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Multi-Attribute Decision-Making Based on Picture Fuzzy Einstein Operator and The TOPSIS Method

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ABSTRACT

Picture Fuzzy Sets (PFSs) denote the extension of conventional fuzzy sets, which capture a broader spectrum of human opinions, encompassing responses such as acceptance, neutrality, rejection, and hesitation. This wider range of responses cannot be accurately accommodated within fuzzy sets as well as intuitionistic fuzzy sets framework. In the realm of Multiple Attribute Group Decision-Making (MAGDM) methods, attributes frequently exhibit conflicts, uncertainties, imprecisions, as well as a lack of commensurability. To tackle the complexities inherent in MAGDM, the Technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS) method has demonstrated its effectiveness. This method is employed in a compromise ranking approach founded on aggregation functions that showcase closeness to the reference points. This study's goal is to instigate a fresh approach to aggregation, referred to as the Picture Fuzzy Einstein Weighted Averaging Distance-based TOPSIS (PFEWAD-TOPSIS) method. To validate the effectiveness of this method in addressing MAGDM problems, a detailed example is conducted.

Keywords: Picture fuzzy sets; Aggregation operator; TOPSIS; Einstein operational rule; Multiple-attribute group decision-making

1 INTRODUCTION

Picture Fuzzy Set (PFS), established by Cuong & Kreinovich [1], refers to a generalization with regard to the intuitionistic fuzzy set (IFS) concept suggested by Atanassov [13]. It provides an alternative approach and serves as a potential tool for representing the preferences of DM as well as accurately characterizing the membership function when decision makers' opinions are uncertain or lacking expertise. The PFS is distinguished by its inclusion of a membership degree, hesitancy degree, non-membership degree, including refusal degree, which is not typically found in fuzzy sets [14] and IFS [13].

Recently, many researchers have studied PFS and its applications. For example, Wei [8] defined some picture fuzzy aggregation operators motivated by the intuitionistic fuzzy number [34;35]. A study by Wang et al [3] formulated certain geometric aggregation techniques within PFSs and employed them to address the Multiple Attribute Group Decision-Making (MAGDM) problem. Here, Wang and Li [22] established a set of advanced operators, including generalized picture hesitant fuzzy prioritized weighted averaging as well as generalized picture hesitant fuzzy prioritized weighted geometric operators, to describe multiple criteria decision-making (MCDM) challenges. In a related study, Wang, Peng, and Wang [17] established a decision-making framework for ranking the risks associated with energy performance contracting projects within a picture fuzzy environment. Meanwhile, Ashraf et al [23] developed a series of picture fuzzy weighted geometric aggregation operators tailored for addressing MAGDM problems. In addition, Liu and Zhang [20] introduced the concept of a picture fuzzy linguistic set as well as established the picture fuzzy linguistic weighted arithmetic averaging operator, utilizing Archimedean operations to address group decision-making scenarios involving multiple criteria. Apart from that, Zeng et al [28] expanded the concepts of ordered weighted averaging, weighted averaging, as well as hybrid averaging operators to the domain of picture linguistic representations, thoroughly exploring their characteristics. Additionally, Zhang et al [31] delved into the linguistic picture aggregation operator together with its practical application in group decision-making. Furthermore, recent works by authors like Xian et al [25], Simic et al [29], as well as Kutlu Gündoğdu et al [6] have shown a strong interest in developing aggregation operators specifically designed for solving decision-making challenges in PFSs environments. Consequently, PFS offers greater adaptability when dealing with scenarios marked by vagueness, uncertainty, as well as imprecision.

The TOPSIS method, originally introduced by Hwang and Yoon [4], has demonstrated its effectiveness, as highlighted by Si et al [26]. The TOPSIS methods are capable of representing uncertainty arising from imprecise assessments of alternatives as well as attribute weights. Their fundamental concept revolves around selecting an alternative nearest to the positive ideal solution as well as farthest from the negative ideal solution. Here, TOPSIS gains particular significance when decision-makers express their preferences employing a broader range of options, such as "yes," "abstain," "no," as well as "refusal." PFS offers a suitable framework to accommodate such decision-making scenarios. Consequently, it's evident that extending TOPSIS with PFN is a natural progression, and this approach finds widespread application in the existing literature. Under these, some researchers, such as Ashraf et al [23], presented the TOPSIS method with PFNs for group decision-making circumstances. Torun and Gördebil [11] utilized the picture fuzzy TOPSIS technique to assess the satisfaction levels of citizens regarding public services. Wang et al [16] introduced the QUALIFLEX method relying on picture fuzzy TOPSIS to address the BEER project selection problem. Meanwhile, M Sarwar Sindhu et al [18] put forward a linear programming model to determine precise weights and create a modified similarity-based distance metric within a picture fuzzy environment, subsequently applying the picture fuzzy TOPSIS method for evaluating Enterprise Resource Planning (ERP) systems. In addition, Kim and Van [15] introduced a TOPSIS-based Analytic Hierarchy Process (AHP) model to showcase the efficiency of evaluating urban development projects using multiple criteria, with input from numerous experts. Besides that, Xue-yang Zhang et al [31[presented picture fuzzy relative projection models for assessing the similarities as well as weights of various decision-maker groups, applying the picture fuzzy TOPSIS method to assess the locations of offshore wind power stations. In a separate study, Cao [7] employed the TOPSIS method and a bi-parametric picture fuzzy distance measure to construct an MCDM model for GSS. Moreover, Si et al [26] expanded the application of the TOPSIS method to address MCDM challenges by incorporating picture fuzzy information. Additionally, Jin et al [12] utilized the picture fuzzy TOPSIS approach for solving issues related to risk management. In a related vein, Liu et al [5] extended the TOPSIS method by introducing a generalized weighted distance measure, including the linguistic picture fuzzy entropy method for determining criterion weights. Table 1 shows some available PFS based on the TOPSIS method.

