04/18	1153.15	38.8192	656.959	795.225	838.976	851.555	860.039	855.358			
05/18	1343.87	168.378	890.638	1057.06	1080.98	1094.22	1108.35				
06/18	1355.12	299.019	942.295	1070.57	1105.54	1133.51					
07/18	1395.34	280.799	937.08	1117.4	1170.66						
08/18	1115.95	147.219	670.267	810.73							
09/18	1902.29	334.382	1348.15								
10/18	2446.88	335.299									

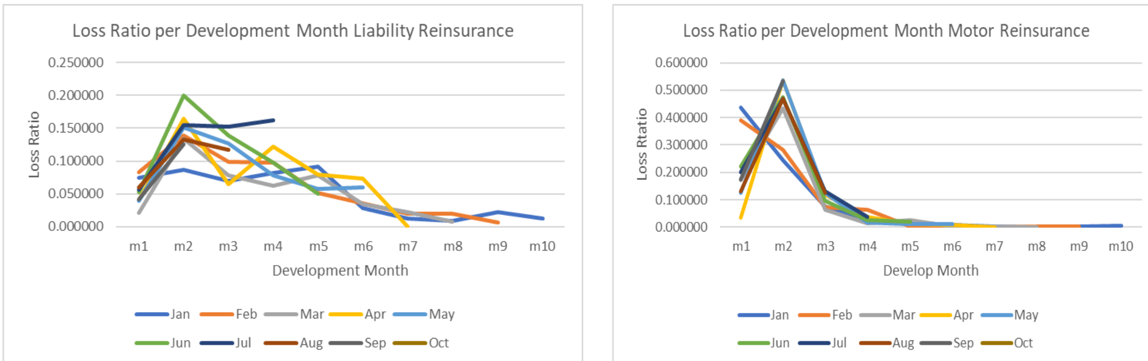


(a) (b)

Figure 1: Claim per Accident Month in: (a) Motor Reinsurance; and (b) Liability Reinsurance

As shown from the plots, there are several differences between claim per accident month in liability reinsurance and motor reinsurance. In motor reinsurance, the highest claim is in September and the lowest is in October. Meanwhile, in liability reinsurance, the highest claimed value is in February and the lowest in October. Lastly, in the case liability reinsurance, we can conclude that there is a significant decrease of the claimed amount at the end of the period.

Figure 2 represents loss ratio per development month for liability reinsurance and motor reinsurance. From both plots, we can infer that there is a significant increase of loss ratio in the second month, which then became the maximum point in each accident months, in exception for accident month January and February in motor reinsurance. Aside from that, shown from the liability reinsurance graph, there are no consistent pattern seen for each accident month, thus leading us to question whether we are able to model both different line of businesses in one go using a multivariate evolutionary generalized linear model framework with adaptive estimation. Furthermore, when the plots shown varying patterns for each accident years, could a multivariate generalized linear model framework with adaptive estimation be the best approach to model this dataset.



(a) (b)

Figure 2: Loss Ratio per Development Month in: (a) Liability Reinsurance; and (b) Motor Reinsurance

3.2 A Multivariable Evolutionary Generalized Linear Model Framework with Adaptive Estimation

A Multivariate Evolutionary Generalized Linear Model is applied to datasets on hand. This research revolves more around the application of GLM with a different Tweedie distribution approach with a different power parameter p so as the model output might give a more flexible dispersion model. In addition, 50.000 samples are used for every time step, filter initiation is also used with static GLM estimation with a mean structure such as:

