App. Math. and Comp. Intel., Vol. 1 (2012) 1–11
http://amci.unimap.edu.my

Unsteady stagnation point flow and heat transfer over a stretching/shrinking sheet with prescribed surface heat flux

M. Suali[a,*], **N. M. A. Nik Long**[a,b], **and A. Ishak**[c]

[a,b] *Department of Mathematics, Faculty of Science*
Universiti Putra Malaysia
43400 UPM Serdang, Selangor, Malaysia.

[b] *Institute for Mathematical Research*
Universiti Putra Malaysia
43400 UPM Serdang, Selangor, Malaysia.

[c] *School of Mathematical Sciences*
Universiti Kebangsaan Malaysia
43600 UKM Bangi, Selangor, Malaysia.

Received: 23 December 2011; Revised: 4 May 2012; Accepted: 14 July 2012

Abstract: *An analysis is carried out to study the unsteady two dimensional stagnation point flow and heat transfer over a stretching/shrinking sheet with prescribed surface heat flux. The governing partial differential equations are converted into nonlinear ordinary differential equations using similarity variables, and solved numerically. The effects of the unsteadiness parameter A, stretching/shrinking parameter ε and Prandtl number Pr on the flow and heat transfer characteristics are studied. It is found that the skin friction $f''(0)$ and the local Nusselt number $\dfrac{1}{\theta(0)}$ increase as the the unsteadiness parameter A increases. Moreover, the velocity and temperature increase as ε and Pr increase.*

Keywords: *Stagnation flow, stretching/shrinking sheet, heat transfer, heat flux, similarity variables, dual solutions.*

PACS: *47.15Cb, 02.60.Lj*

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1 Introduction

The study of flow and heat transfer over a stretching/shrinking sheet is an important problem in many engineering processes with application in industries such as extrusion of plastic

*Corresponding Author: melqv87@yahoo.com (M. Suali)

sheets, wire drawing, hot rolling and glass fiber production. Sakiadis [1, 2] performed the pioneering work of boundary layer flow over a continuous moving surface. Later, his work was verified experimentally by Tsou et al. [3]. Crane [4] studied the flow over a linearly stretching sheet in an ambient fluid and gave a closed form similarity solution for the steady two-dimensional problem. Gupta and Gupta [5] extended the work of Crane [4] by investigating the effect of mass transfer on a stretching sheet with suction or blowing. On the other hand, the stretching boundary problem [2] was extended by Wang [6] to a three-dimensional flow. Mahapatra and Gupta [7] considered the combination of both stagnation flow and stretching surface, and extended to the oblique stagnation flow by Lok et al. [8]. Carragher and Crane [9] investigated the heat transfer flow over a stretching surface in the case of temperature difference between the surface and the ambient fluid is proportional to the power of distance from a fixed point.

Dutta et al. [10] and Grubka and Bobba [11] studied the temperature field in the flow over a stretching surface subject to a uniform heat flux, while Elbashbeshy [12] considered the case of stretching surface with a variable surface heat flux. Lin and Chen [13] presented an exact solution of heat transfer from a stretching surface with a variable heat flux. Magyari and Keller [14, 15] investigated the heat and mass transfer in the boundary layers on an exponentially stretching continuous surface and studied the exact solutions for self-similar boundary-layer flows induced by permeable stretching surface. A few works have been done on flow induced by shrinking sheet. Miklavcic and Wang [16] found that the fluid is stretched toward a slot and the flow characteristic is different from the stretching case. Fang [17] and Fang et al. [18] investigated the flow induced by a shrinking sheet with a constant velocity distribution. Hayat et al. [19] and Sajid [20] extended Fang [17] and Fang et al.'s [18] work with magnetohydrodynamic and rotating flows of viscous fluid. Further, Wang [21] generalized the flow over a shrinking sheet with a stagnation flow. Teipel [22] studied the heat transfer behavior for the three-dimensional unsteady stagnation point flow which focused on the effect of the unsteadiness parameter. Later, Wang [23] investigated the unsteady oblique stagnation point. Recently Bhattacharyya [24] investigated the dual solutions in unsteady stagnation point flow over a shrinking sheet.

The presentation of the paper is in the following order. Section 1 deals with the introduction and some background of the problem. Section 2 discusses the mathematical model and formulation of the flow over a stretching and shrinking sheet with prescribed surface heat flux. Numerical results and discussion are given in Section 3. Conclusion is given in Section 4.

