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Efficient Solving of Nonlinear ODEs: Daftardar-Jafari Method vs Differential Transform Method

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ABSTRACT

This study explores the Daftardar-Jafari Method (DJM) for solving linear and nonlinear ordinary differential equation systems (ODEs). Unlike traditional perturbation methods, DJM does not rely on small parameters, making it highly effective for strongly nonlinear problems. The method constructs a rapidly converging iterative sequence, yielding accurate analytical or approximate solutions with reduced computational costs. We applied DJM to a range of benchmark problems and compared the results with those obtained using the differential transform method (DTM). The DJM provided significantly higher accuracy, demonstrating its superior performance in terms of convergence and computational efficiency. The numerical results, computed using Maple software, reinforce the practical advantages of DJM for solving complex systems of ODEs. In conclusion, DJM is an effective and efficient tool for solving a broad class of ordinary differential equation systems, outperforming traditional methods like the differential transform method in terms of accuracy and reliability.

Keywords: Linear and nonlinear system, ordinary differential equation, Daftardar-Jafari Method, Exact and approximate solution

1 INTRODUCTION

Ordinary differential equations (ODEs) and their nonlinear [1] variants are fundamental tools for modeling dynamic systems across a wide array of scientific and engineering domains. These equations are crucial in disciplines such as physics, biology, engineering, and control theory, where the evolution of a system is determined by both its current state and its historical behavior. The challenge of solving these equations becomes more pronounced when dealing with nonlinearities, as traditional methods often fail to provide exact solutions, particularly for complex, real-world systems. Analytical solutions to these problems are difficult to obtain, prompting the need for more efficient and robust techniques. Among the various methods available to solve nonlinear ordinary differential equations, the Variational Iteration Method (VIM), the Adomian Decomposition Method (ADM) and the Homotopy Perturbation method (HPM) [2-15] have been widely used. However, these methods often require small perturbation parameters or have limitations in terms of convergence when solving strongly nonlinear systems. In contrast, the Daftardar-Jafari Method (DJM) [16-18] does not rely on such assumptions, making it particularly effective in handling nonlinear ODEs without the need for small parameters, which is a significant advantage over traditional perturbation-based methods. This ability to solve nonlinear problems efficiently, without the need for small parameters, sets DJM apart from other established techniques. In recent years, semi-analytical iterative methods have gained significant traction for their ability to handle nonlinear ODEs efficiently while maintaining high computational accuracy. Among these, the Daftardar-Jafari Method (DJM) has emerged as a powerful and reliable approach for solving both linear and nonlinear ODE systems. Unlike traditional perturbation methods, which depend on small parameters, DJM does not require such assumptions, making it highly effective for solving strongly nonlinear problems. This method constructs a rapidly converging series, which ensures accurate and reliable solutions with fewer computational steps. The DJM has been successfully applied to a broad range of problems, including complex nonlinear boundary value problems and fractional differential equations. Its robustness, combined with its ability to reduce the number of iterations, makes it a valuable tool for both theoretical and applied research. The aim of this study is to explore the application of DJM in solving nonlinear ordinary differential equation systems, highlighting its accuracy and computational efficiency. The paper compares the performance of DJM to traditional methods, demonstrating its superior precision and efficiency. While traditional methods like the Laplace transform [1] and Fourier transform [19] are widely used, they are often limited to linear cases. In contrast, DJM effectively overcomes these limitations, particularly in nonlinear scenarios. Furthermore, numerical techniques such as finite difference, spectral methods, and iterative methods provide approximate solutions but can suffer from computational inefficiencies, an issue that DJM addresses with remarkable success. This paper is structured as follows: Section 2 introduces the mathematical framework and theoretical foundations of the Daftardar-Jafari Method. Section 3 focuses on the application of the method to different types of nonlinear ordinary differential equations, providing numerical results and a comparative analysis. Finally, Section 4 concludes the study by summarizing the key findings and suggesting possible directions for future research.

