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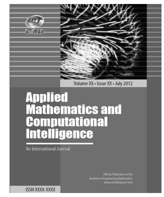


# Thermal radiation effects on MHD convective flow over a plate in a porous medium by perturbation technique

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**Abstract:** *This paper analyzes the influence of thermal radiation on the problem of unsteady magneto-convection flow of an electrically conducting fluid past a semi-infinite vertical porous plate embedded in a porous medium with time dependent suction. Perturbation technique is applied to transform the non-linear coupled governing partial differential equations in dimensionless form into a system of ordinary differential equations. The resulting equations are solved analytically and the solutions for the velocity and temperature fields are obtained. For different values of the flow parameters, the values for Nusselt number and skin-friction co-efficient are calculated. It is observed that the increase in the radiation parameter implies the decrease in the boundary layer thickness and enhances the rate of heat transfer. The velocity decreases as the existence of magnetic field becomes stronger.*

**Keywords:** *Natural convection; Radiation; MHD; Porous medium; Perturbation method.*

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## 1 Introduction

The effect of radiation on MHD flow and heat transfer problems has become industrially more important. Many engineering processes occur at high temperatures and hence the knowledge of radiation heat transfer is essential for designing appropriate equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles and satellites are examples of such processes [1]. When radiative heat transfer takes place, the fluid involved

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can be electrically conducting since it is ionized due to the high operating temperature. Accordingly, examining the effect of magnetic field on the flow becomes more important. In view of these, many authors have made contributions to the study of fluid flow with thermal radiation [2-5], [7]. Radiation effects on unsteady MHD free convection flow of an electrically conducting gray gas near equilibrium in the optically thin limit along an infinite vertical porous plate are studied by Seddeek and Aboeldahab [2]. Unsteady natural convection flow of a viscous and incompressible fluid through a porous medium with high porosity bounded by a vertical infinite stationary plate in the presence of radiation is analyzed by Raptis and Perdikis [3]. They found that both velocity and temperature decrease when the radiation parameter increases. Makinde [4] studied free convection boundary layer flow with thermal radiation and mass transfer past a moving vertical porous plate.

The combined effect of MHD and thermal radiation on steady, free convection over a vertical flat plate embedded in a porous medium is studied by Rashad [5]. It is observed that particle concentration and concentration boundary layer decrease due to increase in either of Lewis number, radiation parameter and buoyancy ratio. Bararnia et al. [6] have applied homotopy analysis method to investigate the MHD natural convection flow of the heat generation fluid driven by a continuously moving permeable surface immersed in a fluid saturated porous medium. Pal and Mondal [7] studied the radiation effects on the combined convection flow of an optically dense viscous incompressible fluid over a vertical surface embedded in a fluid saturated porous medium of variable porosity with heat generation and absorption. They observed that the momentum and thermal boundary layer thickness increases with increase in radiation. Behavior of the polar fluid on steady flow through a vertical infinite plate with the boundary layer has been analyzed by Ferdows et al. [8]. An analytical study for the problem of unsteady mixed convection with thermal radiation and chemical reaction on MHD boundary layer flow of a viscous, electrically conducting fluid past a vertical permeable plate has been presented by Pal and Talukdar[9].

The present work is concerned with the effect of thermal radiation on magnetohydrodynamic convection flow of an unsteady viscous incompressible electrically conducting fluid past a semi-infinite vertical permeable plate embedded in a porous medium. The classical model for radiation effect introduced by Cogley et al. [10] is used. Perturbation technique is applied to convert the governing non-linear partial differential equations into a system of ordinary differential equations, which are solved analytically.

## 2 Mathematical Analysis

Consider a two-dimensional unsteady flow of a laminar, incompressible, electrically conducting and heat absorbing fluid past a semi-infinite vertical porous plate embedded in a uniform porous medium subjected to a thermal radiation field. The physical model and the coordinate system of the problem are shown in Figure 1. The $x^*$ axis is chosen along the plate and the $y^*$ axis is perpendicular to it. A uniform magnetic field of strength $B_0$ in the presence of radiation is imposed transversely in the direction of $y^*$ axis. The induced magnetic field is neglected under the assumption that the magnetic Reynolds number is small. It is assumed that there is no applied voltage which implies the absence of any electrical field. The radiative heat flux in the $x^*$ direction is considered negligible in comparison to that in the $y^*$ direction. The governing equations for this study are based on the conservation of mass, linear momentum and energy. Taking into consideration the assumptions made above, these equations in Cartesian frame of reference are given by Continuity:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