2 Mathematical formulation

Consider the unsteady stagnation point flow over a stretching or shrinking sheet immersed in an incompressible viscous fluid of ambient temperature T_∞. It is assumed that the free stream velocity is in the form $U_\infty(x,t) = ax(1-\lambda t)^{-1}$, the sheet is stretched with velocity $U_w(x,t) = bx(1-\lambda t)^{-1}$ and the surface heat flux is $q_w(x,t) = cx(1-\lambda t)^{-1}$. The x-axis runs along the sheet and y-axis is measured normal to it. These assumptions along with the boundary-layer approximations and neglecting the viscous dissipation, the governing equations are given by Fang et al. [25]

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \qquad (1) \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= \frac{\partial U_\infty}{\partial t} + U_\infty \frac{\partial U_\infty}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \\ \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} \qquad (2) \end{aligned}$$

Now consider the following boundary conditions

$$\begin{aligned} &v = 0, u = U_w, \frac{\partial T}{\partial y} = \frac{-q_w}{k} \quad \text{at} \quad y = 0 \\ &\text{and} \quad u \to U_\infty, T \to T_\infty \quad \text{as} \quad y \to \infty \end{aligned} \tag{3}$$

where u and v are velocity components in x and y directions, respectively, ν is kinematic viscosity, α is thermal diffusivity and T is fluid temperature. Introducing the following similarity transformations

$$\left.\begin{aligned} \psi &= \left(\frac{a\nu}{1-\lambda t}\right)^{1/2} x f(\eta) \\ \eta &= \left(\frac{a}{\nu(1-\lambda t)}\right)^{1/2} y \\ \theta(\eta) &= \frac{k(T-T_\infty)}{q_w}\left(\frac{U_\infty}{\nu x}\right)^{\frac{1}{2}} \quad , \end{aligned}\right\} \tag{4}$$

where η is the similarity variable and ψ is the stream function defined as $u = \dfrac{\partial \psi}{\partial y}$ and $v = -\dfrac{\partial \psi}{\partial x}$. Thus we have

$$u = \frac{ax}{1-\lambda t} f'(\eta) \quad \text{and} \quad v = -\left(\frac{a\nu}{1-\lambda t}\right)^{\frac{1}{2}} f(\eta) \tag{5}$$

where prime denotes differentiation with respect to η. It is easy to see that (5) satisfies Eq. (1) identically. Substituting (4) into Eqs. (2) and (2) yield the nonlinear differential ordinary equations

$$f''' + ff'' + 1 - f'^2 + A\left(1 - f' - \frac{1}{2}\eta f''\right) = 0 \tag{6}$$

$$\frac{1}{Pr}\theta'' + f\theta' - f'\theta - A\left(\theta + \frac{\eta}{2}\theta'\right) = 0 \tag{7}$$

and the boundary conditions (3) becomes

$$f(0) = 0, f'(0) = \frac{b}{a} = \varepsilon, \theta'(0) = -1, \qquad \theta(\infty) \to 0, f'(\infty) \to 1 \tag{8}$$

where $\varepsilon(= \dfrac{b}{a})$ is the stretching/shrinking parameter and the free stream velocity parameter, $Pr = \dfrac{v}{\alpha}$ is the Prandtl number and $A = \dfrac{\lambda}{a}$ is the unsteadiness parameter. The quantities of physical interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as

$$C_f = \frac{\tau_w}{\rho {U_\infty}^2/2}, Nu_x = \frac{x q_w}{k(T_w - T_\infty)} \tag{9}$$

where the surface shear stress τ_w and the surface heat flux q_w are defined as

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

with μ and k being the dynamic viscosity and thermal conductivity, respectively. Substituting (4) into (9) we obtain

$$\frac{1}{2}C_f Re_x^{1/2} = f''(0) \frac{Nu_x}{Re_x^{1/2}} = \frac{1}{\theta(0)}$$

where $Re_x = \dfrac{U_\infty x}{\nu}$ is the local Reynolds number. It should be noticed that Eqs. (6) and (7) are reduced to those of Wang [21] and Nik Long et. al [26] when $A = 0$ (steady-state flow) and $A = 0.1$, respectively.