Table 1: A summary of the available PFS based on TOPSIS

References	Research Focus	Method(s)	Aggregation operator (yes/no). If yes, the name of the operators	Application Type
Ashraf, Mahmood, et al. (2019)	Air quality	TOPSIS	Generalized picture fuzzy weighted geometric (GPFWG) operator	Illustrative example
Torun and Gördebil (2020)	Satisfaction level	Picture fuzzy TOPSIS	No	Case study
Wang et al. (2020)	Hotel BEER project	Picture fuzzy TOPSIS and QUALIFEX	Laplace distribution picture fuzzy order weighted average (LD-PFOWA) operator	Case study
M Sarwar Sindhu et al. (2019)	Enterprise resource planning (ERP) system	Picture fuzzy TOPSIS and Linear programming (LP)	No	Illustrative example
Kim and Van (2021)	Urban development project	Picture fuzzy TOPSIS and AHP	No	Illustrative example
Xue-yang Zhang et al. (2018)	The location of offshore wind power station	Picture fuzzy TOPSIS and relative projection model	Similarity degree-picture fuzzy induced ordered weighted geometric (SD-PFIOWG) operator	Case study
Cao (2020)	Green supplier	Picture fuzzy TOPSIS and bi-parametric distance measure	No	Illustrative example
Si et al. (2020)	Tiger Reserve National Parks selection	Picture fuzzy TOPSIS	Picture fuzzy weighted averaging (PFWA) operator	Illustrative example
Jin et al. (2021)	Risk management	Covering-based picture fuzzy rough set (CPFRS) model and TOPSIS	No	Illustrative example
Liu et al. (2020)	Supplier selection	Linguistic picture fuzzy entropy and TOPSIS	No	Illustrative example

Based on the Table 1, the researcher used an aggregation operator with TOPSIS, mostly based on algebraic operational rules derived from the Archimedean t-norm and t-conorm (ATT) to conduct the combination process deal with the aggregation of criterion values [26]. Currently, the aggregation of information using operators has become a captivating research topic, garnering significant attention from researchers focusing on Einstein operations. The Einstein operation is a distinct mathematical operation employed to represent the intersections as well as unions of different fuzzy contexts. When dealing with intersections, the Einstein product serves as an effective alternative to the algebraic product. Conversely, for unions, the Einstein sum is a favorable alternative to the algebraic sum. These Einstein operations typically possess the capability to provide smooth estimations as well as optimal approximations that closely resemble the desired outcomes [9;19;28]. Thus, it is meaningful to extend the TOPSIS in the framework of PFS based on Einstein operations.

In this paper, a hybrid picture fuzzy Einstein aggregation operator with a technique for order preference by similarity to the TOPSIS method is developed; namely, the picture fuzzy Einstein weighted averaging distance based TOPSIS (PFEWAD-TOPSIS) method for picture fuzzy Multiple Attribute Group Decision Making (MAGDM) problems. Then, the utilization, feasibility, and efficiency of the proposed MAGDM approach is showcased by providing an illustrative example involving the beef supplier selection. The remainder of the paper is structured as follows. Section 2 recalls some fundamental concepts and operators related to PFSs and the traditional method of TOPSIS. In Section 3, extend TOPSIS method-based PFEWA. In Section 4, an illustrative example is provided, which is followed by the examination of the outcomes in Section 5. Finally, the last section gives main conclusion of this study.

2 PRELIMINARIES

This section develops several definitions as well as basic notations related to PFS and picture fuzzy under Einstein operations.

2.1 Picture Fuzzy Sets (PFSs)

This subsection provides a concise introduction to the fundamentals of PFSs. The notion of PFS, as proposed by Cuong [2], is established as an extension of the IFS concept established by Atanassov [13].

Definition 1 [2]. A PFS *P* on a universe *X* denotes an object in regard to the expression by:

$$P=\left\{\left\langle x,\mu_P(x),\eta_P(x),\nu_P(x)\right\rangle \middle| x\in X\right\}, \tag{1}$$

in which $\mu_P(x)$, $\eta_P(x)$, as well as $\nu_P(x)$ $\in[0,1]$ denote the degree of positive membership, neutral membership, as well as negative membership of x to set *P*, accordingly. Furthermore, $\mu_P(x)$, $\eta_P(x)$ and $\nu_P(x)$ for all $x\in X$ abides the condition given below.

$$0\le\mu_P(x)+\eta_P(x)+\nu_P(x)\le 1\ . \tag{2}$$

Moreover, the degree of refusal membership in accordance with *P* is expressed for all *x* as given below.

$$\pi_P(x)=1-(\mu_P(x)-\eta_P(x)-\nu_P(x)). \tag{3}$$

For convenience, the pair $P=(\mu_P(x),\eta_P(x),\nu_P(x))$ is known as a Picture Fuzzy Number (PFN).