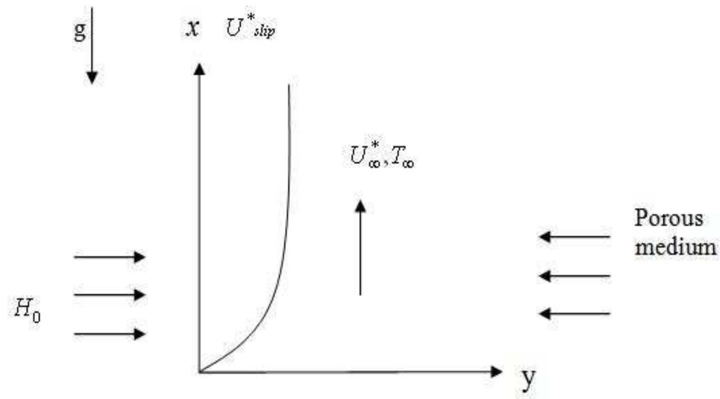


Figure 1. Physical model and the coordinate system.

Momentum:

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial {y^*}^2} - \frac{\sigma B_0^2}{\rho} u^* - \frac{\nu u^*}{K^*} + g\beta(T^* - T_\infty) \tag{2}$$

Energy:

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial {y^*}^2} - \frac{1}{\rho c_p}\frac{\partial q_r^*}{\partial y^*} - \frac{Q_0}{\rho c_p}(T^* - T_\infty) \tag{3}$$

where $u^*$and $v^*$ are the components of the dimensional velocities along $x^*$and $y^*$ directions respectively.
Cogley et al. [10] have shown that, in the optically thin limit for a non-gray gas near equilibrium, the radiative heat flux is represented by the following form:

$$\frac{\partial q_r^*}{\partial y^*} = 4(T^* - T_\infty)I^* \tag{4}$$

where $I^* = \int K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T^*} d\lambda$
Under this assumption, the appropriate boundary conditions for velocity involving slip flow, temperature fields are given by

$$u^* = u^*_{slip} = \frac{\sqrt{K^*}}{\alpha}\frac{\partial u^*}{\partial y^*}, \; T^* = T_w \quad \text{at} \quad y^* = 0 \tag{5}$$

$$u^* \to U^*_\infty = U_0(1 + \varepsilon e^{n^* t^*}), T^* \to T_\infty \quad \text{as} \quad y^* \to \infty \tag{6}$$

Since the suction velocity normal to the plate is a function of time only, it can be taken in the exponential form as

$$v^* = -V_0(1 + \varepsilon A e^{n^* t^*}) \tag{7}$$

where $A$ is a real positive constant, $\varepsilon$ and $\varepsilon A$ are small quantities less than unity and $V_0 > 0$.

Outside the boundary layer, equation (2) gives

$$-\frac{1}{\rho}\frac{dp^*}{dx^*} = \frac{dU^*_\infty}{dt^*} + \frac{\sigma B_0^2}{\rho} U^*_\infty + \frac{\nu}{K^*} U^*_\infty \tag{8}$$

Now, we introduce the dimensionless variables as follows

$$u = \frac{u^*}{U_0}, v = \frac{v^*}{V_0}, y = \frac{V_0 y^*}{\nu}, U_\infty = \frac{U_\infty^*}{U_0}, t = \frac{t^* V_0^2}{\nu}, \theta = \frac{T^* - T_\infty}{T_w - T_\infty},$$
$$n = \frac{n^* \nu}{V_0^2}, K = \frac{K^* V_0^2}{\nu^2}, \Pr = \frac{\mu c_p}{k}, M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, Gr = \frac{\nu \beta g (T_w - T_\infty)}{U_0 V_0^2},$$
$$\phi = \frac{Q_0 \nu}{\rho c_p V_0^2}, F = \frac{4 \nu I^*}{\rho c_p V_0^2} \tag{9}$$

Using (9), the governing equations (2) & (3) reduce to the following non-dimensional form:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + N(U_\infty - u) \tag{10}$$

where $N = M + \frac{1}{K}$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} - F\theta - \phi\theta \tag{11}$$

The boundary conditions (5) and (6) in the dimensionless form can be written as

$$u = u_{slip} = \phi_1 \frac{\partial u}{\partial y}, \theta = 1 \quad \text{at} \quad y = 0 \tag{12}$$

where $\phi_1 = \frac{\sqrt{K}}{\alpha} U_0$

$$u \to U_\infty = 1 + \varepsilon e^{nt}, \theta \to 0 \quad \text{as} \quad y \to \infty \tag{13}$$