2 METHODOLOGY AND FORMULATION

We focus on the following systems of boundary value problems that expressed in the form:

$$\begin{cases} u_1^{(n)} = f_1\left(x, u_1, u_1', \dots, u_1^{(n-1)}, u_2, u_2', \dots, u_2^{(n-1)}, \dots, u_k, u_k', \dots, u_k^{(n-1)}\right) + g_1(x) \\ u_2^{(n)} = f_2\left(x, u_1, u_1', \dots, u_1^{(n-1)}, u_2, u_2', \dots, u_2^{(n-1)}, \dots, u_k, u_k', \dots, u_k^{(n-1)}\right) + g_2(x) \\ \vdots \\ u_k^{(n)} = f_k\left(x, u_1, u_1', \dots, u_1^{(n-1)}, u_2, u_2', \dots, u_2^{(n-1)}, \dots, u_k, u_k', \dots, u_k^{(n-1)}\right) + g_k(x) \end{cases} , x \in [0,1] \qquad (1)$$

where $u_1, \dots, u_k$ are unknown functions and $g_1, \dots, g_k$ are analytical functions.

According to the DJM, by integrating both sides of the system (1) with respect to x.

$$
\begin{cases}
u_1(x) = \underbrace{\int_0^x \dots \int_0^x}_{n} \left(f_1\left(x, u_1, u_1', \dots, u_1^{(n-1)}, u_2, u_2', \dots, u_2^{(n-1)}, \dots, u_k, u_k', \dots, u_k^{(n-1)}\right) + g_1(x)\right) \underbrace{dx \dots dx}_{n} \\
u_2(x) = \underbrace{\int_0^x \dots \int_0^x} \left(f_2\left(x, u_1, u_1', \dots, u_1^{(n-1)}, u_2, u_2', \dots, u_2^{(n-1)}, \dots, u_k, u_k', \dots, u_k^{(n-1)}\right) + g_2(x)\right) \underbrace{dx \dots dx}_{n} \\
\vdots \\
u_k(x) = \underbrace{\int_0^x \dots \int_0^x} \left(f_k\left(x, u_1, u_1', \dots, u_1^{(n-1)}, u_2, u_2', \dots, u_2^{(n-1)}, \dots, u_k, u_k', \dots, u_k^{(n-1)}\right) + g_k(x)\right) \underbrace{dx \dots dx}_{n}
\end{cases} \quad (2)
$$

where the nonlinear part

$$
\begin{cases}
N_1(x) = \underbrace{\int_0^x \dots \int_0^x}_{n} f_1\left(x, u_1, u_1', \dots, u_1^{(n-1)}, u_2, u_2', \dots, u_2^{(n-1)}, \dots, u_k, u_k', \dots, u_k^{(n-1)}\right) \underbrace{dx \dots dx}_{n} \\
N_2(x) = \underbrace{\int_0^x \dots \int_0^x} f_2\left(x, u_1, u_1', \dots, u_1^{(n-1)}, u_2, u_2', \dots, u_2^{(n-1)}, \dots, u_k, u_k', \dots, u_k^{(n-1)}\right) \underbrace{dx \dots dx}_{n} \\
\vdots \\
N_k(x) = \underbrace{\int_0^x \dots \int_0^x} f_k\left(x, u_1, u_1', \dots, u_1^{(n-1)}, u_2, u_2', \dots, u_2^{(n-1)}, \dots, u_k, u_k', \dots, u_k^{(n-1)}\right) \underbrace{dx \dots dx}_{n}
\end{cases}
$$

The solution to the (2) is given in the form

$$
\begin{cases}
u_1 = \sum_{i=0}^{\infty} (u_1)_i \\
u_2 = \sum_{i=0}^{\infty} (u_2)_i \\
\vdots \\
u_k = \sum_{i=0}^{\infty} (u_k)_i
\end{cases} \quad (3)
$$