When utilizing PFNs for real-world problem-solving, it becomes essential to rank these PFNs in the context of MADM problems. In this regard, Garg [10] introduced score as well as accuracy functions, which are defined below.

Definition 2 [10]. Let $P=(\mu_P,\eta_P,\nu_P)$ a PFN. Then, a score function *S* related to a PFN may be described as given below.

$$S(P)=\mu_P-\eta_P-\nu_P,\ S(P)\in[0,1]. \tag{4}$$

Definition 3 [10] . Assume $P=(\mu_P,\eta_P,\nu_P)$ a PFN. The accuracy degree function *H* of a PFN may be expressed as given below.

$$H(P)=\mu_P+\eta_P+\nu_P,\ H(P)\in[0,1]. \tag{5}$$

With the accuracy function *S* as well as the score function *H* in consideration, the relational order between two PFNs is articulated as follows.

Definition 4 [10]. Let $A=(\mu_A,\eta_A,\nu_A)$ as well as $B=(\mu_B,\eta_B,\nu_B)$ be two PFNs, $S(A)=\mu_A-\eta_A-\nu_A$ and $S(B)=\mu_B-\eta_B-\nu_B$ denote the scores of *A* and *B*, accordingly, as well as let $H(A)=\mu_A+\eta_A+\nu_A$ and $H(B)=\mu_B+\eta_B+\nu_B$ denote the accuracy degrees of *A* and *B*, accordingly. Therefore, the comparison rules of *A* and *B* are given as given below.

(i) If $S(A)>S(B)$, then $A>B$,
(ii) If $S(A)<S(B)$, then $A<B$,
(iii) If $S(A)=S(B)$, then
  (a) $H(A)>H(B)$, then $A>B$,
  (b) $H(A)<H(B)$, then $A<B$,
  (c) $H(A)=H(B)$, then $A=B$.

2.2 Einstein operations of PFSs

The operational principles of PFSs that rely on Einstein operations are outlined as follows.

Definition 5 [24]. Let $A=\left(\mu_A(x),\eta_A(x),\nu_A(x)\right)$ as well as $B=\left(\mu_B(x),\eta_B(x),\nu_B(x)\right)$ be a family of two PFSs and $\lambda>0$. Then,

(i) $$A\oplus B=\left(\begin{array}{l}\dfrac{\mu_A(x)+\mu_B(x)}{1+\mu_A(x)\cdot\mu_B(x)},\dfrac{\eta_A(x)\cdot\eta_B(x)}{1+\left(1-\eta_A(x)\right)\cdot\left(1-\eta_B(x)\right)},\\ \dfrac{\nu_A(x)\cdot\nu_B(x)}{1+\left(1-\nu_A(x)\right)\cdot\left(1-\nu_B(x)\right)}\end{array}\right),$$

(ii) $$A\otimes B=\left(\begin{array}{l}\dfrac{\mu_A(x)\cdot\mu_B(x)}{1+\left(1-\mu_A(x)\right)\cdot\left(1-\mu_B(x)\right)},\dfrac{\eta_A(x)+\eta_B(x)}{1+\eta_A(x)\cdot\eta_B(x)},\\ \dfrac{\nu_A(x)+\nu_B(x)}{1+\nu_A(x)\cdot\nu_B(x)}\end{array}\right),$$

(iii) $$\lambda\cdot A=\left(\begin{array}{l}\dfrac{\left(1+\mu_A(x)\right)^{\lambda}-\left(1-\mu_A(x)\right)^{\lambda}}{\left(1+\mu_A(x)\right)^{\lambda}+\left(1-\mu_A(x)\right)^{\lambda}},\dfrac{2\left(\eta_A(x)\right)^{\lambda}}{\left(2-\eta_A(x)\right)^{\lambda}+\left(\eta_A(x)\right)^{\lambda}},\\ \dfrac{2\left(\nu_A(x)\right)^{\lambda}}{\left(2-\nu_A(x)\right)^{\lambda}+\left(\nu_A(x)\right)^{\lambda}}\end{array}\right),$$

(iv) $$A^{\lambda}=\left(\begin{array}{l}\dfrac{2\left(\mu_A(x)\right)^{\lambda}}{\left(2-\mu_A(x)\right)^{\lambda}+\left(\mu_A(x)\right)^{\lambda}},\dfrac{\left(1+\eta_A(x)\right)^{\lambda}-\left(1-\eta_A(x)\right)^{\lambda}}{\left(1+\eta_A(x)\right)^{\lambda}+\left(1-\eta_A(x)\right)^{\lambda}},\\ \dfrac{\left(1+\nu_A(x)\right)^{\lambda}-\left(1-\nu_A(x)\right)^{\lambda}}{\left(1+\nu_A(x)\right)^{\lambda}+\left(1-\nu_A(x)\right)^{\lambda}}\end{array}\right).$$

Khan et al [24] established several Einstein aggregation operators designed for aggregating picture fuzzy information, including the PFEWA operator, as an example.

