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Pricing Writer-Extendable Call Options with Monte Carlo Simulation

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ABSTRACT

Writer-extendable option is an exotic option that can either be exercised at its initial maturity time or be extended to a future maturity time. Within the Black-Scholes environment, this study aims to price writer-extendable call options using the Monte Carlo simulation technique and compare the obtained prices with the closed-form pricing formula. Numerical examples are provided using the closed-form solutions and the Monte Carlo simulation via Euler scheme, which shows that the prices obtained via the latter are accurate.

Keywords: Black-Scholes, Monte Carlo simulation, writer-extendable.

1 INTRODUCTION

A derivative is a financial security whose value is derived from an underlying asset. There are many types of derivatives such as options, swaps, and futures or forward contracts. It is known as a derivative since an option's price is derived from the price of an underlying. The amount per share at which an option is traded is called option pricing. A contract in view of options trading is defined as a compromise by the writer either to but if it is a put, or to sell if it is a call, for a given asset at a prearranged price until a certain time.

The Black-Scholes [1] model gives a big impact to the financial studies. Following this model, many closed-form formulas for different types of options can be derived, such as writer-extendable options. The model assumes constant interest rate and volatility, no dividends are paid out during the life of the option, the returns are normally distributed, no transaction costs purchasing the option, and the market is efficient which cannot be predicted.

Extendable options can be categorized as either a holder-extendable or a writer-extendable option, where its maturity time can be extended. Options that are extended by their writer have no fee and they are more straightforward to price. The extendable options were first studied by [1]. Work [2] derived the closed-form formula for extendable options from an option holder and writer perspective, and [3] priced external writer-extendable option without premium when the option-writer extends the option. In [4], the closed-form formula for extendable options that can be extended n times was derived. While these studies provide closed-form solutions, [5] priced extendable options by applying the fast Fourier transform (FFT) method. Other studies extended the pricing of extendable options by including stochastic volatility [9, 10], jumps [12] under the Merton jump-diffusion model [13], and fractional jump process [14]. The extendable options also gained interest in many studies relating to real situations, such as [8] priced petroleum concessions using a mean-reverting framework including jumps for an extendable option, and [11] studied the features of extendable contracts for product development.

In this study, we consider the pricing problem of writer-extendable call options. The writer-extendable call option is an option that allows its writer to extend the maturity date when the option is at-the-money or out-of-money without any fees. The boundary condition that is satisfied by the writer-extendable call potion at initial maturity date T_1 is:

$$WC(S,K_1,T_1,K_2,T_2)=\begin{cases}\mathrm{C}(\mathrm{S},\mathrm{K}_2,\mathrm{T}_2-\mathrm{T}_1), & \mathrm{S}<\mathrm{K}_1,\\ \mathrm{S}-\mathrm{K}_1, & \mathrm{K}_1\leq \mathrm{S},\end{cases} \tag{1}$$

where S is the underlying asset price, r is the risk-free interest rate, $C(S,K_2,T_2-T_1)$ is the Black-Scholes [1] pricing formula for a vanilla call option which is defined as:

$$C(S,K_2,T_2-T_1)=SN(d+\sigma\sqrt{T_2-T_1})\ -\ K_2e^{-r(T_2-T_1)}\,N(d), \tag{2}$$

where:

$$d\ =\frac{\ln\left(\frac{S}{K_2}\right)+\left(r-\frac{\sigma^2}{2}\right)(T_2-T_1)}{\sigma\sqrt{T_2-T_1}}. \tag{3}$$

The closed-form solution for a writer-extendable call option as given by [2] is:

$$\begin{aligned}&WC(S,K_1,T_1,K_2,T_2)\\&\quad=\ C(S,K_1,T_1)\ +\ S(0)N(d_2+s_2,-d_1-s_1,-\rho)\ -\ K_2e^{-rT_2}\,N(d_2,-d_1,-\rho),\end{aligned} \tag{4}$$

where:

$$d_1=\frac{\ln\left(\frac{S(0)}{K_1}\right)+\left(r-\frac{\sigma^2}{2}\right)T_1}{\sigma\sqrt{T_1}}, \tag{5}$$

and:

$$d_2=\frac{\ln\left(\frac{S(0)}{K_2}\right)+\left(r-\frac{\sigma^2}{2}\right)T_2}{\sigma\sqrt{T_2}}, \tag{6}$$

This study aims to price the writer-extendable call option under the Black-Scholes [1] model using the Monte Carlo simulation, and to compare with the closed-form pricing formula. The Monte Carlo simulation is used to solve option valuation issues and the technique simulates the process by generating the returns on the underlying asset [6]. The rest of the paper is organized as follows.