By expressing the nonlinear part in the form

$$\begin{cases} N_1(u) = \underbrace{\int_0^x \cdots \int_0^x}_{n} f_1\left(x, \sum_{i=0}^{\infty}(u_1)_i, \sum_{i=0}^{\infty}(u'_1)_i, \dots, \sum_{i=0}^{\infty}\left(u_1^{(n-1)}\right)_i, \dots, \sum_{i=0}^{\infty}\left(u_k^{(n-1)}\right)_i\right)\underbrace{dx \dots dx}_{n} \\ N_2(u) = \underbrace{\int_0^x \cdots \int_0^x} f_2\left(x, \sum_{i=0}^{\infty}(u_1)_i, \sum_{i=0}^{\infty}(u'_1)_i, \dots, \sum_{i=0}^{\infty}\left(u_1^{(n-1)}\right)_i, \dots, \sum_{i=0}^{\infty}\left(u_k^{(n-1)}\right)_i\right)\underbrace{dx \dots dx}_{n} \\ \vdots \\ N_k(u) = \underbrace{\int_0^x \cdots \int_0^x} f_k\left(x, \sum_{i=0}^{\infty}(u_1)_i, \sum_{i=0}^{\infty}(u'_1)_i, \dots, \sum_{i=0}^{\infty}\left(u_1^{(n-1)}\right)_i, \dots, \sum_{i=0}^{\infty}\left(u_k^{(n-1)}\right)_i\right)\underbrace{dx \dots dx}_{n} \end{cases} \quad (4)$$

By substituting (3) and (4) into (2) , we obtain:

$$\begin{cases} u_1(x) = \underbrace{\int_0^x \cdots \int_0^x}_{n} \left(f_1\left(x, \sum_{i=0}^{\infty}(u_1)_i, \sum_{i=0}^{\infty}(u'_1)_i, \dots, \sum_{i=0}^{\infty}\left(u_1^{(n-1)}\right)_i, \dots, \sum_{i=0}^{\infty}\left(u_k^{(n-1)}\right)_i\right) + g_1(x)\right)\underbrace{dx \dots dx}_{n} \\ u_2(x) = \underbrace{\int_0^x \cdots \int_0^x} \left(f_2\left(x, \sum_{i=0}^{\infty}(u_1)_i, \sum_{i=0}^{\infty}(u'_1)_i, \dots, \sum_{i=0}^{\infty}\left(u_1^{(n-1)}\right)_i, \dots, \sum_{i=0}^{\infty}\left(u_k^{(n-1)}\right)_i\right) + g_2(x)\right)\underbrace{dx \dots dx}_{n} \\ \vdots \\ u_k(x) = \underbrace{\int_0^x \cdots \int_0^x} \left(f_k\left(x, \sum_{i=0}^{\infty}(u_1)_i, \sum_{i=0}^{\infty}(u'_1)_i, \dots, \sum_{i=0}^{\infty}\left(u_1^{(n-1)}\right)_i, \dots, \sum_{i=0}^{\infty}\left(u_k^{(n-1)}\right)_i\right) + g_k(x)\right)\underbrace{dx \dots dx}_{n} \end{cases}$$

From this, the following results:

$$\begin{cases} (u_0)_1 = \underbrace{\int_0^x \cdots \int_0^x}_{n} g_1(x)\,\underbrace{dx \dots dx}_{n} \\ (u_0)_2 = \underbrace{\int_0^x \cdots \int_0^x} g_2(x)\,\underbrace{dx \dots dx}_{n} \\ \vdots \\ (u_0)_k = \underbrace{\int_0^x \cdots \int_0^x} g_k(x)\,\underbrace{dx \dots dx}_{n} \end{cases}$$

$$\begin{cases} (u_1)_1 = N((u_1)_0) \\ (u_2)_1 = N((u_2)_0) \\ \vdots \\ (u_k)_1 = N((u_k)_0) \end{cases}$$

$$\begin{cases} (u_1)_2 = N((u_1)_0 + (u_1)_1) - N((u_1)_0) \\ (u_2)_2 = N((u_2)_0 + (u_2)_1) - N((u_2)_0) \\ \qquad \vdots \\ (u_k)_2 = N((u_k)_0 + (u_k)_1) - N((u_k)_0) \end{cases}$$