Definition 6 [24]. Let $P_i=\left(\mu_i,\eta_i,\nu_i\right)(i=1,2,\ldots,n)$ represents a collection of PFNs. Here, the PFEWA operator with respect to dimension *n* denotes a function $M^n\rightarrow M$ given by,

$$\text{PFEWA}_{\varpi}\left(P_1,P_2,..,P_n\right)$$

$$=\overset{n}{\underset{i=1}{\oplus}}\left(\varpi_i P_i\right),$$

$$=\left(\begin{array}{l}\dfrac{\prod_{i=1}^{n}\left(1+\mu_{P_i}\right)^{\varpi_i}-\prod_{i=1}^{n}\left(1-\mu_{P_i}\right)^{\varpi_i}}{\prod_{i=1}^{n}\left(1+\mu_{P_i}\right)^{\varpi_i}+\prod_{i=1}^{n}\left(1-\mu_{P_i}\right)^{\varpi_i}},\dfrac{2\prod_{i=1}^{n}\left(\eta_{P_i}\right)^{\varpi_i}}{\prod_{i=1}^{n}\left(2-\eta_{P_i}\right)^{\varpi_i}+\prod_{i=1}^{n}\left(\eta_{P_i}\right)^{\varpi_i}},\\ \dfrac{2\prod_{i=1}^{n}\left(\nu_{P_i}\right)^{\varpi_i}}{\prod_{i=1}^{n}\left(2-\nu_{P_i}\right)^{\varpi_i}+\prod_{i=1}^{n}\left(\nu_{P_i}\right)^{\varpi_i}}\end{array}\right), \tag{6}$$

in which $\varpi_i=\left(\varpi_1,\varpi_2,...,\varpi_n\right)^T$ represents the weight vector of $P_i\left(i=1,2,...,n\right)$, as well as $\varpi_i>0$, $\sum_{i=1}^{n}\varpi_i=1$.

3 AN APPROACH TO MAGDM WITH THE PFEWAD-TOPSIS METHOD

Let $A_i=(A_1,A_2...,A_m)$ denotes a finite set of *m* alternatives, $C_j=(C_1,C_2,...,C_n)$ express a set of *n* attributes, while $D_r=(D_1,D_2,...,D_k)$ denote a set of *k* decision makers. Assuming that all the values assigned to alternatives concerning attributes are represented using a picture fuzzy decision matrix denoted as $H=(h_{ij}^r)_{m\times n}$. The elements $h_{ij}^r=(\mu_{ij}^r,\eta_{ij}^r,\nu_{ij}^r)$, where μ_{ij}^r, η_{ij}^r, as well as ν_{ij}^r signify the degrees satisfied by the alternative A_i satisfy, neutral, as well as not satisfying the attribute C_j as outlined by the decision makers D_r, accordingly. Here, $\mu_{ij}^r\in\left[0,1\right]$, $\eta_{ij}^r\in\left[0,1\right]$, $\nu_{ij}^r\in\left[0,1\right]$ as well as $0\le\mu_{ij}^r+\eta_{ij}^r+\nu_{ij}^r\le1$ for $i=1,2,...,m$; $j=1,2,...,n$; $r=1,2,...,k$. The algorithm with respect to the PFEWAD-TOPSIS method involves the steps given below.

Step 1 Construct the linguistic evaluation matrix by gathering input from the assessment committee of decision makers' preferences.

Decision makers are invited to deliver their judgements and preferences values depending on their responsibilities, knowledge as well as experience on the available alternatives $A_i=(A_1,A_2...,A_m)$ with respect to every attribute $C_j=(C_1,C_2,...,C_n)$ in the picture fuzzy decision matrix $H=(h_{ij}^r)_{m\times n}$, where $h_{ij}^r=(\mu_{ij}^r,\eta_{ij}^r,\nu_{ij}^r)$ $r=1,2,...,k,$ $i=1,2,...,m,$ and $j=1,2,...,n$ using linguistic expression set as in Table 2 [20].

Table 2: The linguistic expression for rating the alternatives

Linguistic Terms	Abbreviation	PFNs
Very Good	VG	(0.90, 0.00, 0.05)
Good	G	(0.75, 0.05, 0.10)
Moderately Good	MG	(0.60, 0.00, 0.30)
Fair	F	(0.50, 0.10, 0.40)
Moderately Poor	MP	(0.30, 0.00, 0.60)
Poor	P	(0.25, 0.05, 0.60)
Very Poor	VP	(0.10, 0.00, 0.85)

Step 2 Convert the linguistic evaluation matrix with respect to decision makers' preferences in terms of picture fuzzy numbers.

Step 3 Construct the normalized decision matrix in the context of picture fuzzy data.

