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A Mathematical Model of Inventory Cash Flow for Deteriorating Items in a Pharmaceutical Industry

Shehu S. L.[1,*], Agaie B. G.[1,2], Akeyede I.[3] and Akpan, C. E[3]

[1]Department of Mathematics, Nasarawa State University Keffi, 1022, Keffi, Nigeria
[2]Department of Mathematics, Federal University Dutse, 720101, Jigawa, Nigeria
[3]Department of Statistics, Federal University of Lafia, Nasarawa, Nigeria

*Corresponding author: shehuslalin@nsuk.edu.ng

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ABSTRACT

Inventory managers are frequently faced with the problem of maintaining inventory system with items that deteriorate. Developing a mathematical model that will lead policy makers to make optimal decisions for such a system is necessary. In this study a pharmaceutical economic order quantity (EOQ) inventory model is proposed in which the dynamics of the inventory are mainly affected by demand and the rate of deterioration. The deterioration rate is taken to be time dependent, and the time due to deterioration is assumed to follow three-parameter Weibull distribution, the demand rate is price dependent, and shortages are allowed and partially backlogged. Ordinary differential equations were used to model and analyze the dynamics of the model. A numerical example was considered to validate the model. Finally, a sensitivity analysis was carried out on some basic parameters and their effects were observed. Our findings showed that the total average cost per unit reduced significantly, while the average quantity demanded per day increased.

Keywords: Inventory Cash Flow, Deteriorating Items, Weibull Distribution.

1.0 INTRODUCTION

Control management of inventories of deteriorating items in a proper manner can be achieved through inventory modeling. Inventory managers are frequently faced with the problem of maintaining inventory system with items that deteriorate. Developing a mathematical model that will lead policy makers to make optimal decisions for such a system is necessary. The aim of any inventory model is to develop criteria (or policy decision) for determining the level of stock that a business organization must maintain to ensure a hitch free operation. The term "inventory" can be defined as the physical quantity of goods, items, raw materials, etc. that are kept in stock by a business enterprise for a smooth and efficient running of its future activities. Sanni [1], puts it that inventory problem occurs when there is at least one cost that decreases as inventory increases. Thus, the essence of inventory control is to reduce costs associated with investment in inventories and ensure that production process is not affected. According to Gupta and Mohan [2], and Taha [3], inventory control policy must answer two basic questions, namely:

1. What is the right quantity to request (purchase or produce) for filling stock items?

2. What is the right time to order for total cost to be minimized?

In order to have a realistic inventory management model for pharmaceutical industries, deterioration is an important factor that needs to be considered. The definition of deterioration is given as loss of utility, evaporation, spoilage, decay, damage and expiration of an item. Generic injectables, tablets, are some pharmaceutical products that usually have a lifetime. Others are caplets, ophthalmics, ointments, creams, liquids, and drugs etc., usually have a specific period of lifetime. Due to faulty items, shortage problems are faced by pharmaceutical inventory system that leads to loss of reputation or profit. Health-care organizations have faced a lot of challenges in recent times and need to continue to improve their services to attain the highest quality at optimal cost. The management of hospitals are faced with inventory challenges that track materials and drugs supply. Health-care facility at every deal is complex and time-consuming during inventory management process

The assumption that items can stay indefinitely (without spoiling) during storage was a common hypothesis adopted in standard inventory models. Most pharmaceutical products can actually spoil when kept for a long time; therefore, the idea of considering their deterioration rate to be constant cannot be realistic as reported by Uthayakumar and Karuppasamy [4]. Drugs expiration in hospitals pharmacies, is a major challenge to professionals in the health industry, since they can no longer be used after their expiration date has elapsed. Hence, consideration of deterioration rate as dependent upon time is very important for any health-care industry. Therefore, it becomes necessary to consider deterioration as a factor in modeling pharmaceutical inventory Rastogi and Singh [5].

2.0 REVIEW OF RELATED LITERATURE

Research has been going on for decades on the effect of deterioration in inventory modeling. According to Rahman [6], deterioration can generally be described as decay or damage item that cannot be used for the purpose intended. Evaporation, loss of utility, decay, damage and spoilage of a product are used to describe it by Uthayakumar and Karuppasamy [7].

Over the years, deteriorating items inventory models have gained attention from many researchers. Tripathi and Chaudhary [8] proposed an inflationary induced EOQ model for Weibull distributed deterioration and trade credits. An inventory model with demand and deteriorating rate that follows a two-parameter Weibull distribution was studied by Sayal et al. [9]. Chowdhury et al. [10] suggested a model for inventory control by considering a demand that depends on both price and level of stock. The model considered deterioration rate as independent of time and no shortages were allowed. Under Health-care Industry models, [7] developed a model where demand for pharmaceutical companies was considered on inventory as time-dependent. The model also assumed deterioration rate to be constant and shortages were allowed. Trade credit policy was investigated by Uthayakumar and Karuppasamy [11] using inventory models for pharmaceutical items. Demand and deterioration rate were considered as time-dependent with shortages not allowed in this study. Under products deterioration with time, a model with two warehouses for inventory was studied by Rastogi et al. [12] where the demand depends on price and shortages was considered as partially backlogged. Also, the cycle time was assumed to be a variable in that model. Chakrabarty et al. [13] worked on a model that controls inventory of deteriorating items in a two warehouse, where inflation as a factor was considered. The demand rate was taken to be an exponential function of time and backlogging rate was assumed to be negative exponential function of waiting time in the model developed. Poswal et al. [14] studied an economic ordering policy fuzzy model to control deteriorating medicinal products of uncertain demand with trade credit for healthcare industries. The signed distance technique is employed to defuzzify the total cost of the system.