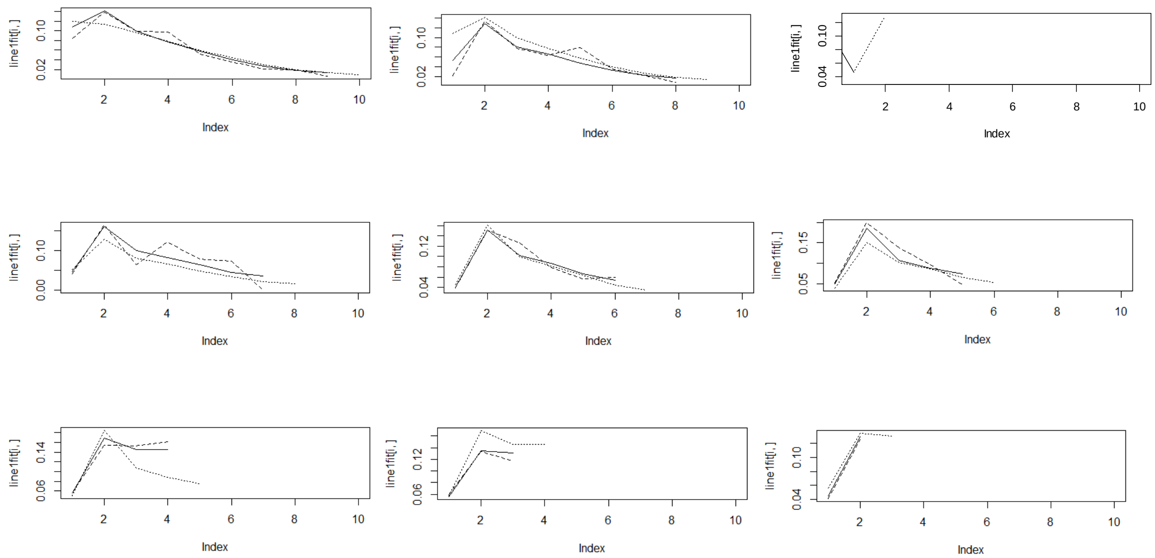
$$a_i^{(n)} + r_i^{(n)} \log j + s_i^{(n)} j + b_{i,1}^{(n)} \mathbb{I}_{\{j=1\}} + b_{i,2}^{(n)} \mathbb{I}_{\{j=2\}} + h_t^{(n)} \tag{9}$$

On the other hand, changes in claim pattern per month are also monitored. To acquire the estimation graph which are to be used in making pattern plots of claim values to development month for every accident year, filtered values from random factors are required, which are in the table below.

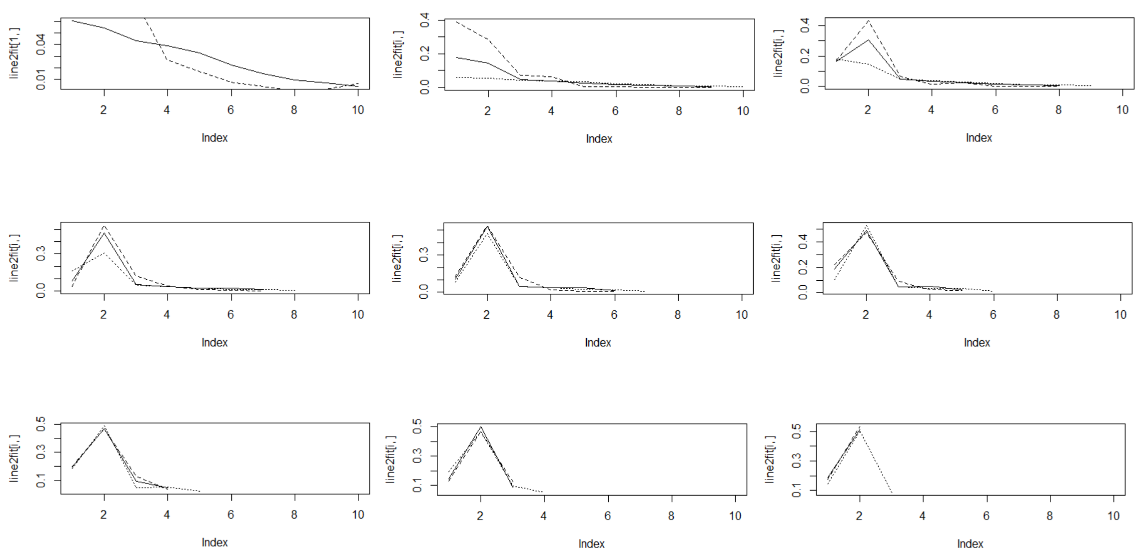
Table 3: Filter Values from Random Factors

