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The Application of Golden Ratio Geometry Analysis to Evaluate Nature Motifs Generated by Gielis' Supershape

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ABSTRACT

Mathematical equations have been used in the design and production of nature motifs, such as jewelry, fashion, furniture, textiles and visual arts. The Gielis' Supershape (GS) is one of the mathematical equations that could be used in the design of nature motifs. GS is a simple and powerful tool for designing nature motifs, but it is not yet extensively used in industries. This could happen because not many researches were conducted to analyze the aesthetic of the nature motif generated by GS. Therefore, this study aims to address this gap by employing the PhiMatrix software to examine the presence of the golden ratio characteristic in nature motifs generated by GS and Enhanced Gielis' Supershapes (EGS). Through a systematic exploration and assessment of these two equations, the study aims to provide valuable insights into enhancing the quality of design and aesthetic appeal in nature motifs. The research seeks to contribute to the field by advocating for the adoption of GS and EGS equations as effective tools in designing nature motifs, thereby expanding creative possibilities across various industries.

Keywords: *Geometric analysis, Gielis' Supershape, golden ratio, nature motif, PhiMatrix.*

1 INTRODUCTION

Nature is a great inspiration for many talented designers. Nature has inspired some of the most famous inventions as well as technical innovations. The inspiration from nature is driving force in architecture, resulting in majestic works of architecture [1], [2]. Designs and images composed entirely of thousands of curves, angles and straight lines. The generated curves and shapes are very common in the field of Computer Aided Geometric Design (CAGD). Mathematical formulas such as fractals and spline curves are commonly used in the creation of curves and shapes. It is difficult to create a mathematical formula that fits entirely with nature motifs. Johan Gielis, had created a superformula and superellipse equation that accommodated the majority of the nature motifs [3], [4], [5].

In order to create designs that are considered satisfactory, designers must consider both the artistic and natural elements of the design. The golden ratio is a widely used method for evaluating the artistic aspect of a design [6]. The golden ratio has been widely used to study the proportions of natural objects and patterns. The purpose of this study is to use the PhiMatrix software to assess the virtuous proportion of nature motif created by GS and EGS. Through this analysis, the study aims to provide a deeper understanding of how the application of the GS equation, both in its original form and enhanced version, contributes to the attainment of virtuous proportion in nature-inspired motifs.

1 Related Literature and Review

1.1 Golden Ratio

The concept of the golden ratio, also known as the "golden number," "golden section," "golden proportion," or "golden mean," has held a significant position throughout history in numerous fields, including geometry, architecture, music, art and philosophy. It has captivated the minds of ancient and contemporary geometric enthusiasts, serving as a symbol of profound fascination [7]. According to [8], the golden ratio is often known as the Fibonacci ratio or the 'divine proportion'. Numerous instances of the phi ratio can be observed in both nature and art. The correlation between the golden ratio and Fibonacci numbers is widely recognized. We can define integer sequences associated with the generalized golden ratios in a similar manner [9]. The relationship between Fibonacci numbers and golden ratio through the drawing of a golden rectangle using Fibonacci numbers and 1:1.618, and the evaluation of $\frac{F_n+1}{F_n}$, when the value of n is getting larger and larger the result gives the value of φ [10].

Besides Fibonacci, golden ratio also can be applied in the arts. According to [11], numerous books have stated that when a rectangle is drawn around the face of Leonardo da Vinci's Mona Lisa, the ratio of its height to width is equivalent to the golden ratio. This ratio was believed to be utilized in achieving a sense of balance and beauty in various Renaissance paintings and sculptures. It has been reported that the divine ratio exists all over the human face. For example, the ratio of the length of the face, 1.618, to the width of the face, 1, demonstrates the golden ratio. Similarly, the ratio of the distance between the lips and the point where the eyebrow meets, 1.618, to the length of the nose, 1, follows the golden ratio. Additionally, the ratio of the length of the face, 1.618, to the distance from the tip of the chin to where the eyebrow meets, 1, adheres to the golden ratio. Moreover, the ratio of the length of the mouth, 1.618, to the width of the nose, 1, and the ratio of the width of the nose, 1.618, to the distance between the nostrils, 1, also exhibit the golden ratio [10]. The golden ratio is considered sacred due to its relationship to nature and even the construction of the universe and the human body. It has been used for centuries in the construction of architectural masterpieces by the great artists, who were being able to see its beauty, used it in their designs and compositions [12].