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$$\begin{cases} (u_1)_2 = N((u_1)_0 + (u_1)_1 + \cdots + (u_1)_{m-1}) - N((u_1)_0 + (u_1)_1 + \cdots + (u_1)_{m-2}) \\ (u_2)_2 = N((u_2)_0 + (u_2)_1 + \cdots + (u_2)_{m-1}) - N((u_2)_0 + (u_2)_1 + \cdots + (u_2)_{m-2}) \\ \qquad \vdots \\ (u_k)_2 = N((u_k)_0 + (u_k)_1 + \cdots + (u_k)_{m-1}) - N((u_2)_0 + (u_2)_1 + \cdots + (u_2)_{m-2}) \end{cases}, \ m = 2,3,\ldots$$

Then the solution of (1) is (3) .

For convergence of the DJM [18]

Lemma [18]. If N is C^{∞} in a neighborhood of u_0 and $\left\|N^{(n)}(u_0)\right\| < L$, for any n and for some real $L > 0$ and $\|u_i\| \leq M < e^{-1}$, $i = 1,2,\ldots$ then the series $\sum_{n=0}^{\infty} u_n$ is absolutely convergent and

$$\|u_i\| \leq LM^n e^{n-1}(e-1), \ \ n = 1,2,\ldots$$

3 TEST PROBLEMS

Example 1

Consider the following system of nonlinear boundary value problems [20]

$$\begin{cases} u^{(4)}(x) + x\,u''(x) + 3v'(x) = 30 \\ v^{(4)}(x) + x^2 v^{(3)}(x) + 6u'(x) = -30 \end{cases}, \quad 0 \leq x \leq 1 \qquad (5)$$

Subjects to the boundary conditions:

$u(0) = 0\,, u'(0) = -1\,, u''(0) = 0\,, u(1) = 0$

$v(0) = 0\,, v'(0) = 2\,, v''(0) = 0\,, v(1) = 1$

According to the DJM, by integrating both sides of the system (5) with respect to x and by using the boundary conditions, yields

$$\begin{cases} u(x) = -x + a\dfrac{x^3}{6} + \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(30 - x\,u''(x) - 3v'(x)\right)\,dx\,dx\,dx\,dx \\ v(x) = 2x + \dfrac{bx^3}{6} + \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(-30 - x^2 v^{(3)}(x) - 6u'(x)\right)\,dx\,dx\,dx\,dx \end{cases}$$

where

$$\begin{cases} u_0(x) = -x + a\dfrac{x^3}{6} + \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x (30)\,dx\,dx\,dx\,dx = -x + a\dfrac{x^3}{6} + \dfrac{5x^4}{4} \\ v_0(x) = 2x + b\dfrac{x^3}{6} + \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x (-30)\,dx\,dx\,dx\,dx = 2x + b\dfrac{x^3}{6} - \dfrac{5x^4}{4} \end{cases}$$

$$\& \begin{cases} N(u) = \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(-x\,u''(x) - 3v'(x)\right)\,dx\,dx\,dx\,dx \\ N(v) = \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(-x^2 v^{(3)}(x) - 6u'(x)\right)\,dx\,dx\,dx\,dx \end{cases}$$

Then

$$\begin{cases} u_1(x) = N(u_0) = \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(-x\,u_0''(x) - 3v_0'(x)\right)\,dx\,dx\,dx\,dx = \dfrac{1}{6}\left(-\dfrac{a}{60} - \dfrac{b}{40}\right)x^6 - \dfrac{x^4}{4} \\ v_1(x) = N(v_0) = \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(-x^2 v_0^{(3)}(x) - 6u_0'(x)\right)\,dx\,dx\,dx\,dx = \dfrac{1}{6}\left(-\dfrac{b}{60} - \dfrac{a}{20}\right)x^6 + \dfrac{x^4}{4} \end{cases}$$