n	i	$a_i^{(n)}$	$r_i^{(n)}$	$s_i^{(n)}$	$b_{i,1}^{(n)}$	$b_{i,2}^{(n)}$
1	1	-2.087543252	1.309957	-0.55933	0.524395	-3.54663
	2	-2.013931471	1.326508	-0.57422	0.370883	-3.38264
	3	-2.124276212	1.386341	-0.60775	-0.23523	-3.18238
	4	-2.059553345	1.428034	-0.57666	-0.39653	-3.25512
	5	-2.015587325	1.401532	-0.56355	-0.53013	-3.09946
	6	-1.958315133	1.426051	-0.57668	-0.36193	-2.98265
	7	-1.831956881	1.465512	-0.52227	-0.39831	-2.85423
	8	-1.858938768	1.471362	-0.58565	-0.32022	-2.86975
	9	-1.848340336	1.464519	-0.61427	-0.48828	-2.8444
	10	-1.82580777	1.430496	-0.62445	-0.76258	-2.87436
2	1	2.160242943	-0.71604	1.463018	0.455415	0
	2	2.047210591	-0.77495	2.157653	1.189497	0.276643
	3	2.049707678	-0.85201	1.837196	1.790101	0.386594
	4	2.304503967	-0.8926	1.153137	2.313895	0.486357
	5	2.25675042	-0.90987	1.364345	2.30707	0.378763

6	2.223863193	-0.91475	1.79176	2.231462	0.417468
7	2.192819508	-0.85437	1.739		0.330494
8	2.363158001	-0.81734	1.494443	1.528353	0.265106
9	2.481085559	-0.80687	1.323609	1.791329	0.648373
10	2.464978385	-0.79961	1.383014	1.840059	0.275103



(a)



(b)

Figure 3: Reveal Loss Ratios Patterns with Development Month every Accident Month in: (a) Liability Reinsurance; and (b) Motor Reinsurance

The plots above reveal loss ratios patterns with development month every accident month for their respective line of businesses as the axis. These plots are used to check every change in loss ratios pattern to development month in each accident month, and by using that line we can conclude whether our model can find a pattern from our dataset. The plots show that in the line of business liability reinsurance, particle filter cannot find a stable pattern from the 1st until the 4th month due to the drastic changes in each month.

But, by the fifth month, the filter particles were able to detect the loss ratios pattern. There are two possibilities. The particle filter can't detect patterns for loss ratios that has too much development year or the particle filter is not good at detecting loss ratios patterns that deviate from the Hoerl curve. But overall, we can summarize filter particles are sufficient in detecting loss ratios patterns by development month. To clarify that the filter particles can find loss ratios to development patterns in each accident month for the respective line of businesses, heatmaps are used to show the comparison between the real data with the top triangle made with the model we just made.

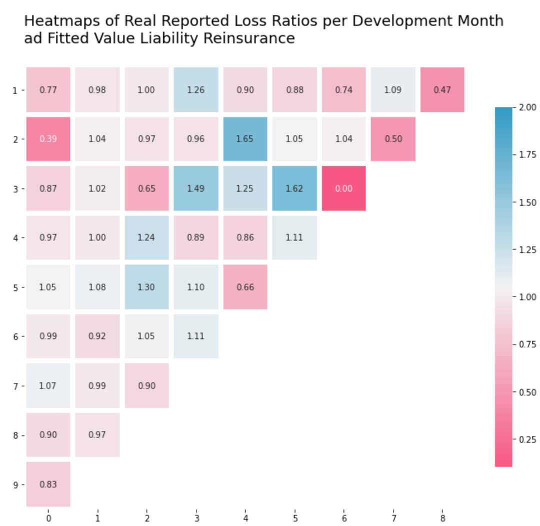


Figure 4: Heatmaps of Real Reported Loss Ratios per Development Month and Fitted Value Liability Reinsurance

According to the heatmaps, the accuracy of the Multivariate Generalized Linear Model Framework with Adaptive Estimation for each element in the line of business liability reinsurance is still around 1. This shows that the filter particle from the said method can find loss ratios to development month patterns excellently, thus it will produce a good loss ratio prediction. In addition, we can concur that the accuracy rate gets better in the lower region of the triangle.