3 Results and discussion

Table 1: Values of $f''(0)$ for different values of ε

ε	Wang [21]	Nik Long et. al [26]	Present	
	A=0	A=0.1	A=0	A=0.1
4		-7.130017	-7.086378	-7.130017
3			-4.276545	-4.308713
0.2	1.05113	1.072329	1.051130	1.072329
0.1	1.14656	1.171193	1.146561	1.171193
-0.2			1.373886	1.410656
-0.5	1.49567	1.549006	1.495672	1.549006
-1.15	1.08223		1.082232	1.255264

Table 2: Values of $\dfrac{1}{\theta(0)}$ for different values of ε when $A = 0$ and $Pr = 1.0$

ε	Wang [21]	Present
4		2.116738
3		1.870671
0.2	0.91330	0.913303
0.1	0.86345	0.863452
-0.2	0.50145	0.501448
-0.5	0.50145	0.501448
-1.15	-0.297995	-0.2979953

The nonlinear ordinary differential Eqs. (6) and (7) subject to the boundary conditions (8) was solved numerically using the shooting method where we convert the boundary value problem into an initial value problem. The results are given to carry out a parametric study showing the influence of the non-dimensional parameters, namely the unsteadiness parameter A, stretching/shrinking parameter ε and Prandtl number Pr. For the validation of the numerical results, the values of $f''(0)$ and $\dfrac{1}{\theta(0)}$ for the case $A = 0$ (steady state flow) and $A = 0.1$ with $Pr = 1.0$ and $Pr = 0.7$, respectively have been considered and compared with those of Wang [21] and Nik Long et. al [26]. The quantitative comparison are shown in Tables 1 and 2 for the values of $f''(0)$ and $\dfrac{1}{\theta(0)}$, respectively for different values of ε, and found that they are in a favorable agreement.

We observed that, when the value of the unsteadiness parameter $A = 0.01$, it has dual solutions for $-1.2536 \leq \varepsilon \leq -1$, the solution is unique for $\varepsilon > -1$ and there is no solution for $\varepsilon < -1.2536$. Whereas, when the unsteadiness parameter $A = 0.1$ and $A = 0.3$, the dual

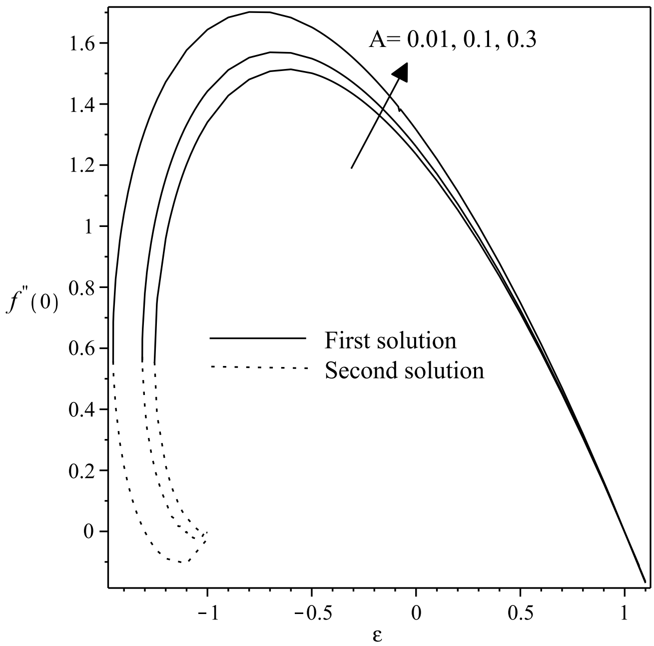

Figure 1: Skin friction coefficient $f''(0)$ as a function of ε for different values of A when $Pr = 0.7$

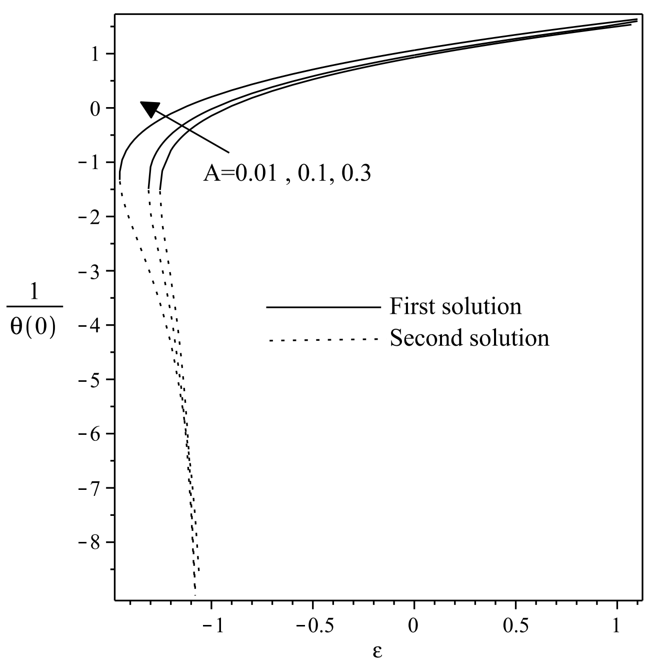

Figure 2: Local Nusselt number $\dfrac{1}{\theta(0)}$ as a function of ε for different values of A when $Pr = 0.7$