By using the boundary conditions $u(1) = 0$ & $v(1) = 1$

Then we have $a = b = 0$

Then the exact solution is

$$\begin{cases} u(x) = \displaystyle\sum_{i=0}^{\infty} u_i = -x + x^4 \\ v(x) = \displaystyle\sum_{i=0}^{\infty} v_i = 2x - x^4 \end{cases}$$

Example 2

Consider the following system of nonlinear boundary value problems [21].

$$\begin{cases} u^{(4)}(x) - u''(x) - u^{(3)}(x)v'(x) = 24x\sin x - 12x^2 + 26 \\ v^{(4)}(x) - 24x\, v''(x) - u^{(3)}(x)\, v(x) = \cos x \end{cases} \quad , \quad 0 \le x \le 1 \qquad (6)$$

Subjects to the boundary conditions:

$u(0) = 0\,, u'(0) = 0\,, u''(0) = -2\,, u(1) = 0$
$v(0) = 1\,, v'(0) = 0\,, v''(0) = -1\,, v(1) = \cos(1)$

According to the DJM, by integrating both sides of the system (6) with respect to x and by using the boundary conditions, yields

$$\begin{cases} u(x) = -x^2 + a\dfrac{x^3}{6} + \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(24x\sin x - 12x^2 + 26 + u''(x) + u^{(3)}(x)v'(x)\right) dx\,dx\,dx\,dx \\ v(x) = 1 - \dfrac{x^2}{2} + \dfrac{bx^3}{6} + \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(\cos x + 24x\, v''(x) + u^{(3)}(x)\, v(x)\right) dx\,dx\,dx\,dx \end{cases}$$

where

$$\begin{cases} u_0(x) = -x^2 + a\dfrac{x^3}{6} + \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x (24x\sin x - 12x^2 + 26)\ dx\,dx\,dx\,dx \\ \qquad = -96 + 23x^2 + \dfrac{ax^3}{6} + \dfrac{13x^4}{12} - \dfrac{x^6}{30} + 96\cos x + 24x\sin x \\ v_0(x) = 1 - \dfrac{x^2}{2} + \dfrac{bx^3}{6} + \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x (\cos x)\ dx\,dx\,dx\,dx = \dfrac{bx^3}{6} + \cos x \end{cases}$$

$$\& \quad \begin{cases} N(u) = \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(u''(x) + u^{(3)}(x)v'(x)\right) dx\,dx\,dx\,dx \\ N(v) = \displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(24x\, v''(x) + u^{(3)}(x)\, v(x)\right) dx\,dx\,dx\,dx \end{cases}$$

Then

$$\begin{cases} u_1(x) = N(u_0) = \int_0^x\int_0^x\int_0^x\int_0^x \left(u_0''(x) + u_0^{(3)}(x)v_0'(x)\right) dx\,dx\,dx\,dx \\ = -288bx^2 + \frac{ax^2}{2} - \frac{bx^{10}}{7560} - \frac{17x^3}{6} - 488\sin x + \frac{13bx^8}{5040} - a + 3840b - 2160bx\sin x - 4bx^4\cos x \\ +68bx^3\sin x + 528bx^2\cos x - \frac{x^5}{10} + a\cos x + \frac{abx^7}{5040} + \frac{9}{2}\cos x\sin x + 242x\cos x - 3840b\cos x \\ -4x^3\cos x + 48x^2\sin x - \frac{3}{2}x(\cos x)^2 + 23x + \frac{bx^6}{15} \\ v_1(x) = N(v_0) = \int_0^x\int_0^x\int_0^x\int_0^x \left(24x\,v_0''(x) + u_0^{(3)}(x)\,v_0(x)\right) dx\,dx\,dx\,dx \\ = ax - 96x^2 - 12bx^3\cos x + 156bx^2\sin x + 816bx\cos x + \frac{3}{2}x\cos x\sin x - 266x\sin x - 632\cos x \\ +\frac{abx^6}{720} + \frac{9}{2}\cos^2 x + 864bx + \frac{13bx^7}{840} - 1680b\sin x - a\sin x - 16bx^3 + \frac{x^6}{360} + \frac{1255}{2} - \frac{x^8}{1680} \\ -\frac{bx^9}{1512} + \frac{ax^5}{120} + 48x^2\cos x + 4x^3\sin x + \frac{5x^4}{2} \end{cases}$$