It is a known fact that customers behavior towards demand can change due to competition as well as intensive advertisements; therefore, it is not true the assumption on general demand rate for a product will be constant. In modeling inventory, different patterns of demand such as price dependent, constant, time-dependent and stock-dependent have been considered. But demand is mostly affected by selling prices, that is the reason why firms used to regulate the prices of their products. Thus, the selling price increase usually results in decreasing repeated demand. Therefore, it becomes imperative for managers to consider selling price as an important factor that affects profit significantly. A combined inventory model on production was taken into consideration by Singh et al. [15] in which the demand rate was considered as selling price dependent. Constant rate of deterioration was assumed by the authors in the model developed. [11] proposed an inventory model for pharmaceutical industries in which delay in payments was permitted. They considered demand as time dependent and allowed shortages to be partially backlogged. Recently, Rastogi and Singh [16] investigated an inventory model for production in which inflation was incorporated. Also, the production demand rate was taken to be price dependent, and shortages were allowed. An inventory model for non-instantaneous deteriorating items was investigated by Udayakumar et al. [17]. In their model, they considered money inflation and time discounting, where delay in payment as an alternative to price discount given by the supplier was also considered. Furthermore, they also considered salvage value for deteriorating items in their model.

Performing a particular task repeatedly by workers can improve their experience and consequently help in reducing labour time, the cost of production and the number of defective units. This is referred to as "learning effect". Improvement can be attained by repeating a particular procedure or product several times. Singhal and Singh [18] have developed an EPQ model for deteriorating items with constant deterioration rate, learning effects associated with all inventory costs and volume flexibility. A reverse logistic inventory model was presented by Singh and Rathore [19] under the effect of learning for deteriorating products. The study focuses mainly on the computation of optimal value for total cost of the system and rate of deterioration was assumed constant. Agarwal et al. [20] studied a model with deterioration rate that follows Weibull distribution. Their findings showed that holding cost gradually decreases, when learning effect was incorporated on holding cost. Sharma et al. [21] studied an EOQ model for seasonal products (like fashion commodities, electronic goods, seasonal fruits, vegetables, etc.) with ramp-type demand under the consideration of permissible delay provided by supplier to the retailer [5], presented a pharmaceutical inventory model for varying deteriorating items with price sensitive demand and partial backlogging under the effect of learning.

During shortages, researchers mostly assume all demand that accrued is either lost or totally backlogged. However, in real sense there are some customers who are ready to wait for the items till the next replenishment. Inventory system on pharmaceuticals, cannot be an exception. They also have this challenge of shortages which can make them lose profit or their reputation. For instance, when there is a shortage of drugs in a retail pharmacy, some customers may not be patient to wait.

Such customers may decide to buy somewhere else, and this will make the pharmacy lose business. Mishra and Tripathy [22] studied inventory models for deteriorating items where salvage value was considered for the items. Deterioration rate of the model followed a Weibull distribution with three-parameters and shortages were not allowed. [15] presented a model on inventory of deteriorating items where seasonal pattern was considered, and demand rate was taken as stock dependent. Shortages were allowed and the authors also discussed different backlogging conditions of the model. Rastogi, et al. [23] studied a model where credit limit was assumed. The model considered holding cost that is time dependent and partially backlogged shortages were allowed. The backlogging rate was considered as function that depends on waiting time. Sundararajan and Uthayakumar [24] considered an EOQ model where instantaneous deteriorating items were studied on delay in payment. Furthermore, the study assumed both cost and deterioration rate as constant. An EOQ model with demand that is selling price dependent for deteriorating items and shortages for production inventory was proposed by Rastogi and Singh [25].

In this study, we shall consider a model for deteriorating inventory in a pharmaceutical industry in which deterioration rate follows Weibull distribution with three parameters and a demand function that depends on selling price. Also, learning effect will be applied to both ordering as well as holding cost in the proposed model.

3.0 METHODOLOGY

Differential equations will be used to model and analyze the dynamics of the inventory. The method of (Rastogi and Singh, 2019), shall be adopted with modification of the deterioration parameter.

3.1 Assumptions and Notations of the Model

The following assumptions and notations are used in deriving the model.

3.1.1 Assumptions of the Study

The development of the new model will be based on the assumptions given below:
1. The demand function depends on the selling price of the product and is given by $D(p) = a - bp > 0$.

2. Three parameter Weibull distribution is assumed as product rate of deterioration given as

$\alpha\beta(t-\gamma)^{\beta-1}$where $0 < \alpha < 1$is the scale parameter, $\beta > 0$ is the shape parameter, $t > 0$ is the time of deterioration and $\gamma \leq t$is the location parameter.

