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Common Fixed Point Theorem for Compatible Mappings of Type (A-1) in Intuitionistic Fuzzy Metric Space

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ABSTRACT

In this paper, common fixed point results for two pairs of compatible mappings of type (A-1) satisfying contractive condition on intuitionistic fuzzy metric space are established.

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1. INTRODUCTION

The human reasoning involves the use of variable whose values are fuzzy sets. Description of system behavior in the language of fuzzy rules, lowers the need for precision in data gathering and data manipulation, and may be viewed as a form of data compression. But there are situations when description by a (fuzzy) linguistic variable given in terms of a membership function only, seems too rough. The use of linguistic variables represents a physical significant paradigm shift in system analysis.

Atanassov [1] introduced the notion of intuitionistic fuzzy sets by generalizing the notion of fuzzy set by treating membership as a fuzzy logical value rather than a single truth value. For an intuitionistic set, the logical value has to be consistent (in the sense $\gamma A(x) + \mu A(x) \geq 1$). $\gamma A(x)$ and $\mu A(x)$ denotes degree of membership and degree of non-membership, respectively. All results which hold for fuzzy sets can be transformed into Intuitionistic fuzzy sets but converse need not be true. Intuitionistic fuzzy set can be viewed in the context as a proper tool for representing hesitancy concerning both membership and non-membership of an element to a set. To be more precise, a basic assumption of fuzzy set theory that, if we specify the degree of membership of an element in a fuzzy set as a real number from [0, 1], say 'a', then the degree of its non-membership is automatically determined as '(1 – a)', need not hold for intuitionistic fuzzy sets. In intuitionistic fuzzy set theory, it is assumed that non-membership should not be more than (1 – a). For instant, lack of knowledge (hesitancy concerning both membership and non-membership of an element to a set) and the temperature of a patient changes and other symptoms are not quite clear. Intuitionistic fuzzy set theory has been used to extract information by reflecting and modelling the hesitancy present in real-life situations. The application of intuitionistic fuzzy sets instead of fuzzy sets means the introduction of another degree of freedom into a set description. By employing intuitionistic fuzzy sets in databases, we can express a hesitation concerning examined objects.

Coker [2] introduced the concept of intuitionistic fuzzy topological spaces. Alaca *et al.* [3] proved the well-known fixed point theorems of Banach [4] in the setting of intuitionistic fuzzy metric spaces. Later on, Turkoglu *et al.* [5] proved Jungck's [6] common fixed point theorem in the setting of intuitionistic fuzzy metric space. No wonder that intuitionistic fuzzy fixed point theory has become an area of interest for specialists in fixed point theory as intuitionistic fuzzy mathematics has covered new possibilities for fixed point theorists.

Sessa[7] has introduced the concept of weakly commuting and Jungck [8] initiated the concept of compatibility. Cho [9] introduced the concept of compatible maps of type (α) and compatible maps of type (β) in fuzzy metric space.

The concept of type A-compatible and S-compatible was given by Pathak and Khan [10]. Pathak et. al. [11] renamed A-compatible and S-compatible as compatible mappings of type (A-1) and compatible mappings of type (A-2) respectively.

Singh et. al. [12] proved fixed point theorems in fuzzy metric space and menger space using the concept of semicompatibility, weak compatibility and compatibility of type (β) respectively.

2. PRELIMINARIES

Definition 1. Let X be any set. A fuzzy set A in X is a function with domain X and values in [0,1].

Definition 2. A binary operation $*$: [0,1]×[0,1] →[0,1] is continuous t-norm if $*$ satisfies the following conditions for all $a,b,c,d \in [0,1]$,

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$.

Definition 3. A binary operation $\Diamond$: [0,1]×[0,1] → [0,1] is continuous t-conorm if $\Diamond$ satisfies the following conditions for all $a,b,c,d \in [0,1]$,

(i) $\Diamond$ is commutative and associative;
(ii) $\Diamond$ is continuous;
(iii) $a \Diamond 0 = a$;
(iv) $a \Diamond b \geq c \Diamond d$ whenever $a \leq c$ and $b \leq d$.