[13] also mentioned that the golden ratio appears in all modalities of nature and science, in the human body in a variety of its functional and structural performances. According to [14], the golden ratio, $\varphi = \frac{x}{y} = \frac{x+y}{x}$, where *x* and *y* are the golden sections of their sum, has long been known to operate in a variety of the creations of nature. The selection and use of systems of proportions has always been an important concern for artists and architects. There were not only specific ratios used, but also some systems of proportions were preferred. Some systems of proportions were based on the musical intervals, the human body and the golden ratio [15]. Next, [16] presented a new model for

the description of Arabidopsis seed shape based on the comparison of the outline of its longitudinal section with a transformed cardioid and the transformation consists of scaling the horizontal axis by a factor equal to the golden ratio.

1.2 Gielis's Supershape

GS was the modification of curve formula developed by the French mathematician, Gabriel Lamẻ in the year 1818, and the inventive mathematician Piet Hein in the year 1959. Gabriel Lame defined the curves in the xy-plane by the following formula:

$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1 \tag{1}$$

where a, b, and n are any positive real numbers [17]. Then, Piet Hein has revived the Lame curves by creating a superellipse with $n > 2$. [18] then innovated the formula created by Lame and Hein to produce a superformula:

$$r(\theta) = \frac{1}{\sqrt[n_1]{\left|\frac{1}{a}cos\left(\frac{1}{4}m\theta\right)\right|^{n_2} + \left|\frac{1}{b}sin\left(\frac{1}{4}m\theta\right)\right|^{n_3}}} \tag{2}$$

Superformula allows an improved understanding of botanical shapes in relation to optimization and evolutionary trends in plants and plant modeling. In the superformula framework, every shape has its own particular symmetry. The original Gielis equation (2) cannot reflect some diverse shapes due to limitations of its power-law hypothesis [19]. [20] then introduced a new function $f(\theta) = cos\left(\frac{m}{2}\theta\right)$. This function is important to make the curve more precisely but stay symmetry. The improvement formula is defined as follows:

$$r(\theta) = \frac{1}{\sqrt[n_1]{\left|\frac{1}{a}cos\left(\frac{1}{4}m\theta\right)\right|^{n_2} + \left|\frac{1}{b}sin\left(\frac{1}{4}m\theta\right)\right|^{n_3}}}.f(\theta) \tag{3}$$

2 MATERIAL AND METHODS

This study involves two types of Gielis' superformula; GS as shown in equation (3) and EGS introduced by [21] as defined by the following formula.

$$r(\theta) = \frac{cos\left(\frac{m}{2}\theta\right)sin\left(\frac{m}{2}\theta\right)}{\sqrt[n_1]{\left|\frac{1}{a}cos\left(\frac{1}{4}m\theta\right)\right|^{n_2} + \left|\frac{1}{b}sin\left(\frac{1}{4}m\theta\right)\right|^{n_3}}} \tag{4}$$

The rotational symmetry is controlled by the parameter m. The parameters n_1, n_2 and n_3 control the curvature of the sides, while the values of *a* and *b* control the size of the shape [22]. The study process encompasses three key steps as illustrated in Figure 1.