## 3 Solution of the Problem

To solve the equations (10) and (11), we assume the solution in the following form:

$$u = f_0(y) + \varepsilon e^{nt} f_1(y) + O(\varepsilon^2) \tag{14}$$
$$\theta = g_0(y) + \varepsilon e^{nt} g_1(y) + O(\varepsilon^2) \tag{15}$$

Substituting (14) and (15) into the equations (10) and (11) and equating the harmonic and non-harmonic terms, neglecting the coefficient of $O(\varepsilon^2)$ , we get the following pairs of equations for $(f_0, g_0)$and $(f_1, g_1)$.

$$f_0'' + f_0' - N f_0 = -N - Gr g_0 \tag{16}$$
$$f_1'' + f_1' - (N + n) f_1 = -A f_0' - Gr g_1 - (N + n) \tag{17}$$
$$g_0'' + \Pr g_0' - \Pr(F + \phi) g_0 = 0 \tag{18}$$
$$g_1'' + \Pr g_1' - \Pr(F + \phi + n) g_1 = -A \Pr g_0' \tag{19}$$

where the primes denote the differentiation with respect to $y$.
The corresponding boundary conditions can be written as

$$f_0 = \phi_1 f_0', \; f_1 = \phi_1 f_1', \; g_0 = 1, \; g_1 = 0 \quad \text{at} \quad y = 0 \tag{20}$$
$$f_0 = 1, \; f_1 = 1, \; g_0 \to 0, \; g_1 \to 0 \quad \text{as} \quad y \to \infty \tag{21}$$

The solutions of equations (16)-(19) which satisfy the boundary conditions (20) and (21) are given by

$$f_0(y) = 1 + D_3 e^{-m_3 y} + D_2 e^{-m_1 y} \tag{22}$$
$$f_1(y) = 1 + D_7 e^{-m_4 y} + D_4 e^{-m_3 y} + D_5 e^{-m_2 y} + D_6 e^{-m_1 y} \tag{23}$$
$$g_0(y) = e^{-m_1 y} \tag{24}$$
$$g_1(y) = -D_1 e^{-m_2 y} + D_1 e^{-m_1 y} \tag{25}$$

where

$$m_1 = \frac{\Pr + \sqrt{\Pr^2 + 4\Pr(F+\phi)}}{2}, \qquad m_3 = \frac{1}{2}(1+\sqrt{1+4N}),$$
$$m_2 = \frac{\Pr + \sqrt{\Pr^2 + 4\Pr(F+\phi+n)}}{2}, \qquad m_4 = \frac{1}{2}(1+\sqrt{1+4(n+N)}),$$
$$D_1 = \frac{A\Pr m_1}{m_1^2 - \Pr m_1 - (F+\phi+n)\Pr}, \qquad D_2 = \frac{-Gr}{m_1^2 - m_1 - N},$$
$$D_3 = \frac{-(1+D_2+\phi_1 m_1 D_2)}{1+\phi_1 m_3}, \qquad D_4 = \frac{AD_3 m_3}{m_3^2 - m_3 - (N+n)},$$
$$D_5 = \frac{GrD_1}{m_2^2 - m_2 - (N+n)}, \qquad D_6 = \frac{AD_2 m_1 - GrD_1}{m_1^2 - m_1 - (N+n)},$$
$$D_7 = \frac{-(1+D_4+D_5+D_6) - \phi_1(D_4 m_3 + D_5 m_2 + D_6 m_1)}{1+\phi_1 m_4}$$

Substituting equations (22)-(25) in equations (14) and (15), we obtain the velocity and temperature distributions in the boundary layer as follows:

$$\begin{aligned} u(y,t) &= 1 + D_3 e^{-m_3 y} + D_2 e^{-m_1} + \varepsilon e^{nt}(1 + D_7 e^{-m_4 y} + D_4 e^{-m_3 y} \\ &\quad + D_5 e^{-m_2 y} + D_6 e^{-m_1 y}) \qquad (26) \\ \theta(y,t) &= e^{-m_1 y} + \varepsilon e^{nt}(-D_1 e^{-m_2 y} + D_1 e^{-m_1 y}) \qquad (27) \end{aligned}$$