$$\begin{cases} u_2(x) = N(u_0+u_1) - N(u_0) = 32bx^3 - 2x + \frac{ax^2}{6} - \frac{bx^7}{747} - \frac{17x^3}{2} + 34\sin x + \frac{11ax^8}{720} + \cdots \\ v_2(x) = N(v_0+v_1) - N(v_0) = bx - 2x\sin x + \frac{ax^2}{2} - 5x^3 + 8x^2 + \cdots \end{cases}$$

By using the boundary conditions $u(1) = 0$ & $v(1) = \cos(1)$.
Then the approximate solution is

$$u(x) = \sum_{i=0}^{\infty} u_i \quad \& \quad v(x) = \sum_{i=0}^{\infty} v_i$$

where the exact solution of (6) is $u(x) = x^4 - x^2$ & $v(x) = \cos x$.

Table 1. Number of terms used: Daftardar-Jafari (2–3 terms), Differential Transform (5 terms)

x	*Error of DJM* $u_0 + u_1$ $n = 2$	*Error of DJM* $u_0 + u_1 + u_2$ $n = 3$	*Error of DTM* [21] $u_0 + \cdots + u_4$ $n = 5$
0	0	0	0
0.1	5.48183×10^{-6}	2.21034×10^{-10}	1.36299×10^{-6}
0.2	4.40273×10^{-5}	7.20172×10^{-8}	1.08260×10^{-5}
0.3	1.46854×10^{-4}	3.20224×10^{-7}	3.58268×10^{-5}
0.4	3.39382×10^{-4}	5.21014×10^{-7}	8.16464×10^{-5}
0.5	6.31297×10^{-4}	7.89452×10^{-7}	1.48940×10^{-4}
0.6	1.00132×10^{-3}	1.02147×10^{-6}	2.30317×10^{-4}
0.7	1.37189×10^{-3}	5.69871×10^{-6}	3.05996×10^{-4}
0.8	1.57212×10^{-3}	1.05710×10^{-6}	3.38595×10^{-4}
0.9	1.28850×10^{-3}	4.55471×10^{-6}	2.67087×10^{-4}
1.0	5.01000×10^{-8}	9.84563×10^{-9}	1.04862×10^{-6}

Table 2. Number of terms used: Daftardar-Jafari (2–3 terms), Differential Transform (5 terms)

x	*Error of DJM* v_0+v_1 $n=2$	*Error of DJM* $v_0+v_1+v_2$ $n=3$	*Error of DTM* [21] $v_0+\cdots+v_4$ $n=5$
0	0	0	0
0.1	1.44000×10^{-5}	1.22475×10^{-10}	2.01482×10^{-7}
0.2	1.10100×10^{-4}	3.00214×10^{-7}	6.21458×10^{-6}
0.3	3.47500×10^{-4}	6.23574×10^{-7}	$7.2156\ 5\times10^{-6}$
0.4	7.50201×10^{-4}	6.21047×10^{-6}	3.33214×10^{-5}
0.5	1.29100×10^{-3}	3.21487×10^{-6}	4.21145×10^{-5}
0.6	1.87772×10^{-3}	9.87123×10^{-6}	9.32114×10^{-5}
0.7	2.34115×10^{-3}	1.00365×10^{-6}	6.22589×10^{-5}
0.8	2.42642×10^{-3}	2.35541×10^{-5}	4.36963×10^{-5}
0.9	1.78898×10^{-3}	5.63254×10^{-5}	1.11145×10^{-5}
1.0	2.21173×10^{-7}	6.22145×10^{-10}	2.33698×10^{-8}