There are two attributes' types in the actual decision: benefit attributes as well as cost attributes. A benefit criterion (the larger the values, the better) and a cost criterion (the lower the values, the better) are of polar opposite sorts. To prevent dimensional differences during the evaluation procedure, the evaluation must be unified and relieved of the influence with regard to different attribute types. This can be done by converting the attribute values of the cost type into the benefit type using the equation given below. Note that the transformed decision matrices remain represented by $H=(h_{ij}^{r})_{m\times n}=(\mu_{ij}^{r},\eta_{ij}^{r},\nu_{ij}^{r})_{m\times n}\ r=1,2,\dots,k,\ \ i=1,2,\dots,m,$ and $j=1,2,\dots,n$.

$$H=\begin{cases}(h_{ij}^{r})_{m\times n}=(\mu_{ij}^{r},\eta_{ij}^{r},\nu_{ij}^{r})_{m\times n} & \text{for benefit attribute } C_j \\ (h_{ij}^{r})_{m\times n}=(\nu_{ij}^{r},\eta_{ij}^{r},\mu_{ij}^{r})_{m\times n} & \text{for cost attribute } C_j\end{cases} \tag{7}$$

Step 4 Aggregate the preference values given by all decision-makers in obtaining the overall preference values.

Individual opinions of the decision makers are aggregated into a picture fuzzy decision matrix employing the PFEWA aggregation operator as described in Eq. (6). Then, the aggregated results are illustrated in the form of $\tilde{H}_{ij}$.

$$\tilde{H}_{ij}=\left[\tilde{h}_{ij}\right]_{m\times n}=\begin{bmatrix}\left(\mu_{\tilde{h}_{11}},\eta_{\tilde{h}_{11}},\nu_{\tilde{h}_{11}}\right) & \left(\mu_{\tilde{h}_{12}},\eta_{\tilde{h}_{12}},\nu_{\tilde{h}_{12}}\right) & \cdots & \left(\mu_{\tilde{h}_{1n}},\eta_{\tilde{h}_{1n}},\nu_{\tilde{h}_{1n}}\right) \\ \left(\mu_{\tilde{h}_{21}},\eta_{\tilde{h}_{21}},\nu_{\tilde{h}_{21}}\right) & \left(\mu_{\tilde{h}_{22}},\eta_{\tilde{h}_{22}},\nu_{\tilde{h}_{22}}\right) & \cdots & \left(\mu_{\tilde{h}_{2n}},\eta_{\tilde{h}_{2n}},\nu_{\tilde{h}_{2n}}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\mu_{\tilde{h}_{m1}},\eta_{\tilde{h}_{m1}},\nu_{\tilde{h}_{m1}}\right) & \left(\mu_{\tilde{h}_{m2}},\eta_{\tilde{h}_{m2}},\nu_{\tilde{h}_{m2}}\right) & \cdots & \left(\mu_{\tilde{h}_{mn}},\eta_{\tilde{h}_{mn}},\nu_{\tilde{h}_{mn}}\right)\end{bmatrix},$$

where $\mu_{h_{ij}}$, $\eta_{h_{ij}}$, and $\nu_{h_{ij}}$ accordingly express denote the degree of positive membership, degree of neutral membership, and degree of negative membership with regards to the alternative $A_i \in A$ and the criteria C_j, accordingly.

Step 5 Obtain the picture fuzzy Einstein positive ideal solution (PFEPIS), M^+ as well as Picture Fuzzy Einstein Negative-Ideal Solution (PFENIS), M^- as follows.

$$M^+ = \left\{f_1^+, f_2^+, \ldots, f_n^+\right\} \text{ where } f_j^+ = \left(\max_i\left(\mu_{\tilde{h}_{ij}}\right), \min_i\left(\eta_{\tilde{h}_{ij}}\right), \min_i\left(\nu_{\tilde{h}_{ij}}\right)\right);$$
$$i = (1,2,\ldots,m) \text{ and } j = (1,2,\ldots,n). \tag{8}$$

$$M^- = \left\{f_1^-, f_2^-, \ldots, f_n^-\right\} \text{ where } f_j^- = \left(\min_i\left(\mu_{\tilde{h}_{ij}}\right), \max_i\left(\eta_{\tilde{h}_{ij}}\right), \max_i\left(\nu_{\tilde{h}_{ij}}\right)\right);$$
$$i = (1,2,\ldots,m) \text{ and } j = (1,2,\ldots,n). \tag{9}$$

Step 6 Calculate the distance between the alternative and M^+, the distance between the alternative and M^-, respectively, as given below.

$$D_i^+ = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\varpi_j\left(\left(\mu_{f_j^+}(x_i) - \mu_{\tilde{h}_{ij}}(x_i)\right)^2 + \left(\eta_{f_j^+}(x_i) - \eta_{\tilde{h}_{ij}}(x_i)\right)^2 + \left(\nu_{f_j^+}(x_i) - \nu_{\tilde{h}_{ij}}(x_i)\right)^2\right)} \tag{10}$$

$$D_i^- = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\varpi_j\left(\left(\mu_{\tilde{h}_{ij}}(x_i) - \mu_{f_j^-}(x_i)\right)^2 + \left(\eta_{\tilde{h}_{ij}}(x_i) - \eta_{f_j^-}(x_i)\right)^2 + \left(\nu_{\tilde{h}_{ij}}(x_i) - \nu_{f_j^-}(x_i)\right)^2\right)} \tag{11}$$

where $i = (1,2,\ldots,m)$ and $j = (1,2,\ldots,n)$.