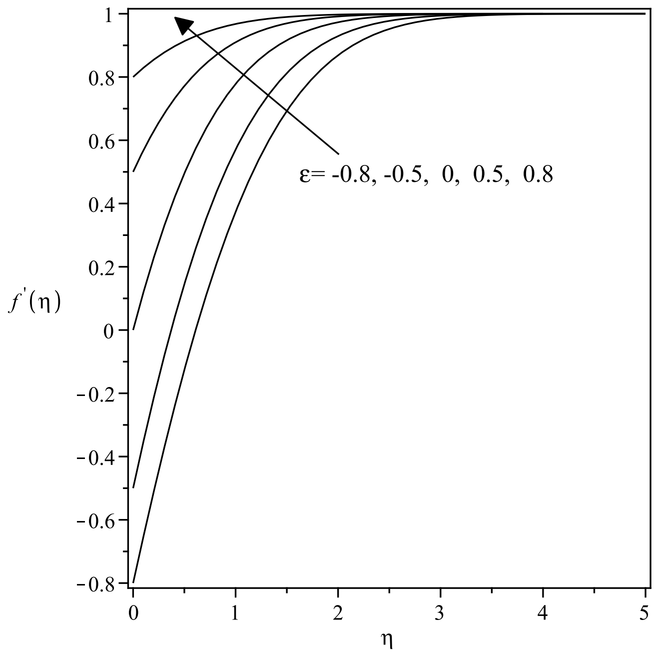

Figure 3: Velocity profiles $f'(\eta)$ for different values of ε when $A = 0.01$ and $Pr = 0.7$

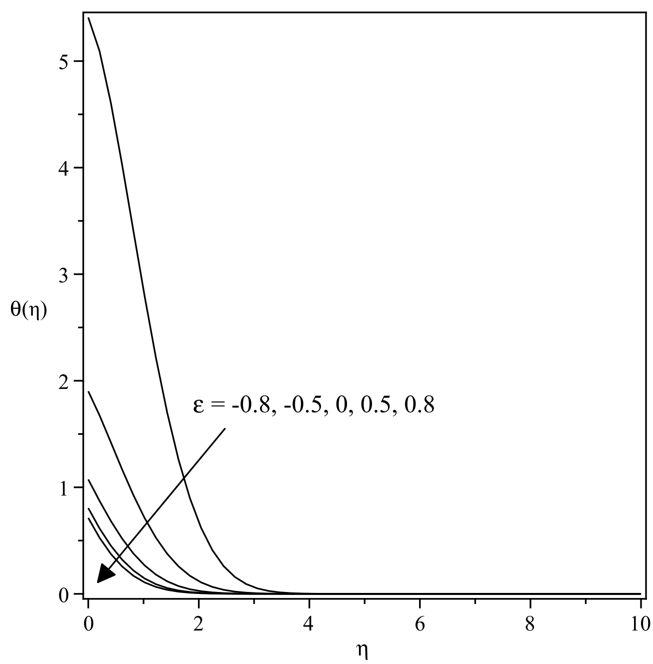

Figure 4: Temperature profiles $\theta(\eta)$ for different values of ε when $A = 0.01$ and $Pr = 0.7$

solutions exist for $-1.3118 \leq \varepsilon \leq -1$ and $-1.4520 \leq \varepsilon \leq -1$, respectively. The solution