Section 2 briefly describes the methodology, and Section 3 documents the numerical examples. Section 4 concludes the paper.

2 PRICING WITH MONTE CARLO SIMULATION

Monte Carlo simulation [15], also known as probability simulation, is a technique that is used to study the impact of risk and uncertainties in a model. This technique relies on repeated random sampling where it provides generally approximate solutions. This technique separates the time interval into small time steps and randomly sampling possible paths for the variable, then this process will be repeated to predict all possible future outcome of the random variable.

A writer-extendable call can be exercised at its initial maturity T_1 or extended to maturity T_2 if out-of-money at T_1. Suppose the process that an underlying asset $S(t)$ follows is a geometric Brownian motion as such:

$$dS(t) = rS(t)dt + \sigma S(t)dW(t), \tag{7}$$

where W is a standard Brownian motion, σ is the volatility, and r is the risk-free interest rate. Then, the process (7) can be expressed as follows:

$$S(T) = S(t)e^{\left(r-\frac{\sigma^2}{2}\right)(T-t)+\sigma\sqrt{T-t}Z}, \tag{8}$$

where Z is a standard normal random variable. Consequently, we have:

$$\ln S(T) \sim \phi\left[\ln S(t) + \left(r - \frac{\sigma^2}{2}\right)(T-t), \sigma\sqrt{T-t}\right], \tag{9}$$

which shows that $\ln S(t)$ is normally distributed with mean $\ln S(t) + \left(r - \frac{\sigma^2}{2}\right)(T-t)$ and variance $\sigma^2(T-t)$; therefore, $S(t)$ is log-normally distributed.

The Monte Carlo estimator for the price of a writer-extendable call option can be written as:

$$WC(t, \hat{s}_{T_1}) = \frac{e^{-r(T_1-t)}}{n}\sum_{j=1}^{n}\left[w_{T_{1,j}} + e^{\hat{s}^j_{T_1}} - K_1\right], \tag{10}$$

where $w_{T_{1,j}}$ is the vanilla call option price with extended strike price K_2, and extended maturity T_2 for the j^{th} path, while n is the number of simulations. The asset paths are evaluated using the Euler scheme:

$$S_{j+1} = S_j + \left(r - \frac{\sigma^2}{2}\right)\Delta t + v\Delta W_j, \tag{11}$$

where $t_j = j\Delta t$, $\Delta W_j = W_{t_{j+1}} - W_{t_j} = Z\sqrt{\Delta t}, Z \sim N(0,1)$, and $t = t_0 < t_1 < \cdots < t_z = T$ is the division of the interval $[t, T]$ for the time where there are z equal segments. The Box-Muller algorithm is used to generate random variable for normal distribution with mean 0 and variance 1.

3 NUMERICAL EXAMPLES

This section documents numerical illustrations of the Monte Carlo simulation technique for pricing writer-extendable call options, in addition to the closed-form pricing formula. The computations were implemented in C++ Programming and conducted on an AMD A9-9420 RADEON R5 \@ 3.00GHz machine, running under Windows 10 Home with 4.00GB RAM. Table 1 documents the input parameters.

Table 1: Parameters input

Parameters	**Value**
Strike price, K_1	100
Maturity time, T_1	1
Extended strike price, K_2	105
Extended maturity time, T_2	1.5
Risk-free interest rate, r	0.08
Volatility, σ	0.25

To measure the accuracy of the Monte Carlo simulation, we measure the absolute error, relative error, and percentage relative error, defined as follows, respectively:

$$Absolute\ Error = |WC_{exact} - WC_{mcs}|, \tag{12}$$

$$Relative\ Error = \left|\frac{WC_{exact} - WC_{mcs}}{WC_{exact}}\right|, \tag{13}$$

$$Percentage\ Relative\ Error = \left|\frac{WC_{exact} - WC_{mcs}}{WC_{exact}}\right| \times 100\%, \tag{14}$$

where WC_{exact} is the price from Equation (4), and WC_{mcs} is the price obtained from Equation (10). The prices obtained via closed-form solution and the Monte Carlo simulation are tabulated in Table 2.