3. Shortages are allowed and partially backlogged, and the rate of backlogging is assumed to be afunction which depends on waiting time.

4. Deteriorated items are not being replaced.

5. Learning effect is being applied to both ordering and holding costs.

6. It is assumed that the time taken between placing and receiving an order (lead time) is zero.

3.1.2 Notations

The following notations were adopted for the development of this model.

T	Time per cycle
t_1	Time for which zero inventory level is attained
α, β and γ	Coefficients of deterioration
a, b	Parameters for demand
n	Cycle number
c_1	Per unit purchase cost
p	Per unit selling price, $p > c_1$
Q	Ordered quantity
Q_1	Inventory initial level at $t = 0$
Q_2	Amount of shortage
O_0	Part of ordering cost that is fixed per order
O_1	Variable part of ordering cost per order that can be reduced owing to learning effect
h_0	Part of holding cost that is fixed per unit
h_1	Variable part of holding cost per unit which can be reduced owing to learning effect
d	Cost of deterioration per unit
s	Cost of shortage per unit
l	Cost of lost sale per unit
u	Coefficient of backlogging,u>1
λ	Positive coefficient for backlogging
Δ_1, Δ_2	Positive constants
T.A.C.	Total average cost

3.2 Mathematical Formulation of the Model

Figure 1below gives the system's behavior with time. When t = 0, items are received by the retailer and the inventory level becomes I1(0) = Q1. Thereafter, the level of inventory started decreasing due to deterioration of items and market demand during the time [0, t1] until it reaches zero at t = t1. During [t1, T] shortages are allowed to exist and the demand that occurs within the period is backlogged partially or lost, and the rate of backlogging is assumed to be a function that depends on waiting time.

The differential equations describing the behavior of pharmaceutical inventory at any time t are as follows:

$$\frac{dI_1(t)}{dt} = -(a - bp) - \alpha\beta(t-\gamma)^{\beta-1}I_1(t),\ 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = -(a - bp),\ t_1 \le t \le T \tag{2}$$

With boundary conditions $I_1(t_1) = I_2(t_1) = 0$ (3)

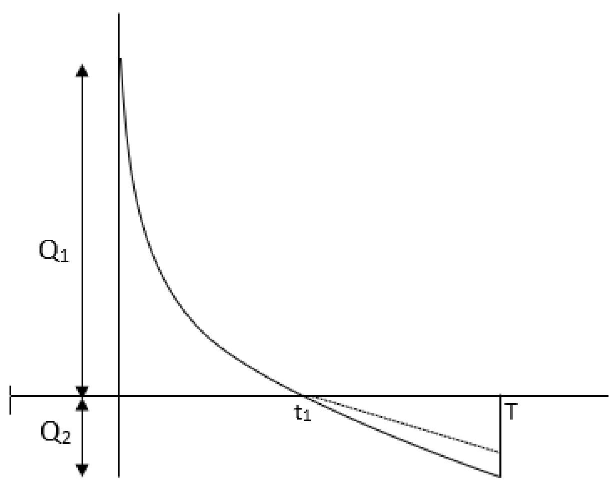

Figure 1: Inventory time graph

Solving (1), we have

$$I_1(t) = (a - bp)\left[(t_1 - t) + \frac{\alpha}{\beta+1}\left((t_1-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}\right) - \alpha(t_1 - t)(t-\gamma)^{\beta}\right],\ \ 0 \le t \le t_1 \tag{4}$$

Solving (2) gives

$$I_2(t_1) = (a - bp)(t_1 - t)\ \ , t_1 \le t \le T \tag{5}$$

During a replenishment cycle, the total quantity Q is

$$Q = Q_1 + Q_2 \tag{6}$$

Using equation (4), the initial order quantity Q_1 is given by letting $t = 0$

$$\Longrightarrow Q_1 = I_1(0)$$

$$Q_1 = (a - bp)\left[t_1 + \frac{\alpha}{\beta+1}\left((t_1 - \gamma)^{\beta+1} - (-\gamma)^{\beta+1}\right) + \alpha t_1(-\gamma)^{\beta}\right] \tag{7}$$

Now the back ordered quantity

$$Q_2 = \int_{t_1}^{T} u^{-\lambda t}\,(a - bp)dt = (a - bp)\left(\frac{u^{-\lambda t_1} - u^{-\lambda T}}{\lambda \log(u)}\right) \tag{8}$$

Using equations (7) and (8) in (6), the total ordering quantity Q is given by

$$Q = (a - bp)\left[t_1 + \frac{\alpha}{\beta+1}\left((t_1 - \gamma)^{\beta+1} - (-\gamma)^{\beta+1}\right) - \alpha t_1(-\gamma)^{\beta}\right] + (a - bp)\left(\frac{u^{-\lambda t_1} - u^{-\lambda T}}{\lambda \log(u)}\right) \tag{9}$$

Purchasing cost:

The purchasing quantity is given by equation (9), therefore, the purchasing cost is

$$P.C. = c_1(a - bp)\left\{\left\{\left[t_1 + \frac{\alpha}{\beta+1}\left((t_1 - \gamma)^{\beta+1} - (-\gamma)^{\beta+1}\right) - \alpha t_1(-\gamma)^{\beta}\right] + \left(\frac{u^{-\lambda t_1} - u^{-\lambda T}}{\lambda \log(u)}\right)\right\}\right\} \tag{10}$$

Holding cost:

The inventory cost during the interval $[0,\ t_1]$ is

$$H.C. = \left(h_0 + \frac{h_1}{n^{\Delta_2}}\right)\left\{\int_0^{t_1} I_1(t)dt\right\}$$

$$= \left(h_0 + \frac{h_1}{n^{\Delta_2}}\right)(a - bp)\left\{\frac{{t_1}^2}{2} + \frac{\alpha}{(\beta+1)}\left[t_1(t_1 - \gamma)^{\beta+1} - \frac{(t_1-\gamma)^{\beta+2}}{(\beta+2)}\right] - \frac{\alpha}{(\beta+1)(\beta+2)}(t_1 - \gamma)^{\beta+2}\right\} \tag{11}$$

Deterioration cost:

The total demand during $[0,\ t_1]$ is

$$\int_0^{t_1}(a - bp)dt$$

Therefore, the number of deteriorating units = $I_1(0)$ − total demand

$$= I_1(0) - \int_0^{t_1}(a - bp)dt$$

Hence, the deterioration cost is given by

$$D.C. = d(a-bp)\left\{\frac{\alpha}{\beta+1}\left[(t_1-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right]-\alpha(-\gamma)^{\beta}t_1\right\} \quad (12)$$

Shortage cost:

The shortage cost during the interval $[t_1,\ T]$ is

$$S.C. = s\int_{t_1}^{T}(a-bp)dt = s(a-bp)(T-t_1) \quad (13)$$

Lost sale cost:

The lost sale cost during the interval $[t_1,\ T]$ is

$$L.S.C. = l\int_{t_1}^{T}\left(1-u^{-\lambda t}\right)(a-bp)dt = l(a-bp)\left\{(T-t_1)+\frac{1}{\lambda \log(u)}\left(u^{-\lambda T}-u^{-\lambda t_1}\right)\right\} \quad (14)$$

Ordering cost:

Ordering cost $O.C. = O_0 + \frac{O_1}{n^{\Delta_1}}$ (15)

The total average cost T. A. C. of the system is given by

$$T.A.C. = \frac{1}{T}\{O.C.+P.C.+H.C.+D.C.+S.C.+L.S.C.\}$$

$$= \frac{1}{T}\left\{\left(O_0+\frac{O_1}{n^{\Delta_1}}\right)+c_1(a-bp)\left\{\left[t_1+\frac{\alpha}{\beta+1}\left((t_1-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right)-\alpha t_1(-\gamma)^{\beta}\right]+\left(\frac{u^{-\lambda t_1}-u^{-\lambda T}}{\lambda\log(u)}\right)\right\}+\left(h_0+\frac{h_1}{n^{\Delta_2}}\right)(a-bp)\left\{\frac{{t_1}^2}{2}+\frac{\alpha}{(\beta+1)}\left[t_1(t_1-\gamma)^{\beta+1}-\frac{(t_1-\gamma)^{\beta+2}}{(\beta+2)}\right]-\frac{\alpha}{(\beta+1)(\beta+2)}(t_1-\gamma)^{\beta+2}\right\}+d(a-bp)\left\{\frac{\alpha}{\beta+1}\left[(t_1-\gamma)^{\beta+1}-(-\gamma)^{\beta+1}\right]-\alpha(-\gamma)^{\beta}t_1\right\}+s(a-bp)(T-t_1)+l(a-bp)\left\{(T-t_1)+\left(\frac{u^{-\lambda T}-u^{-\lambda t_1}}{\lambda\log(u)}\right)\right\}\right\} \quad (16)$$

The necessary condition for minimizing T.A.C. is $\frac{dT.A.C}{dt_1}=0$

Now, $\frac{\delta T.A.C}{\delta t_1} = \frac{(a-bp)}{T}\left\{c_1\left\{1+\alpha\left[(t_1-\gamma)^{\beta}-(-\gamma)^{\beta}\right]\right\}-\left(u^{-\lambda t_1}\right)+\left(h_0+\frac{h_1}{n^{\Delta_2}}\right)\left\{t_1+\alpha\left[t_1(t_1-\gamma)^{\beta}-\frac{1}{(\beta+1)}(t_1-\gamma)^{\beta+1}\right]\right\}+d\left\{\alpha\left[(t_1-\gamma)^{\beta}+\gamma\right]\right\}-s+l\left(u^{-\lambda t_1}-1\right)\right\}=0$

Solving $\frac{dT.A.C}{dt_1}=0$, we obtain the optimal value of t_1 which minimize T.A.C. provided it satisfy $\frac{\delta^2 T.A.C}{\delta t_1^2} > 0$.

$$\frac{d^2T.A.C}{dt_1^2} = \frac{(a-bp)}{T}\left\{c_1\left[\alpha\beta(t_1-\gamma)^{\beta-1}\right]+u^{-\lambda t_1}\lambda\log(u)+\left(h_0+\frac{h_1}{n^{\Delta_2}}\right)\left[1+\alpha\beta t_1(t_1-\gamma)^{\beta-1}+\right] + d\left[\alpha\beta(t_1-\gamma)^{\beta-1}\right]-l\left(u^{-\lambda t_1}\lambda\log(u)\right)\right\}>0$$

Algorithm

Step 1: Start.