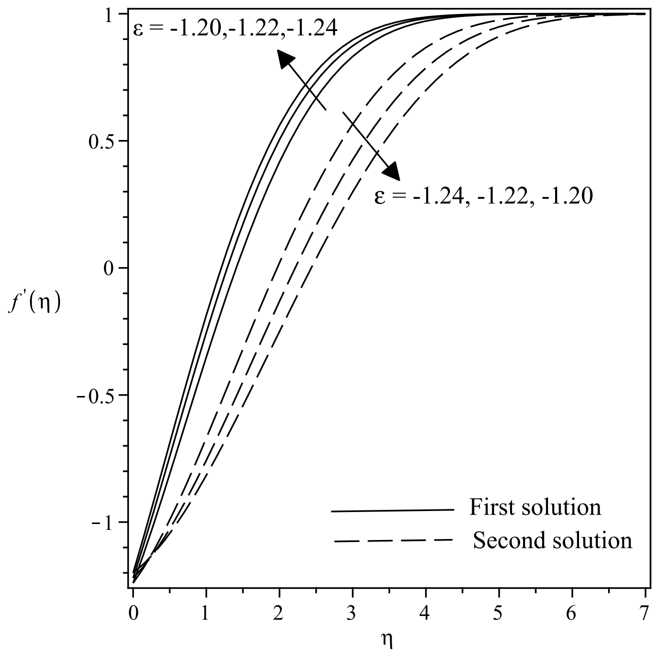

Figure 5: The dual solutions of velocity profiles $f'(\eta)$ for different values of ε when $A = 0.01$ and $Pr = 0.7$

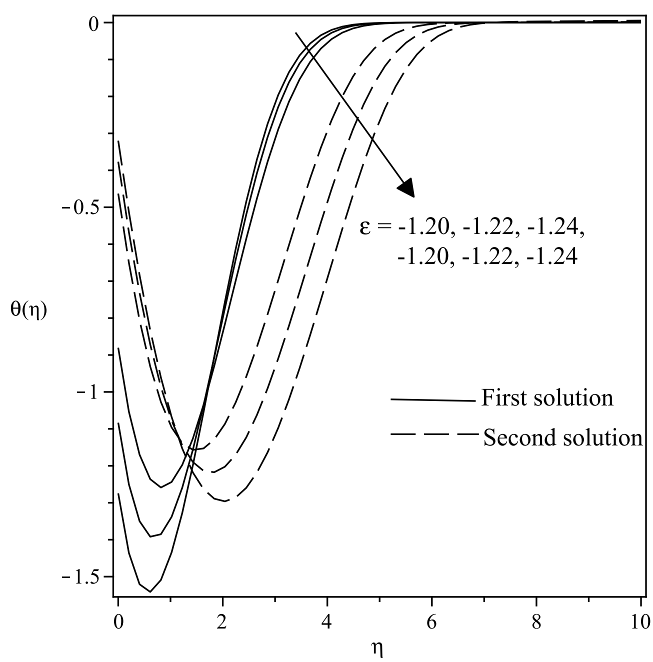

Figure 6: The dual solutions of temperature profiles $\theta(\eta)$ for different values of ε when $A = 0.01$ and $Pr = 0.7$

for both values of A is unique for $\varepsilon > -1$ and there are no solution for $\varepsilon < -1.3118$ and

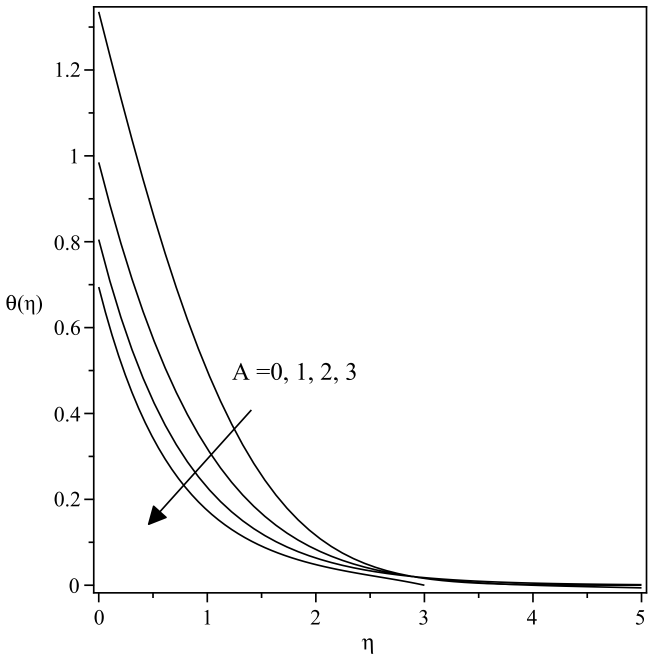

Figure 7: Temperature profiles $\theta(\eta)$ for different values of A when $\varepsilon = 0.1$ and $Pr = 0.7$