Definition 4. A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2\times[0,\infty)$ satisfying the following conditions :

(i) $M(x,y, t) + N(x,y, t) \leq 1$ for all $x,y \in X$ and $t > 0$;
(ii) $M(x,y,0) = 0$ for all $x,y \in X$;
(iii) $M(x,y, t) = 1$ for all $x,y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x,y, t) = M(y,x, t)$ for all $x,y \in X$ and $t > 0$;
(v)$M(x,y, t) * M(y, z, s) \leq M(x, z, t+s)$ for all $x,y, z \in X$ and $s, t > 0$;
(vi) for all $x,y \in X$, $M(x,y, .) : [0, \infty) \to [0,1]$ is left continuous;
(vii) $\lim_{t\to\infty} M(x,y,t) = 1$ for all $x,y \in X$ and $t > 0$;
(viii) $N(x,y,0) = 1$ for all $x,y \in X$;
(ix) $N(x,y, t) = 0$ for all $x,y \in X$ and $t > 0$ if and only if $x = y$;
(x) $N(x,y, t) = N(y,x, t)$ for all $x,y \in X$ and $t > 0$;
(xi) $N(x,y, t) \Diamond N(y, z, s) \geq N(x, z, t+s)$ for all $x,y, z \in X$ and $s, t > 0$;
(xii) for all $x,y \in X$, $N(x,y, .) : [0, \infty) \to [0,1]$ is right continuous;
(xiii) $\lim_{t\to\infty} N(x,y,t) = 0$ for all $x,y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t respectively.

Remark 1. Every fuzzy metric space $(X,M,*)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \Diamond)$ such that t-norm $*$ and t-conorm $\Diamond$ are associated as $x\Diamond y = 1-((1-x) * (1-y))$ for all $x,y \in X$.

Remark .2. In intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$, $M(x,y,.)$ is non-decreasing and $N(x,y,.)$ is non-increasing, for all $x,y \in X$.

Alaca *et al.* [3] introduced the following notions:

Definition 5. Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Then
(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$.
(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$, $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$.

Definition 6. An intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Example 1. Let $X = \left\{\frac{1}{n} : n = 1,2,3,..\right\} \cup \{0\}$ and let $*$ be the continuous t-norm and $\Diamond$ be the continuous t-conorm defined by $a * b = ab$ and $a\Diamond b = \min\{1,a+b\}$ respectively, for all $a,b \in [0,1]$. For each $x,y \in X$ and $t > 0$, define (M,N) by $M(x,y,t) = \frac{t}{t+|x-y|}$ if $t > 0$, $M(x,y,0) = 0$ and $N(x,y,t) = \frac{|x-y|}{t+|x-y|}$ if $t > 0$, $N(x,y,0) = 1$. Clearly, $(X, M, N, *, \Diamond)$ is complete intuitionistic fuzzy metric space.

Definition 7. Two self mappings P and Q of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ are said to be compatible , if $\lim_{n\to\infty} M(PQx_n,QPx_n,t) = 1$ and $\lim_{n\to\infty} N(PQx_n,QPx_n,t) = 0$ for all $t > 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$, for some z in X.

Definition 8. Two self mappings P and Q of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ are said to be compatible of type (A) , if $\lim_{n\to\infty} M(PQx_n,QQx_n,t) = \lim_{n\to\infty} M(QPx_n,PPx_n,t) = 1$ and $\lim_{n\to\infty} N(PQx_n,QQx_n,t) = \lim_{n\to\infty} N(QPx_n,PPx_n,t) = 0$ for all $t > 0$, whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$, for some z in X.

Definition 9. Two self mappings P and Q of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ are said to be compatible of type (A-1) , if $\lim_{n\to\infty} M(QPx_n,PPx_n,t) = 1$ and $\lim_{n\to\infty} N(QPx_n,PPx_n,t) = 0$ for all $t > 0$,whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$, for some z in X.