Table 2: Writer-extendable call option prices

Underlying Asset S	**Closed-Form Formula**	**MCS**	**Bias**
80	4.6892	4.6921	0.002870
90	8.9253	8.9285	0.003221
100	14.7450	14.7497	0.004700
110	21.9245	21.9292	0.004700
120	30.1361	30.1424	0.006300

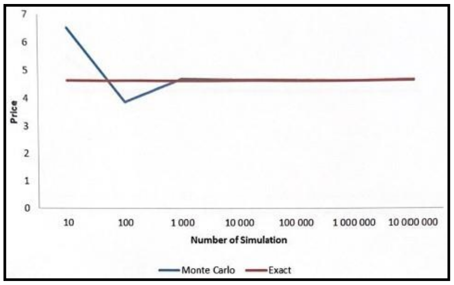
Table 3: Error measurements for writer-extendable call option prices

Underlying Asset S	**Absolute Error**	**Relative Error**	**Percentage Error**
80	0.002870	0.000612	0.061206
90	0.003221	0.000361	0.036083
100	0.004700	0.000319	0.031873
110	0.004700	0.000214	0.021436
120	0.006300	0.000209	0.020905

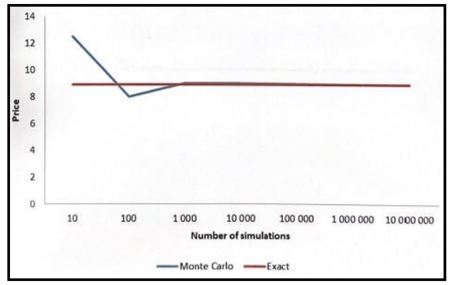
It can be seen from Table 2 that the biases are relatively small between the prices from closed-form formula and the Monte Carlo simulation (MCS). Error measurements tabulated in Table 3 also show that the percentage errors are less than 1% which implies the Monte Carlo simulation is an accurate technique in pricing options with extended maturity.

Additionally, we test for convergence of the Monte Carlo simulation for $S = \{80,90,100,110,120\}$. The plots are given in Figures 1(a), 1(b), 1(c), 1(d), and 1(e). It can be observed that as the number of simulations increases, the prices of the writer-extendable call options converge to the price from the closed-form formula.

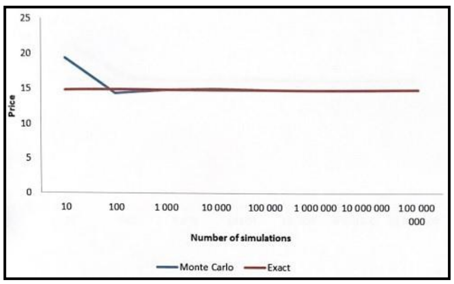
According to [7], the Euler scheme has fewer terms and the same order of weak convergence. Another observation is that as the number of simulations increases, the computational time increases as well. Nevertheless, the price is more accurate.



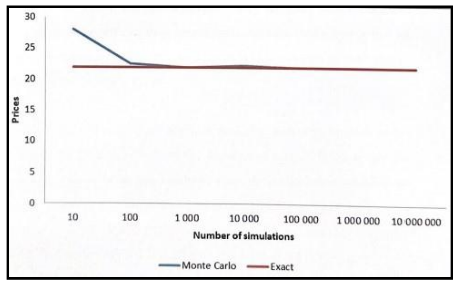
(a) Convergence for $S = 80$



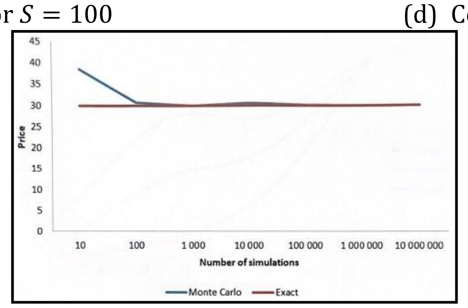
(b) Convergence for $S = 90$



(c) Convergence for $S = 100$



(d) Convergence for $S = 110$



(e) Convergence for $S = 120$

Figure 1: Writer-extendable call price convergence using Monte Carlo simulation

4 CONCLUSION

In this study, we derive the closed-form formula for a call option that may be extended by the option writer under the Black-Scholes model. The closed-form formula is as given in [2]. We also model the writer-extendable call options with the Monte Carlo simulation via Euler. Numerical results show that the Monte Carlo simulation produces high accuracy of approximated prices as the number of simulations increases more than 1,000.

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