Firstly, nature motifs were generated using the GS equation GS with $f(\theta) = \cos\theta$, $f(\theta) = \sin\theta$ and an EGS formula, to compare the best formula that fits the golden ratio characteristic of both formulas. The six free parameters in both GS and EGS are set to the same values. Subsequently, a comprehensive analysis was conducted using the PhiMatrix software to assess the presence of virtuous proportion within the designed motifs, with a particular emphasis on the golden ratio characteristic. In this analysis, we chose to use the pentagram of the regular pentagon and circle views provided by PhiMatrix. Skinner's study as cited in [23] described that regular pentagons and pentagrams have had such an impact on art and philosophy. The pentagon is also a symbol of Sacred Geometry, a discipline that assigns sacred meanings to geometric shapes and proportions. The regular pentagon is made up of a variety of wonderful figures that are frequently used in works of art. The law of the "golden cup," which was used by architects and goldsmiths, was well known in ancient Egypt and classical Greece [24]. Finally, a comparative evaluation was undertaken to discern the superior set of nature motifs between GS and the EGS equation in terms of virtuous proportion and aesthetic appeal.

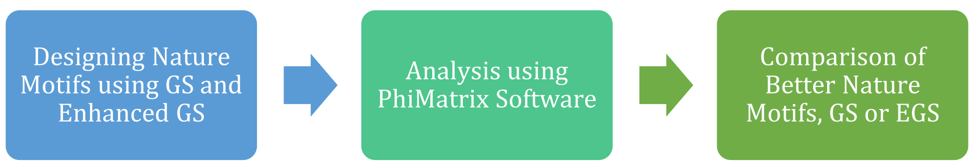
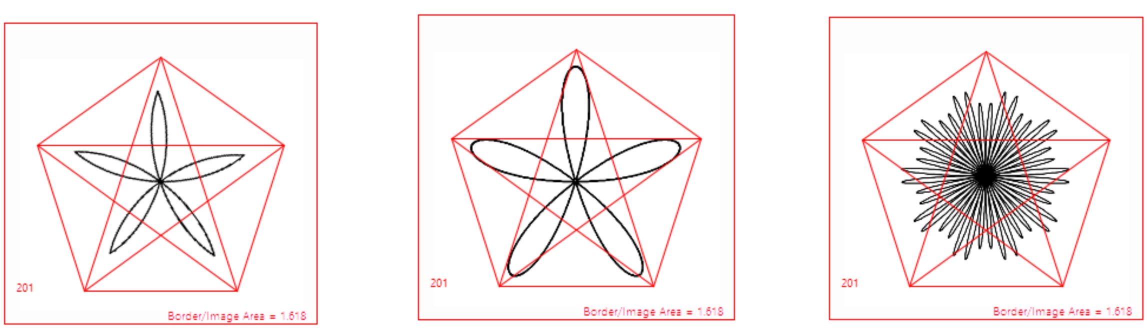


Figure 1: The Study Process of Golden Ratio Geometric Analysis.

3 RESULTS AND DISCUSSION

Figure 2 depicts a nature motif in the shape of a flower generated by GS with $f(\theta) = \cos\theta$, $f(\theta) = \sin\theta$ and EGS. The observation shows that the motif generated by GS with $f(\theta) = \cos\theta$ and $f(\theta) = \sin\theta$ fits the pentagram's golden ratio characteristic. All the flower petals were generated inside the pentagram. This, however, has not occurred for flowers generated by EGS. The petals do not lie in the pentagram.

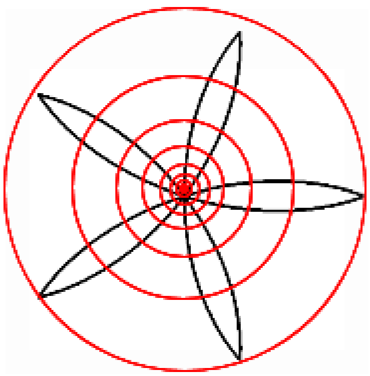


GS with $f(\theta) = \cos\theta$ GS with $f(\theta) = \sin\theta$ EGS

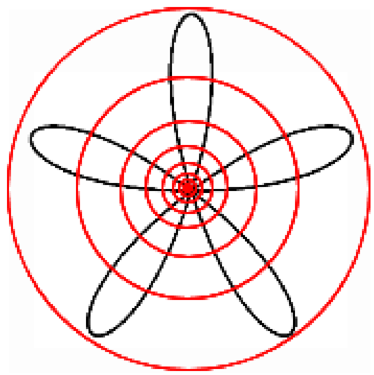
$a=1, b=1, m=10, n_1=1, n_2=1, n_3=1$

Figure 2 : Golden Ratio Geometric Analysis using Pentagram PhiMatrix Analysis.