Skin-friction at the wall is given by

$$C_{f_x} = \frac{\tau_w}{\rho U_0 V_0} = \left.\frac{\partial u}{\partial y}\right|_{y=0}$$
$$C_{f_x} = -(D_3 m_3 + D_2 m_1) - \varepsilon e^{nt}(D_7 m_4 + D_4 m_3 + D_5 m_2 + D_6 m_1) \qquad (28)$$

We calculate the heat transfer coefficient in terms of Nusselt number as follows:

$$Nu_x = x\frac{\left.\frac{\partial T}{\partial y^*}\right|_w}{T_w - T_\infty} \Rightarrow Nu_x/Re_x = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} \qquad (29)$$

$$Nu_x/Re_x = -m_1 + \varepsilon e^{nt} D_1(m_2 - m_1) \qquad (30)$$

where $Re_x = \dfrac{V_0 x}{\nu}$ is the Reynolds number.

## 4 Results and Discussion

The problem of the influence of radiation on magneto-hydrodynamic unsteady convective heat transfer past a semi-infinite vertical porous plate by perturbation technique is dealt. Based on these solutions, we have carried out numerical computations for the velocity and temperature for various values of the material parameters. The numerical values for skin-friction and Nusselt number are computed for various values of the parameters M, K, $\phi$,$\phi_1$, F, $Gr$ and $Pr$. These results are presented in Table 1. It is seen from the table that the effect of increasing values of M, K, $\phi$, $\phi_1$, F and Pr is to decrease skin-friction coefficient whereas increasing Grashof number increases skin-friction coefficient. Further, no effect of M, K, $\phi_1$ and $Gr$ is seen on Nusselt number. But Nusselt number decreases on increasing the values of $\phi$, F and Pr.

Table 1. Skin friction and nusselt number for various values of $fm, f, g$ & $Pr$ with $n = 0.1, t = 1, \varepsilon = 0.2$ and $a = 0.5$

| M | K | $\phi$ | $\phi_1$ | F | G | Pr | $C_f$ | $Nu/Re_x$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 2 | 0.7 | 0.82829 | -1.63299 |
| 1 | | | | | | | 0.77572 | -1.63299 |
| 3 | | | | | | | 0.73675 | -1.63299 |
| 5 | | | | | | | 0.72180 | -1.63299 |
| 2 | 1 | 1 | 1 | -1 | 2 | 0.7 | 0.84852 | -0.76866 |
| | | | | 0 | | | 0.77871 | -1.30943 |
| | | | | 3 | | | 0.72036 | -2.10586 |
| 2 | 1 | 1 | 1 | 1 | 0 | | 0.53619 | -1.63299 |
| | | | | | 3 | | 0.64356 | -1.63299 |
| | | | | | 5 | | 1.07303 | -1.63299 |
| | | | | | 10 | | 1.60988 | -1.63299 |
| 2 | 1 | 1 | 1 | 1 | 2 | 0.054 | 0.90075 | -0.35993 |
| | | | | | | 1 | 0.72509 | -2.07288 |
| | | | | | | 6.7 | 0.60419 | -8.92813 |
| 2 | 0.001 | 1 | 1 | 1 | 2 | 0.7 | 0.75727 | -1.63299 |
| | 0.01 | | | | | | 0.72690 | -1.63299 |
| | 0.1 | | | | | | 0.70891 | -1.63299 |
| 2 | 1 | 0 | 1 | 1 | 2 | 0.7 | 0.77871 | -1.30943 |
| | | 1 | | | | | 0.75093 | -1.63299 |
| | | 5 | | | | | 0.70204 | -2.47407 |
| 2 | 1 | 1 | 3 | 1 | 2 | 0.7 | 0.31576 | -1.63299 |
| | | | 5 | | | | 0.19991 | -1.63299 |

Figure 2 illustrates the effect of Grashof number $Gr$ on the velocity distribution. The numerical results show that the effect of increasing values of Grashof number leads to an increase in velocity. In addition, the curves show that the peak value of velocity increases rapidly near the wall as Grashof number increases, and then decays to the relevant free stream velocity. Figure 3 plots the velocity profiles against the span-wise coordinate y for different magnetic parameters. This illustrates that the velocity decreases as the existence of magnetic field becomes stronger. This conclusion agrees wish the fact that the magnetic field exerts retarding force on the free-convection flow. Figure 4 illustrates the effect of radiation on the velocity in the boundary layer. We note from this graph that there is a decrease in the velocity with the increase in radiation parameter F. The increase of the radiation parameter F leads to decrease the boundary layer thickness and to enhance the heat transfer rate in the presence of thermal buoyancy force.