Step 2: Assign a value for each parameters a, b, p, α, β, γ, λ, c_1, s, O_0, O_1, h_0, h_1, T ,d,l, u, n, Δ_1, Δ_2.

Step 3: Solve the equation $\frac{dT.A.C}{dt_1} = 0$ and obtain the value of t_1 using step-2.

Step 4: Evaluate $\frac{d^2T.A.C}{dt_1^2}$

Step 5: If the value of $\frac{d^2T.A.C}{dt_1^2}$is greater than zero at the obtained value of t_1 of step-3, then obtained value of t_1 will be optimal value which is denoted by t_1^* and corresponding to this value, the obtained value *T.A.C.* by Eq. (16) will be optimal total average cost (*T.A.C**). Otherwise go to step-2 and choose another set of values of the parameters.

Step 6: Repeat step-3 to step-5 until we get t_1^* and *T.A.C**.

Step 7: End.

Numerical Example:

The cost of purchasing a medicine from a pharmaceutical company by a pharmacist is ₦10 per unit. The ordering cost incurred by the retailerfor the stock is O_0naira per order and $h_0$naira per unit for holding the stock. Due to the repetition of the cycle, the retailer can improve the deficiency of handling facilities. So, h_1 part of the holding cost per order can be reduced. The selling price of the retailer is ₦20 per unit. Due to deterioration the retailer bears a loss of ₦11 per unit. When the stock is finished, some customers return for purchase and some impatient customers may purchase from other places that results in the loss of business. The backlogging of customers during shortages is dependent upon waiting time. A less in waiting time results more of backlogging. Due to impatient customers the lost sale cost per unit is estimated at ₦6 per unit. Find the optimal time interval and optimal ordering quantity that will minimize the total average cost.

In order to solve this problem a numerical example is given as follows:

Let T = 30, a = 350 units, b = 0.2, p = ₦20/unit, $\gamma = 0.8$, $\alpha = 0.01$, $\beta = 1.2$, $\lambda = 1.5$, c_1= ₦10/unit, s = ₦5/unit, O_0 = ₦150/order, O_1 = ₦100, h_0 = ₦0.3/unit, h_1 = ₦0.2/unit, d = ₦11/unit, l = ₦6/unit, u = 1.2, n = 3, $\Delta_1 = 0.5$, $\Delta_2 = 0.2$

Using Matlab software, the optimal value of t_1 and T.A.C. are:

$t_1^* = 3.511$ days and $T.A.C^* =$ ₦3957.67 respectively and the corresponding ordering quantity