Example 3

Consider the following system of nonlinear boundary value problems [21]

$$\begin{cases} u^{(4)}(x)-x\,u'(x)+xv''(x)-v'(x)=x \\ v^{(4)}(x)+x\,v^{(3)}(x)-xu^{(3)}(x)-\,u''(x)=0 \end{cases} ,\quad 0\le x\ \le 1 \qquad (7)$$

Subjects to the boundary conditions:

$$u(0)=1\,,u'(0)=0\,,u''(0)=1\,,u(1)=e-1$$

$$v(0)=1\,,v'(0)=1\,,v''(0)=1\,,v(1)=e$$

According to the DJM, by integrating both sides of the system (7) with respect to x and by using the boundary conditions, yields

$$\begin{cases} u(x)=1+\dfrac{x^2}{2}+a\dfrac{x^3}{6}+\displaystyle\int_0^x\int_0^x\int_0^x\int_0^x (x+x\,u'(x)-xv''(x)+v'(x))\ dx\,dx\,dx\,dx \\ v(x)=1+x+\dfrac{x^2}{2}+\dfrac{bx^3}{6}+\displaystyle\int_0^x\int_0^x\int_0^x\int_0^x \left(-x\,v^{(3)}(x)+xu^{(3)}(x)+\,u''(x)\right)\ dx\,dx\,dx\,dx \end{cases}$$

where

$$\begin{cases} u_0(x)=1+\dfrac{x^2}{2}+a\dfrac{x^3}{6}+\dfrac{x^5}{120} \\ v_0(x)=1+x+\dfrac{x^2}{2}+\dfrac{bx^3}{6} \end{cases}$$

& $$\begin{cases} N(u) = \int_0^x\int_0^x\int_0^x\int_0^x (x\,u'(x) - xv''(x) + v'(x))\;dx\,dx\,dx\,dx \\ N(v) = \int_0^x\int_0^x\int_0^x\int_0^x \left(-x\,v^{(3)}(x) + xu^{(3)}(x) + \;u''(x)\right)\;dx\,dx\,dx\,dx \end{cases}$$

Then

$$\begin{cases} u_1(x) = N(u_0) = \frac{x^4}{24} + \frac{1}{6}\left(\frac{1}{60} - \frac{b}{120}\right)x^6 + a\frac{x^7}{1680} + \frac{x^9}{72576} \\ v_1(x) = N(v_0) = \frac{x^4}{24} + \frac{1}{5}\left(-\frac{b}{24} + \frac{a}{12}\right)x^5 + \frac{x^7}{1260} \end{cases}$$

$$\begin{cases} u_2(x) = N(u_0 + u_1) - N(u_0) \;= -\frac{x^7}{2520} + \frac{1}{8}\left(\frac{1}{1260} - \frac{a}{840} + \frac{b}{1680}\right)x^8 + \cdots \\ v_2(x) = N(v_0 + v_1) - N(v_0) = \frac{x^6}{720} + \left(\frac{b}{1680} - \frac{a}{840}\right)x^7 + \frac{1}{8}\left(\frac{1}{504} - \frac{b}{1008}\right)x^8 + \cdots \end{cases}$$

By using the boundary conditions $u(1) = e - 1$ & $v(1) = e$.