The changes in velocity profile due to different permeability of the porous medium are plotted in Figure 5. This figure shows that when the permeability parameter $\phi_1$ increases from 0, the velocity increases gradually and it attains its maximum peak value when $\phi_1$=0.5. The horizontal velocity profile in the boundary layer for different values of time is depicted in Figure 6. It is observed that the horizontal velocity slowly attains the peak value close to the porous boundary and then it decreases till it reaches the minimum value at the end of the boundary layer for all the values of time. It is noticed that the velocity increases as the time increases. It is observed the Figure 7 that the velocity increases for the increasing values of $\varepsilon$ and reaches its maximum peak value for a larger $\varepsilon$.

The temperature profiles for different Prandtl numbers are given in Figure 8 which shows that, increasing values of Prandtl number implies the decrease in temperature profile. This leads to decrease the thermal boundary layer thickness. Figure 9 represents the temperature distribution for different values of radiation parameter F. It is observed that increase in the radiation parameter decreases the temperature distribution in the thermal boundary layer. Figure 10 has been plotted to depict the variation of temperature profiles for different values of heat source parameter $\phi$. From this graph, we observe that temperature decreases with increase in $\phi$ because when heat is absorbed, the buoyancy force decreases the temperature profile.

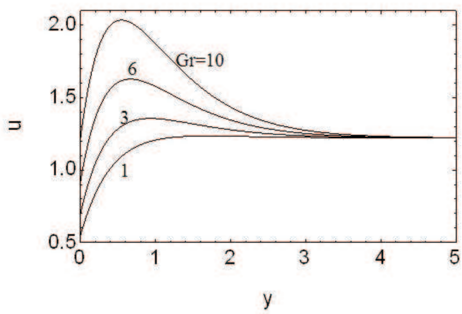


Fig. 2. Velocity profiles for different Grashof numbers with Pr=0.7, M=1, K=1,$\phi = 0.5$, F=1, n=0.1, t=1, $\varepsilon = 0.2$, A=1 and $\phi_1 = 0.3$

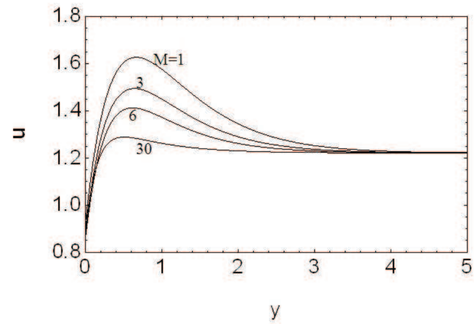


Fig. 3. Velocity profiles for different values of M with Gr=6, Pr=0.7, K=1, $\phi = 0.5$, F=1, n=0.1 , t=1, $\varepsilon = 0.2$, A=1 and $\phi_1 = 0.3$

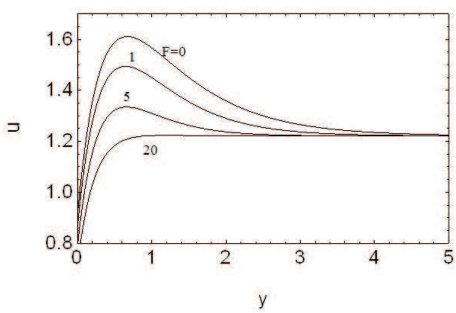


Fig. 4. Velocity profiles for different radiation parameters with Gr=6, Pr=0.7, K=1, $\phi$=0.5,M=3, n=0.1 , t=1, $\varepsilon = 0.2$, A=1 and $\phi_1$=0.3

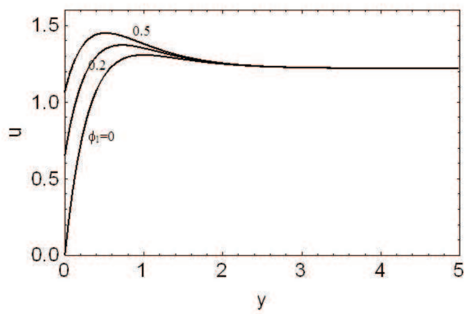


Fig. 5. Velocity profiles for different permeability of porous medium with Gr=6, Pr=1,K=1,$\phi$=0.5, M=3, n=0.1, t=1, $\varepsilon = 0.2$ and A=1