$Q^* = 1654.6$ units

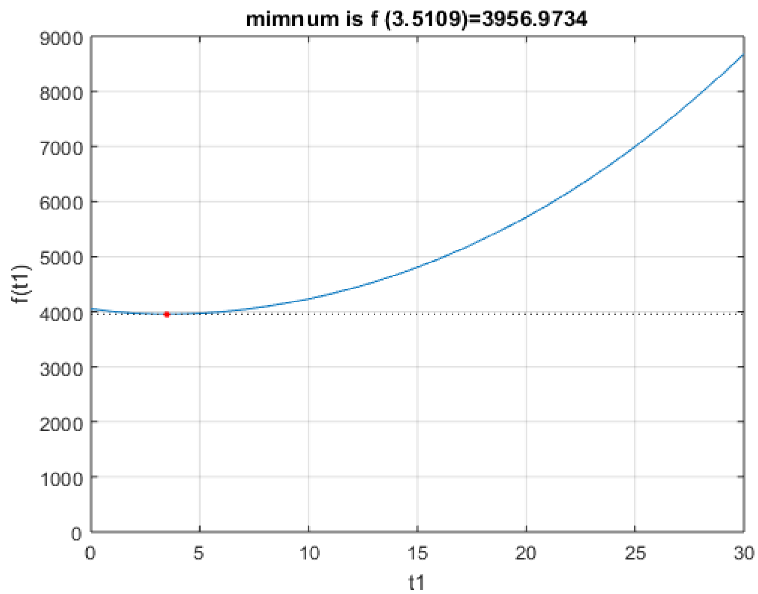

Figure 2: Graph of T.A.C against t_1

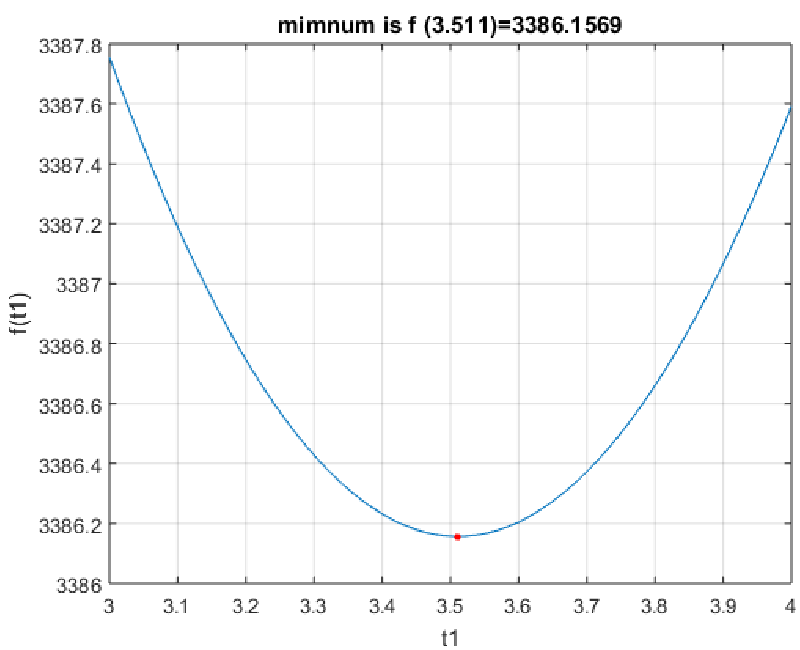

Figure 3: Graphical representation of convexity of the T.A.C. with respect to t_1

Sensitivity Analysis

In this section, a sensitivity analysis is carried out on different parameters by considering the variation in a single parameter at a time while other variables remain constant.

4.0 RESULTS AND DISCUSSION

The following were observed based on the numerical results and sensitivity analysis. Table 1 shows the effect of demand parameter "a" on t_1, T.A.C. and Q. From the table, it is observed that as demand parameter "a" increase, $t_1$remains constant T.A.C and Q increases. This effect can also be observed in Fig. 4.

Table 1: Effect of distinct values of demand parameter a on optimal solution

% change in a	a	t_1	T.A.C.	Q
-25	*225*	3.511	2529.93	1232.04
-20	240	3.511	2701.18	1315.66
-15	255	3.511	2872.42	1399.29
-10	270	3.511	3043.67	1482.91
-5	285	3.511	3214.91	1566.53
0	300	3.511	3386.16	1650.15
5	*315*	3.511	3557.40	1733.78
10	330	3.511	3728.65	1817.40
15	345	3.511	3899.89	1901.02
20	360	3.511	4071.14	1984.65
25	375	3.511	4242.38	2068.27

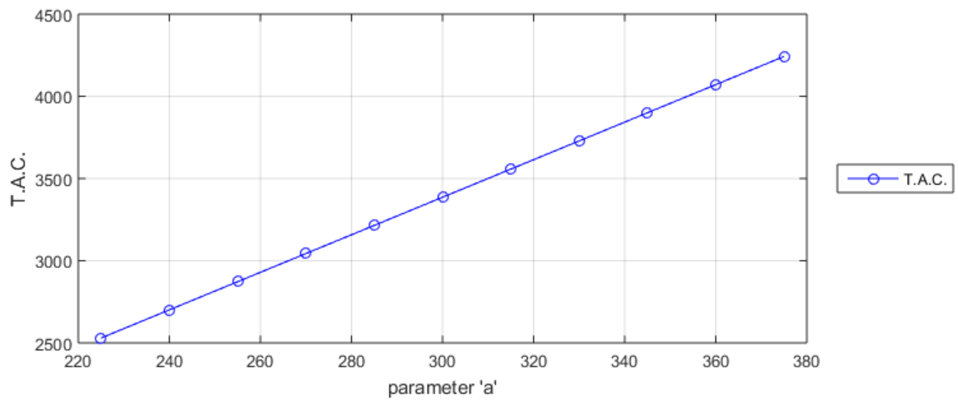

Figure 4: Variation of overall average cost with respect to demand parameter "a"

Table 2: Effect of distinct values of selling price "p" on optimal solution

% change in p	p	t_1	T.A.C.	Q
-25	*15*	3.511	3397.57	1655.73
-20	16	3.511	3395.29	1654.61
-15	17	3.511	3393.01	1653.50
-10	18	3.511	3390.72	1652.38
-5	19	3.511	3388.44	1651.23
0	20	3.511	3386.16	1650.15
5	*21*	3.511	3383.87	1649.04
10	22	3.511	3381.59	1647.92
15	23	3.511	3379.31	1646.81
20	24	3.511	3377.02	1645.69
25	25	3.511	3374.74	1644.58

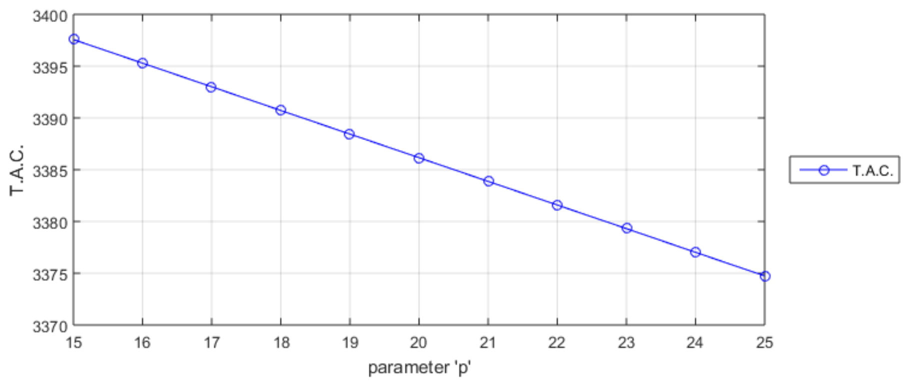

Figure 5: Variation of overall average cost with respect to price "p"