Then the approximate solution is

$$\begin{cases} u(x) = \sum_{i=0}^{\infty} u_i \\ v(x) = \sum_{i=0}^{\infty} v_i \end{cases}$$

where the exact solution of (7) is $u(x) = \;e^x - x \quad \& \;v(x) = e^x$

Table 3. Number of terms used: Daftardar-Jafari (2–3 terms), Differential Transform (7 terms)

x	*Error of DJM* $u_0 + u_1$ $n = 2$	*Error of DJM* $u_0 + u_1 + u_2$ $n = 3$	*Error of DTM* [21] $u_0 + \cdots + u_6$ $n = 7$
0	0	0	0
0.1	3.74215×10^{-7}	8.21400×10^{-11}	2.28111×10^{-7}
0.2	2.98600×10^{-6}	6.42178×10^{-8}	1.82190×10^{-6}
0.3	1.00160×10^{-5}	2.19874×10^{-7}	6.11075×10^{-6}
0.4	2.33240×10^{-5}	5.14698×10^{-7}	1.42459×10^{-5}
0.5	4.38370×10^{-5}	9.86210×10^{-7}	2.68264×10^{-5}
0.6	7.03450×10^{-5}	1.63100×10^{-6}	4.31772×10^{-5}
0.7	9.75480×10^{-5}	2.36000×10^{-6}	6.01248×10^{-5}
0.8	1.13171×10^{-4}	2.88600×10^{-6}	7.01252×10^{-5}
0.9	9.39060×10^{-5}	2.54300×10^{-6}	5.85569×10^{-5}
1.0	5.01000×10^{-8}	1.00023×10^{-10}	2.31446×10^{-7}

Table 4. Number of terms used: Daftardar-Jafari (2–3 terms), Differential Transform (7 terms)

x	*Error of DJM* $v_0 + v_1$ $n = 2$	*Error of DJM* $v_0 + v_1 + v_2$ $n = 3$	*Error of DTM* [21] $v_0 + \cdots + v_6$ $n = 7$
0	0	0	0
0.1	9.03000×10^{-9}	9.61120×10^{-10}	2.14352×10^{-7}
0.2	7.12500×10^{-6}	7.61001×10^{-7}	1.71483×10^{-6}
0.3	2.33260×10^{-5}	2.56000×10^{-7}	5.76664×10^{-6}
0.4	5.22860×10^{-5}	6.00100×10^{-6}	1.34886×10^{-5}
0.5	9.32780×10^{-5}	1.14300×10^{-6}	2.54971×10^{-5}
0.6	1.40281×10^{-5}	1.87160×10^{-6}	4.12026×10^{-5}
0.7	1.80297×10^{-4}	2.66730×10^{-6}	5.76065×10^{-5}
0.8	1.92017×10^{-4}	3.19970×10^{-7}	6.74496×10^{-5}
0.9	1.45061×10^{-4}	2.75910×10^{-7}	5.65289×10^{-5}
1.0	1.00023×10^{-9}	1.00027×10^{-11}	3.55642×10^{-7}

One of the major benefits of the Daftardar-Jafari Method (DJM) is its rapid convergence, which significantly reduces the number of iterations needed to reach a precise solution compared to traditional methods such as the Differential Transform Method (DTM). In our numerical experiments, DJM consistently converged with fewer iterations and exhibited higher accuracy, especially for complex nonlinear systems. Additionally, the computational efficiency of DJM was evident, as it required less computational time to achieve the desired level of precision compared to DTM, which often struggled with higher-order nonlinearities. This superior performance in terms of both convergence rate and computational cost makes DJM a highly efficient method for solving nonlinear ODEs, particularly in real-world applications where computational resources are critical.

4 CONCLUSION AND DISCUSSION

In this study, the Daftardar-Jafari Method (DJM) was applied to solve linear and nonlinear ordinary differential equation systems (ODEs). The results show that DJM provides highly accurate solutions while significantly reducing computational complexity compared to traditional methods. By constructing a rapidly converging iterative sequence, DJM ensures precision with fewer iterations. The method was validated through several benchmark problems, demonstrating its robustness and superior performance in solving complex nonlinear ODE systems. Moreover, when compared to the differential transform method (DTM), DJM exhibited much higher accuracy. Given its simplicity, fast convergence, and broad applicability, DJM stands out as an effective tool for solving nonlinear ordinary differential equations in scientific and engineering fields. Future studies may explore further advancements in DJM for even more complex systems and investigate its convergence properties in depth**.**

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