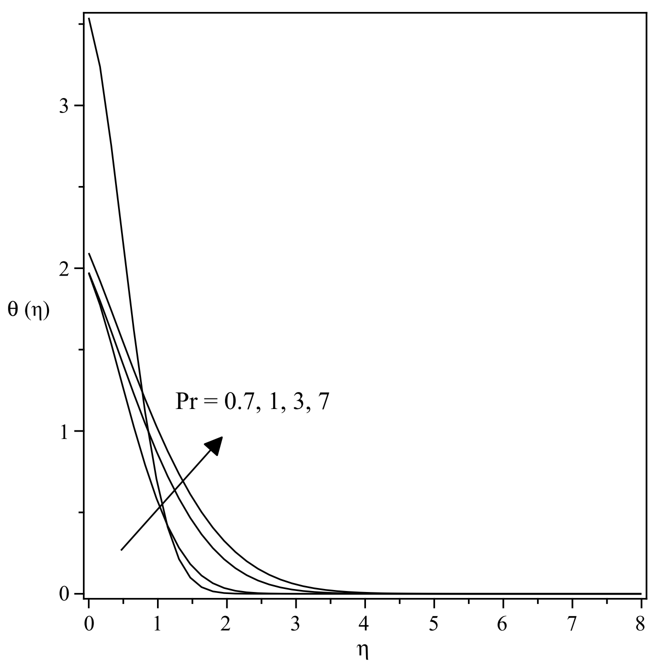

Figure 8: The temperature profiles $\theta(\eta)$ for different values of Pr when $A = 0.01$ and $\varepsilon = -0.5$

Table 3: Range of ε for unique solution,dual solutions and non-existence of the solution for different values of A

A	Unique solution	Dual solutions	Non-existence
0.01	$(-\infty, -1.2536)$	$[-1.2536, -1.0000]$	$(-1, \infty)$
0.1	$(-\infty, -1.3118)$	$[-1.3118, -1.0000]$	$(-1, \infty)$
0.3	$(-\infty, -1.4520)$	$[-1.4520, -1.0000]$	$(-1, \infty)$

$\varepsilon < -1.4520$, respectively. These results are presented in concise form in Table 3. From the table, it can be concluded that the solution exists up to a critical value (ε_c) of ε, which depends on A. Based on our computations, $\varepsilon_c = -1.2536$, -1.3118 and -1.4520 for $A = 0.01$, 0.1 and 0.3, respectively. Figures 1 and 2 show the relationship between the skin friction coefficient and the local Nusselt number with different values of unsteadiness parameters A when $Pr = 0.7$. As seen in these figures, as the unsteadiness parameter A increases, the skin friction $f''(0)$ and local Nusselt number $\dfrac{1}{\theta(0)}$ increase as well. The samples of velocity and temperature profiles are given in Figures 3-8, respectively. These figures show the boundary conditions (8) for Eqs. (6) and (7) are satisfied and approached infinity asymptotically, which support the numerical results presented in Figures 1 and 2. Figures 3 and 4 show the velocity and temperature $f'(\eta)$ and $\theta(\eta)$ for different values of ε. It is seen that the velocity increases as ε increases. However, the surface temperature decreases as ε increases. Figures 5 and 6 show the dual solutions of velocity and temperature profiles for different values of ε. These figures exhibit the existence of the dual solution for the dealt problem. It is observed that for the dual solutions of velocity and temperature fields, the boundary layer thickness for the first solution is smaller than the second solution. Meantime, the temperature profiles for different values of A are presented in Figure 7. These figure shows the temperature decreases monotonously with increasing values of A. The effect of Pr on the temperature profile is illustrated in Figure 8 for some values of Pr when $A = 0.01$ and $\varepsilon = -0.5$, which show the temperature inside the boundary layer increases with Pr. However, the surface temperature $\theta(0)$ decreases, which increases the local Nusselt number $\dfrac{1}{\theta(0)}$. Thus, the heat transfer rate at the surface increases as Pr increases.

4 Conclusion

A numerical study is performed for the problem of unsteady two-dimensional stagnation point flow and heat transfer over a stretching and shrinking sheets with prescribed heat flux. The similarity transformation is employed to reduce the partial differential equations into non-linear ordinary differential equations. The effects of the governing parameters A, ε and Pr on the fluid flow and heat transfer characteristics was discussed and the numerical results obtained are comparable well with the previously reported cases. The numerical results indicated that the parameter A increases the skin friction coefficient $f''(0)$ and the local Nusselt number $\dfrac{1}{\theta(0)}$. Moreover, the velocity and temperature increase as the values ε and Pr increase but leads to a decrease in surface temperature.

Acknowledgments

The second author would like to thanks Ministry of Higher Education (MOHE) for the research grant FRGS, project no: 03-11-08-670FR.

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