Table 2 shows the variation per unit of selling price "p" and it is observed that an increase per unit selling price shows a decrease in T.A.C. and Q while $t_1$remains unaffected. This is in line with the law of demand which states that "the higher the price, the lower the quantity demanded". Therefore, an increase in per unit price will make the retailer to order lesser quantity. This effect can also be seen in Fig. 5.

Tables 3 and 4 show the variation in backlogging parameters "λ and u" at distinct points. From these tables, it is observed that an increase in backlogging parameters results in a decrease of T.A.C. which is also realistic. This effect can be seen in Figs. 6 and 7.

Table 3 Effect of distinct values of backlogging parameter λ on optimal solution

% change in λ	λ	t_1	T.A.C.	Q
-25	1.125	3.857	3423.48	1935.38
-20	1.200	3.779	3413.95	1861.78
-15	1.275	3.706	3405.63	1797.89
-10	1.350	3.637	3398.33	1742.14
-5	1.425	3.573	3391.88	1693.24
0	1.500	3.511	3386.16	1650.15
5	1.575	3.453	3381.05	1612.04
10	1.650	3.397	3376.48	1578.18
15	1.725	3.345	3372.37	1548.00
20	1.800	3.294	3368.65	1521.02
25	1.875	3.246	3365.28	1496.81

Table 4: Effect of distinct values of backlogging parameter u on optimal solution

% change in u	u	t_1	T.A.C.	Q
-25	-	-	-	-
-20	-	-	-	-
-15	1.05	4.998	3676.49	3914.03
-10	1.10	4.284	3490.88	2461.04
-5	1.14	3.908	3430.18	1987.32
0	1.20	3.511	3386.16	1650.15
5	1.26	3.230	3364.17	1488.89
10	1.32	3.019	3351.43	1400.82
15	1.38	2.854	3343.31	1348.46
20	1.44	2.720	3337.76	1315.42
25	1.50	2.610	3333.80	1293.64

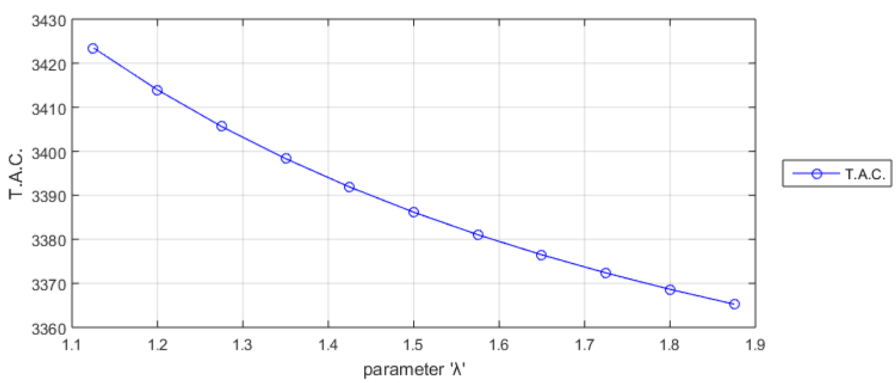

Figure 6: Variation of overall average cost with respect to backlogging parameter $'\lambda'$

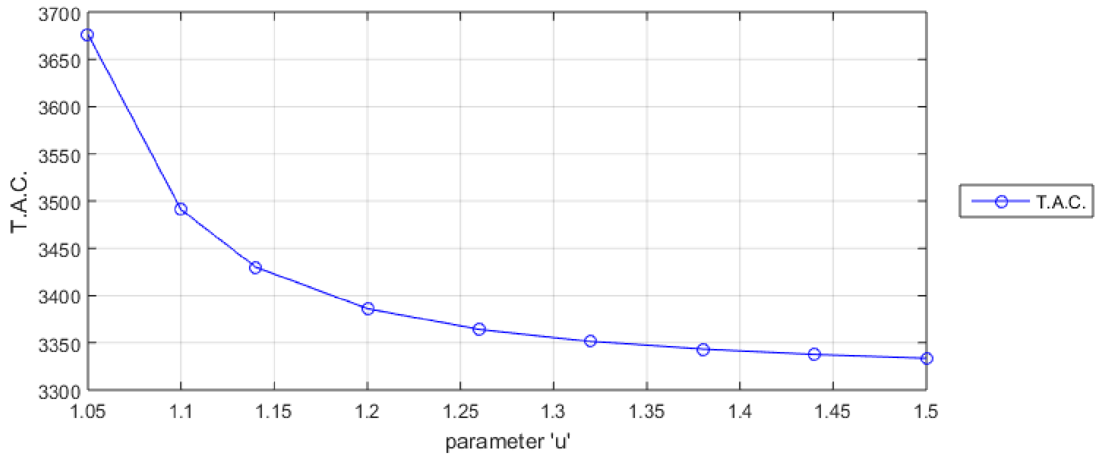

Figure 7: Variation of overall average cost with respect to backlogging parameter ‘u’

From Table 5, it is observed that as deterioration parameter “α” increases, T.A.C. also increases and Q increases while $t_1$decreases. This is because as deterioration increases, it decreases the overall profit of the system and that is the reason for the increase in the overall average cost of the system. Therefore, it becomes imperative for managers to take care of the deterioration parameter as it can quickly affect profit.