Table 4: Predicted Claims per Month (decrement) for Liability Reinsurance

	Premium	**Jan-18**	**Feb-18**	**Mar-18**	**Apr-18**	**May-18**	**Jun-18**	**Jul-18**	**Aug-18**	**Sep-18**	**Oct-18**
01/18	2668.12	125.31	325.99	228.85	222.59	169.24	108.75	36.653	32.39	35.22	35.000
02/18	2579.27	138.76	360.99	253.42	246.49	187.40	120.43	40.588	35.86	39.00	
03/18	2412.58	106.14	276.13	193.84	188.54	143.35	92.120	31.046	27.43		
04/18	2003.16	111.51	290.10	203.65	198.08	150.60	96.780	32.616			
05/18	1727.32	93.667	243.67	171.06	166.38	126.50	81.293				
06/18	1329.15	83.355	216.85	152.23	148.06	112.57					

07/18	1212.89	88.611	230.52	161.83	157.40
08/18	1112.92	63.898	166.23	116.69	
09/18	1156.29	53.437	139.01		
10/18	1500.18	49.000			

Table 5: Predicted Claims per Month (cumulative) for Motor Reinsurance

	Premium	Jan-18	Feb-18	Mar-18	Apr-18	May-18	Jun-18	Jul-18	Aug-18	Sep-18	Oct-18
01/18	2668.12	344.577	693.435	158.704	51.355	22.855	10.107	23.317	0.725	1.982	10.000
02/18	2579.27	350.985	706.330	161.655	52.310	23.280	10.295	23.751	0.739	2.018	
03/18	2412.58	245.112	493.269	112.893	36.531	16.258	71.895	16.586	0.516		
04/18	2003.16	229.009	460.862	105.476	34.131	15.189	67.171	15.497			
05/18	1727.32	295.019	593.703	135.879	43.969	19.568	86.533				
06/18	1329.15	306.455	616.718	141.146	45.673	20.326					
07/18	1212.89	323.905	651.833	149.183	48.274						
08/18	1112.92	232.932	468.758	107.283							
09/18	1156.29	444.816	895.157								
10/18	1500.18	335.000									

3.3 Forecast

To find out how much claim reserves estimation the Swiss Re Group needs to prepare, in particular for the motor reinsurance and liability reinsurance, we can see the table below.

Table 7: Claim Reserves Estimation for Liability Reinsurance

	Jan-18	Feb-18	Mar-18	Apr-18	May-18	Jun-18	Jul-18	Aug-18	Sep-18	Oct-18	Total
01/18											
02/18										35	35
03/18									33	33	66
04/18								27	30	30	87
05/18							27	25	28	27	107
06/18						77	25	23	26	25	176
07/18					107	71	23	21	24	23	269
08/18				130	98	65	21	19	22	21	376
09/18			123	119	90	60	19	18	20	20	469
10/18		159	113	110	83	55	18	16	18	18	590

Table 8: Claim Reserves Estimation for Motor Reinsurance

	Jan-18	Feb-18	Mar-18	Apr-18	May-18	Jun-18	Jul-18	Aug-18	Sep-18	Oct-18	Total
01/18											0
02/18										3	3
03/18									1	3	4
04/18								25	1	4	30
05/18							9	1	1	4	15
06/18						10	9	1	1	4	25
07/18					22	10	9	1	1	4	47
08/18				53	23	10	10	1	1	4	102
09/18			163	55	24	11	10	1	1	4	269
10/18		741	169	57	24	11	11	1	1	5	1020

4 CONCLUSION

According to this research, we can summarize filter particles yang that we have obtained A Multivariate Evolutionary Generalised Linear Model Framework with Adaptive Estimation are sufficient in detecting loss ratios patterns by development month. However, if we look at the RMSE generated for each line of business, the RMSE generated by the GLM Simple Method with the Tweedie Family Distribution Approach is smaller than the RMSE generated by the Multivariate Evolutionary GLM Framework with Adaptive Estimation so we can conclude that the GLM Simple Method with the Tweedie Family Distribution Approach is more suitable for us to use in calculating Swiss Re Group's claim reserves, especially for motor reinsurance and liability reinsurance.

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