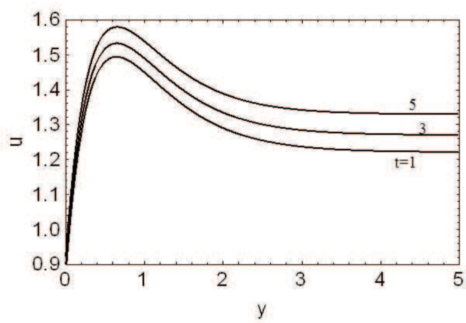


Fig. 6. Velocity profiles for different times with Gr=6, Pr=0.7, K=1, $\phi$=0.5, F=1, M=3, n=0.1, $\varepsilon = 0.2$, A=1 and $\phi_1$=0.3

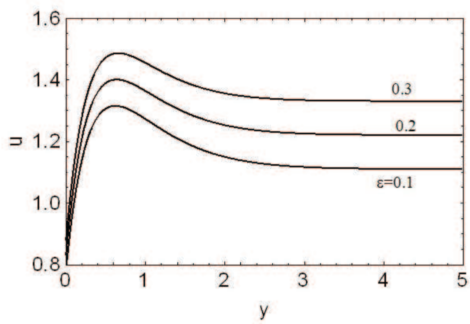


Fig. 7. Velocity profiles for different $\epsilon$ with Gr=6, Pr=1, K=1, $\phi$=0.5, M=3, n=0.1, t=1, A=1 and $\phi_1$=0.3

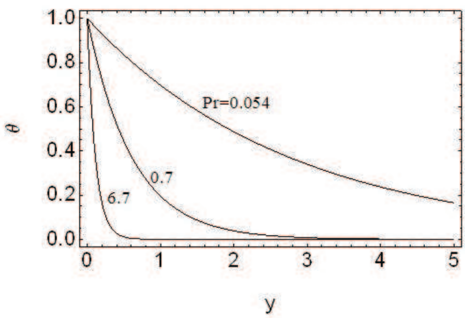


Fig. 8. Temperature profiles for different Prandtl numbers with $\phi$=1, F=1, K=1, n=0.1, t=1, $\varepsilon$=0.2 and A=0.5

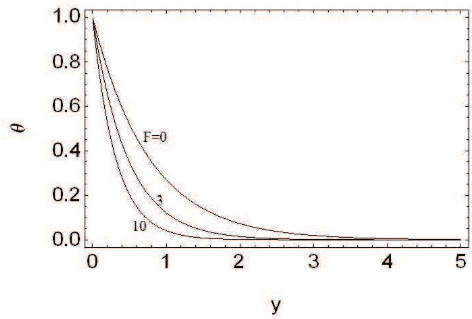


Fig. 9. Temperature profiles for different radiation parameters with Pr=0.7, $\phi$=1, K=1, n=0.1, t=1, $\varepsilon = 0.2$ and A=0.5

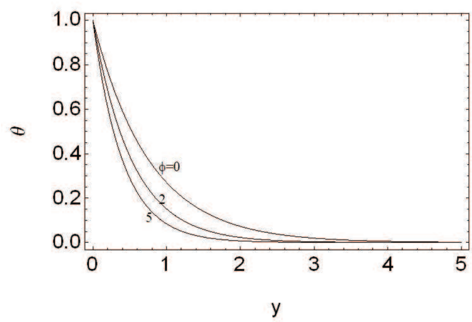


Fig. 10. Temperature profiles for different $\phi$ with Pr=0.7, F=1, n=0.1, t=1, $\varepsilon$=0.2 and A=0.5

## 5 Conclusions

The problem of the influence of radiation on magneto-hydrodynamic unsteady convective heat transfer past a semi-infinite vertical porous plate is studied and the solution is obtained by perturbation technique. Based on these solutions, numerical computations for various values of the material parameters are carried out. The fundamental parameters found to have an influence on the problem under consideration are magnetic field parameter, radiation parameter, porous permeability, heat source parameter, Grashof number and Prandtl number. It is found that the effect of magnetic parameter and radiation parameter reduces the velocity while the effect of porous permeability enhances it. For increasing values of the Grashof number, the velocity increases but it decreases on increasing the Prandtl number. The effect of increasing values of magnetic parameter, radiation parameter and Prandtl number tend to decrease the skin friction coefficient. An increase in radiation parameter and Prandtl number results in lowering the temperature steadily and hence a decrease in thermal boundary layer thickness is observed. Further, the temperature increases as $\varepsilon$ increases, whereas it decreases when $\phi$ increases.

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