Table 5: Effect of distinct values of deterioration parameter α on optimal solution

% change in α	α	t_1	T.A.C.	Q
-25	*0.0075*	3.714	3382.35	1646.23
-20	0.0080	3.671	3383.15	1647.02
-15	0.0085	3.629	3383.93	1647.80
-10	0.0090	3.588	3384.69	1648.57
-5	0.0095	3.549	3385.43	1649.37
0	0.0100	3.511	3386.16	1650.15
5	*0.0105*	3.474	3386.87	1650.94
10	0.0110	3.438	3387.56	1651.72
15	0.0115	3.406	3388.24	1652.51
20	0.0120	3.3697	3389.62	1658.6
25	0.0125	3.3368	3390.29	1659.6

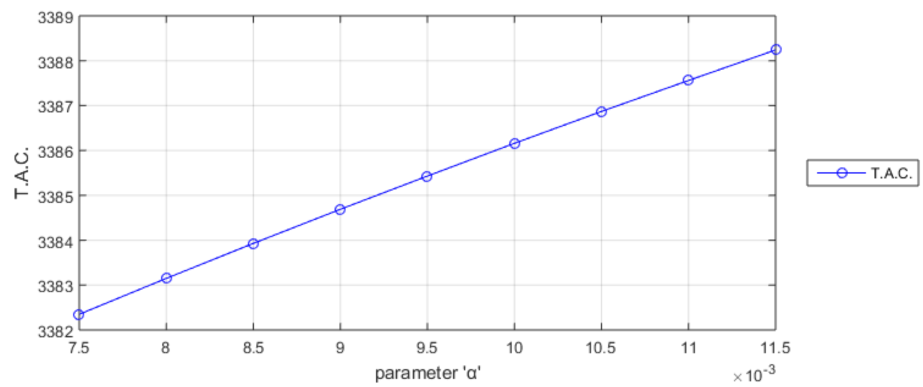

Figure 8: Variation of overall average cost with respect to deterioration parameter 'α'

From Table 6 we observed that due to learning effect, as the value of repetition increases, the overall system average cost "T.A.C." decreases continuously. This effect can also be seen depicted in Fig.

Table 6: Effect of distinct values of n on optimal solution

change in n	t_1	T.A.C.
1	3.399	4246.75
2	3.471	4243.85
3	3.511	4242.38
4	3.538	4241.42
5	3.558	4240.72
6	3.574	4240.18
7	3.587	4239.73
8	3.598	4239.36
9	3.608	4239.04
10	3.6159	4238.77

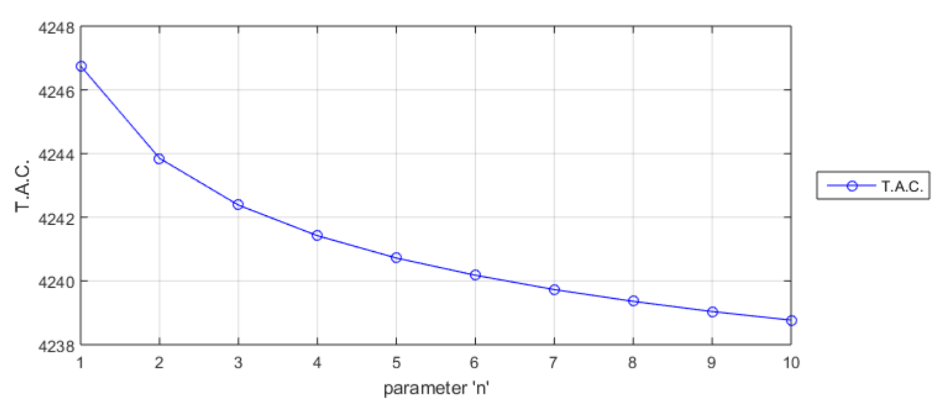

Figure 9: Variation of overall average cost with respect to 'n'

4.0 CONCLUSION

A mathematical model for deteriorating items is presented for application in pharmaceutical industries by considering price dependent demand and variable rate of deterioration that follows a three parameter Weibull distribution. Learning effect was applied to ordering and holding costs of the system and the results showed a significant decrease in the overall total average cost of the system. The model also considered shortages and partial backlogging of unsatisfied customers' demand, where the rate of backlogging was considered as a function that depends on waiting time of the next replenishment. A numerical example was presented, and sensitivity analysis was carried out to illustrate the effect of some basic parameters on the total average cost of the system. Results obtained from the sensitivity analysis show the applicability of the model in real life and are presented in tables and graphs.

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