Step 7 Calculate the closeness coefficient R_i to the picture fuzzy Einstein ideal solution of each alternative as given below.

$$D_i^- = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\varpi_j\left(\left(\mu_{\tilde{h}_{ij}}(x_i) - \mu_{f_j^-}(x_i)\right)^2 + \left(\eta_{\tilde{h}_{ij}}(x_i) - \eta_{f_j^-}(x_i)\right)^2 + \left(\nu_{\tilde{h}_{ij}}(x_i) - \nu_{f_j^-}(x_i)\right)^2\right)} \tag{12}$$

Step 8 Rank all alternatives according to the closeness coefficient R_i. The greater the R_i value indicates a better alternative rank.

4 NUMERICAL EXAMPLE

This section adopts numerical examples from a research paper by Meksavang et al. (2019), which shows the potential evaluation of ten sustainable beef suppliers. The example utilizes the proposed method to illustrate the process effectively. Suppose there are ten beef suppliers $A_i = (A_1, A_2 \dots, A_{10})$ are to be ranked and select the best one after their beef samples analysis against the sustainable attributes. Seven attributes $C_j = (C_1, C_2, \dots, C_7)$ are selected by the decision makers $D_r = (D_1, D_2, D_3)$ to evaluate the beef supplier: (1) C_1 denotes the quality of meat; (2) C_2 denotes the age of the cattle; (3) C_3 represents the diet fed to cattle; (4) C_4 is the average weight of cattle; (5) C_5 is the traceability; (6) C_6 is the carbon footprint; (7) C_7 is the price of cattle. All these attributes are benefit attributes except C_7 is the cost attribute. Assume $\varpi = (0.15, 0.15, 0.10, 0.10, 0.15, 0.15, 0.20)^T$ denote the weight vector for the attribute $C_j\,(j = 1,2,\dots,7)$, in which $\varpi_j \in [0,1]$ and $\sum_{j=1}^{n} \varpi_j = 1.$ To choose the most acceptable beef supplier, the PFEWAD-TOPSIS operator is utilized to MAGDM problem with picture fuzzy information as defined in the steps given below.

Step 1. Gather the decision-maker preferences of all ten alternatives against the seven attributes depicted in Table 3.

Table 3: Linguistic evaluation matrix provided by the decision-makers

Attributes	Decision Makers	Alternatives									
		A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
	D_1	VG	F	P	MG	F	G	MP	P	P	MG
C_1	D_2	VG	MG	MG	MG	MG	G	F	VG	MG	MG
	D_3	VG	F	G	MG	MG	G	G	VG	F	G
	D_1	MP	G	P	F	VG	F	MP	G	F	MP
C_2	D_2	MP	G	MP	F	VG	MP	MP	MG	MP	F
	D_3	MP	G	F	F	VG	P	MP	F	P	MG
	D_1	VG	F	MG	VG	VVG	G	MP	F	MP	MG
C_3	D_2	VG	F	G	G	G	G	F	MG	F	MG
	D_3	VG	F	VG	MG	G	G	MG	G	MG	MG
	D_1	G	MG	MG	P	P	G	P	P	MG	VG
C_4	D_2	G	G	MG	P	P	G	MP	MP	G	VG
	D_3	G	VG	MG	P	MP	G	F	F	VG	VG
	D_1	G	VG	G	VG	VP	VG	P	P	MG	VG
C_5	D_2	VG	VG	MG	G	P	VG	P	P	MG	VG
	D_3	VG	VG	F	VG	MP	VG	MP	P	MG	VG
	D_1	MP	G	MG	VG	G	F	MP	VG	F	VG
C_6	D_2	F	MG	MG	G	G	MG	F	G	F	VG
	D_3	MG	F	MG	VG	G	G	MG	MG	F	VG
	D_1	F	F	P	MP	F	F	F	F	F	P
C_7	D_2	F	F	MP	F	F	MP	MP	MP	F	F
	D_3	F	MP	F	MP	P	F	F	F	F	F

Step 2. Transform the linguistic evaluation matrix to picture fuzzy decision matrix presented as well as illustrated in Table 4.