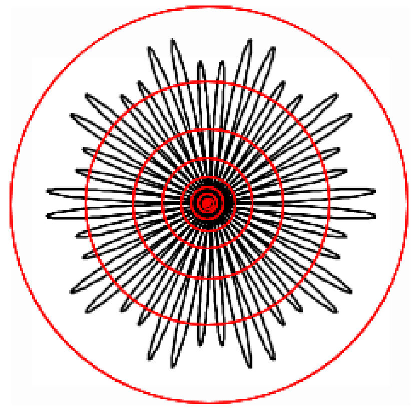
We continue the analysis by examining various interpretations of the golden ratio using the circle characteristic. The circle encircles everything and forms the golden ratio relations within all the basic geometric shapes it encloses [25]. Figure 3 reveals that the flower generated by EGS exhibits the circle characteristic of the golden ratio because it contains two lengths of petals which form two circles with different radii. While the GS flower does not have two circles, the ratio of 1:1.618 cannot be developed for a picture with only one circle. Curvature allows the circle and logarithmic spiral to be defined as two curves with opposing tendencies, which explains why these two curves are fundamental in natural sciences [26].



GS with $f(\theta) = \cos\theta$



GS with $f(\theta) = \sin\theta$

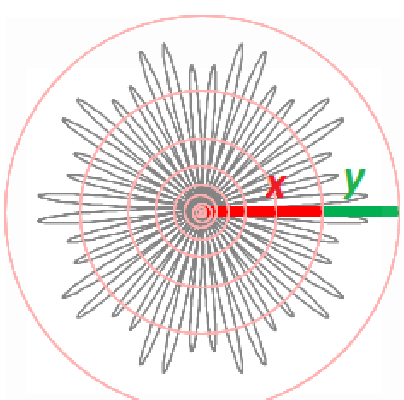


EGS

$a=1, b=1, m=10, n_1=1, n_2=1, n_3=1$

Figure 3: Golden Ratio Geometric Analysis using Circle PhiMatrix Analysis.

The calculation of conformant to golden section rule can also be proven mathematically as:



$$x = 0.5cm$$
$$y = 0.3cm$$
$$\frac{x+y}{x} = \frac{0.5+0.3}{0.5} = \frac{0.8}{0.5} = 1.6$$

The calculation proves that the flower created by EGS with parameter value $a = 1, b = 1$, $m = 10, n_1 = n_2 = n_3 = 1$ conforms to the golden section rule. The proportional ratio, 1.6 close to the golden number, 1.618.

Both GS and EGS exhibit characteristics of the golden ratio. Figure 2 illustrates the pentagram characteristics of GS, while Figure 3 showcases the circular characteristics of EGS. Despite being symmetrical, the shapes generated using GS and EGS still maintain the distinctive golden ratio characteristic. In addition, [27] emphasizes that the golden ratio plays a crucial role in creating a harmonious composition in successful design, thereby enhancing design aesthetics.

4 CONCLUSION

This study presents a systematic approach to exploring the potential of the GS and EGS equation in the design of nature motifs. Through the utilisation of the PhiMatrix software, the research provides insights into the presence of virtuous proportion and aesthetic appeal within the designed motifs. The comparative evaluation between GS and the EGS equation offers valuable guidance to industries seeking alternative design methods for incorporating nature-inspired elements into their products. Ultimately, this research contributes to expanding the creative possibilities and enhancing the visual appeal of nature motifs in various industrial applications. In future research, the generator or researcher should do further analysis to find the best parameter in either GS or EGS so that the shape that we want to generate satisfies the golden ratio criteria.

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