Table 4: Picture fuzzy evaluation of the alternatives by decision-makers

Attributes	Decision Makers	Alternatives									
		A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
C_1	D_1	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)	(0.50,0.10, 0.40)	(0.75,0.05, 0.10)	(0.30,0.00, 0.60)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)
	D_2	(0.90,0.00, 0.05)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)
	D_3	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.75,0.05, 0.10)
C_2	D_1	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.25,0.05, 0.60)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)
	D_2	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.30,0.00, 0.60)	(0.30,0.00, 0.60)	(0.60,0.00, 0.30)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)
	D_3	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.25,0.05, 0.60)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)
C_3	D_1	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.75,0.05, 0.10)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)	(0.60,0.00, 0.30)
	D_2	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.60,0.00, 0.30)	(0.50,0.10, 0.40)	(0.60,0.00, 0.30)
	D_3	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)
C_4	D_1	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.75,0.05, 0.10)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)
	D_2	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.75,0.05, 0.10)	(0.30,0.00, 0.60)	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.90,0.00, 0.05)
	D_3	(0.75,0.05, 0.10)	(0.90,0.00, 0.05)	(0.60,0.00, 0.30)	(0.25,0.05, 0.60)	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)
C_5	D_1	(0.75,0.05, 0.10)	(0.90,0.00, 0.05)	(0.75,0.05, 0.10)	(0.90,0.00, 0.05)	(0.10,0.00, 0.85)	(0.90,0.00, 0.05)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)
	D_2	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.25,0.05, 0.60)	(0.90,0.00, 0.05)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)
	D_3	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.30,0.00, 0.60)	(0.90,0.00, 0.05)	(0.30,0.00, 0.60)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)
C_6	D_1	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)
	D_2	(0.50,0.10, 0.40)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.50,0.10, 0.40)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)
	D_3	(0.60,0.00, 0.30)	(0.50,0.10, 0.40)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)
C_7	D_1	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.25,0.05, 0.60)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.25,0.05, 0.60)
	D_2	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)	(0.30,0.00, 0.60)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)
	D_3	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)	(0.25,0.05, 0.60)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)

Step 3. Since all attributes are benefit attributes except C_7 is the cost attribute, normalization is required to remove the impact of various attributes. Table 5 displays the normalized evaluation matrix in the context of picture fuzzy data.

Table 5: Picture fuzzy normalized evaluation matrix

Attributes	Decision Makers	Alternatives									
		A1	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*A8*	*A9*	*A10*
C1	*D1*	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)	(0.50,0.10, 0.40)	(0.75,0.05, 0.10)	(0.30,0.00, 0.60)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)
	D2	(0.90,0.00, 0.05)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)
	D3	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.75,0.05, 0.10)
C2	*D1*	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.25,0.05, 0.60)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)
	D2	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.30,0.00, 0.60)	(0.30,0.00, 0.60)	(0.60,0.00, 0.30)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)
	D3	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.25,0.05, 0.60)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)
C3	*D1*	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.75,0.05, 0.10)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)	(0.60,0.00, 0.30)
	D2	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.60,0.00, 0.30)	(0.50,0.10, 0.40)	(0.60,0.00, 0.30)
	D3	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)
C4	*D1*	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.75,0.05, 0.10)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)
	D2	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.75,0.05, 0.10)	(0.30,0.00, 0.60)	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.90,0.00, 0.05)
	D3	(0.75,0.05, 0.10)	(0.90,0.00, 0.05)	(0.60,0.00, 0.30)	(0.25,0.05, 0.60)	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)
C5	*D1*	(0.75,0.05, 0.10)	(0.90,0.00, 0.05)	(0.75,0.05, 0.10)	(0.90,0.00, 0.05)	(0.10,0.00, 0.85)	(0.90,0.00, 0.05)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)
	D2	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.25,0.05, 0.60)	(0.90,0.00, 0.05)	(0.25,0.05, 0.60)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)
	D3	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)	(0.30,0.00, 0.60)	(0.90,0.00, 0.05)	(0.30,0.00, 0.60)	(0.25,0.05, 0.60)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)
C6	*D1*	(0.30,0.00, 0.60)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.30,0.00, 0.60)	(0.90,0.00, 0.05)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)
	D2	(0.50,0.10, 0.40)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.50,0.10, 0.40)	(0.75,0.05, 0.10)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)
	D3	(0.60,0.00, 0.30)	(0.50,0.10, 0.40)	(0.60,0.00, 0.30)	(0.90,0.00, 0.05)	(0.75,0.05, 0.10)	(0.75,0.05, 0.10)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.50,0.10, 0.40)	(0.90,0.00, 0.05)
C7	*D1*	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)	(0.60,0.05, 0.25)	(0.60,0.00, 0.30)	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)	(0.60,0.05, 0.25)
	D2	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)	(0.60,0.00, 0.30)	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.60,0.00, 0.30)	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)
	D3	(0.40,0.10, 0.50)	(0.60,0.00, 0.30)	(0.40,0.10, 0.50)	(0.60,0.00, 0.30)	(0.60,0.05, 0.25)	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)	(0.40,0.10, 0.50)

Step 4. As per Table 5, all PFNs are aggregated employing the PFEWA operator and the aggregating results are presented in Table 6. Let $\varpi = (0.30, 0.40, 0.30)^T$ denote the weight vector for the decision-makers $D_r = (D_1, D_2, D_3)$ in which $\varpi_r \in [0,1]$ and $\sum_{r=1}^{k} \varpi_r = 1$.

Table 6: Aggregated picture fuzzy decision matrix

Alternatives	**Attributes**						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	(0.90,0.00,0.05)	(0.30,0.00,0.60)	(0.90,0.00,0.05)	(0.75,0.05,0.10)	(0.87,0.00,0.06)	(0.48,0.00,0.42)	(0.40,0.10,0.50)
A_2	(0.54,0.00,0.36)	(0.75,0.05,0.10)	(0.50,0.10,0.40)	(0.78,0.00,0.11)	(0.90,0.00,0.05)	(0.63,0.00,0.24)	(0.47,0.00,0.43)
A_3	(0.57,0.00,0.28)	(0.35,0.00,0.53)	(0.78,0.00,0.11)	(0.60,0.00,0.30)	(0.63,0.00,0.24)	(0.60,0.00,0.30)	(0.55,0.00,0.33)
A_4	(0.60,0.00,0.30)	(0.50,0.10,0.40)	(0.78,0.00,0.11)	(0.25,0.05,0.60)	(0.85,.0.00,0.07)	(0.85,0.00,0.07)	(0.53,0.00,0.37)
A_5	(0.57,0.00,0.33)	(0.90,0.00,0.05)	(0.81,0.00,0.08)	(0.27,0.00,0.60)	(0.22,0.00,0.67)	(0.75,0.05,0.10)	(0.47,0.08,0.41)
A_6	(0.75,0.05,0.10)	(0.35,0.00,0.53)	(0.75,0.05,0.10)	(0.75,0.05,0.10)	(0.90,0.00,0.05)	(0.63,0.00,0.24)	(0.49,0.00,0.41)
A_7	(0.54,0.00,0.31)	(0.30,0.00,0.60)	(0.48,0.00,0.42)	(0.35,0.00,0.53)	(0.27,0.00,0.60)	(0.48,0.00,0.42)	(0.49,0.00,0.41)
A_8	(0.80,0.00,0.11)	(0.63,0.00,0.24)	(0.63,0.00,0.24)	(0.35,0.00,0.53)	(0.25,0.05,0.60)	(0.78,0.00,0.11)	(0.49,0.00,0.41)
A_9	(0.48,0.00,0.41)	(0.35,0.00,0.53)	(0.48,0.00,0.42)	(0.78,0.00,0.11)	(0.60,0.00,0.30)	(0.50,0.10,0.40)	(0.40,0.10,0.50)
A_{10}	(0.65,0.00,0.22)	(0.48,0.00,0.42)	(0.60,0.00,0.30)	(0.90,0.00,0.05)	(0.90,0.00,0.05)	(0.90,0.00,0.05)	(0.47,0.08,0.41)

Step 5 Obtain the Picture Fuzzy Einstein Positive Ideal Solution (PFEPIS) and Picture Fuzzy Einstein Negative-Ideal Solution (PFENIS) as indicated in Table 7.

Table 7: Aggregated picture fuzzy decision matrix

Ideal Solution	Attributes						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
PFEPIS	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.90,0.00, 0.05)	(0.55,0.00, 0.33)
PFENIS	(0.48,0.00, 0.41)	(0.30,0.00, 0.60)	(0.50,0.10, 0.40)	(0.25,0.05, 0.60)	(0.22,0.00, 0.67)	(0.50,0.10, 0.40)	(0.40,0.10, 0.50)

Step 6 The distance between the alternative and the PFEPIS and the distance between the alternative and the PFENIS were calculated in Table 8.

Table 8: The distance and the closeness coefficient of each alternative

Alternatives	Distance		Closeness Coefficient	Ranking Order
	D+	D-		
A_1	0.0508	0.0746	0.5951	4
A_2	0.0295	0.0833	0.7386	2
A_3	0.0400	0.0338	0.4581	6
A_4	0.0561	0.0563	0.5009	5
A_5	0.0837	0.0486	0.3674	8
A_6	0.0334	0.0730	0.6864	3
A_7	0.0863	0.0026	0.0287	10
A_8	0.0670	0.0271	0.2878	9
A_9	0.0528	0.0402	0.4323	7
A_{10}	0.0263	0.0884	0.7705	1

Step 7 Rank the alternatives.

The closeness coefficient was calculated, and the ten alternatives were arranged in descending order based on their closeness coefficients, resulting in the ranking of alternatives as $A_{10} > A_2 > A_6 > A_1 > A_4 > A_3 > A_9 > A_5 > A_8 > A_7$. Hence, the best beef supplier is A_{10} while the worst one is A_7.

5 CONCLUSIONS

This study determines an extension of the TOPSIS method in the PFSs context, namely Picture Fuzzy Einstein Weighted Averaging Distance-based TOPSIS (PFEWAD-TOPSIS), as well as employing it in a practical instance pertinent to the beef supplier ranking. The primary advantage of the suggested approach is that a compromise solution may assess the uncertainty inherent in decision-making problems, particularly when different perspectives or viewpoints are involved, such as yes, abstain, no, and refusal. The integration of the TOPSIS method with PFSs holds great promise for addressing MAGDM challenges, particularly in scenarios where decision-makers opinions involve a degree of vagueness. Consequently, in the future, this PFEWAD-TOPSIS method can find practical applications in various real-life contexts such as personnel selection, manufacturing systems, project selection, as well as various other management decision problems.

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