Alaca [3] proved the following results:

Lemma 1. Let $(X, M, N,*,\Diamond)$ be intuitionistic fuzzy metric space and for all $x,y \in X$, $t > 0$ and if for a number $k > 1$ such that $M(x,y,kt) \geq M(x,y,t)$ and $N(x,y,kt) \leq N(x,y,t)$ then $x = y$.

Lemma 2. Let $(X, M, N,* \Diamond)$ be intuitionistic fuzzy metric space and for all $x,y \in X$, $t > 0$ and if for a number $k > 1$ such that $M(yn+2,yn+1, t) \geq M(yn+1,yn,kt)$, $N(yn+2,yn+1, t) \leq N(yn+1,yn,kt)$, Then $\{yn\}$ is a cauchy sequence in X.

Next, we give some properties of compatible mappings of type (A-1) which will be used in our main theorem.

Proposition 1. Let S and T be self maps of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$. If the pair (S, T) are compatible of type (A-1) and $Sz = Tz$ for some z in X then $STz = TTz$.

Proposition 2. Let S and T be self maps of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ with $t * t > t$ and $(1-t) \Diamond (1-t) \leq (1-t)$ for all t in [0,1]. If the pair (S,T) are compatible of type (A-1) and $Sx_n, Tx_n \to z$ for some z in X and a sequence $\{x_n\}$ in X then $TTx_n \to Sz$ if S is continuous at z.

Proposition 3. Let S and T be self maps of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$. If the pair (S, T) are compatible of type (A-1) and $Sz = Tz$ for some z in X then $TSz = SSz$.

3. COMMON FIXED POINT THEOREMS

In this section, we study common fixed point theorems for compatible mappings of type (A-1) .

Theorem 1. Let $(X, M, N, *, \Diamond)$ be a complete intuitionistic fuzzy metric space with $t * t \geq t$ and $(1-t) \Diamond (1-t) \leq (1-t)$. Let A, B, S and T be selfmappings of X such that the following conditions are satisfied:

(i) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$,

(ii) S and T are continuous,

(iii) There exists $k \in (0,1)$ such that for every $x, y \in X$, and $t > 0$

$$M(Ax,By,kt) \geq \{M(Sx,Ty,t) * M(Ax,Sx,t) * M(By,Ty,t) * M(Ax,Ty,t)\} \quad (1)$$

$$N(Ax,By,kt) \leq \{ N(Sx,Ty,t) \Diamond N(Ax,Sx,t) \Diamond N(By,Ty,t) \Diamond N(Ax,Ty,t)\} \quad (2)$$

If the pair (A,S) and (B, T) are compatible mappings of type (A-1) , then A, B, S and T have a unique common fixed point in X.

Proof Let x_0 be an arbitrary point in X. Since $A(X) \subset T(X)$ and $B(X) \subset S(X)$, there exist $x_1, x_2 \in X$ such that $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$. Inductively, we construct the sequences $\{y_n\}$ and $\{x_n\}$ in X such that

$$y_{2n+1} = Ax_{2n} = Tx_{2n+1}, y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$$

for n = 0,1, 2,….Now from (1) and (2) for $x = x_{2n}$, $y = x_{2n+1}$, we have

$M(Ax_{2n},Bx_{2n+1},kt) \geq \{M(Sx_{2n},Tx_{2n+1},t) * M(Ax_{2n},Sx_{2n},t) * M(Bx_{2n+1},Tx_{2n+1},t) * M(Ax_{2n},Tx_{2n+1},t) \}$

that is

$M(y_{2n+1}, y_{2n+2},kt) \geq \{M(y_{2n}, y_{2n+1},t) * M(y_{2n+1}, y_{2n},t) * M(y_{2n+2}, y_{2n+1},t) * M(y_{2n+1}, y_{2n+1},t)\}$

$M(y_{2n+1}, y_{2n+2},kt) \geq \{M(y_{2n}, y_{2n+1},t) * M(y_{2n+1}, y_{2n+2},t)\}$

$M(y_{2n+1}, y_{2n+2},kt) \geq M(y_{2n}, y_{2n+1},t)$

and

$N(Ax_{2n},Bx_{2n+1},kt) \leq \{N(Sx_{2n},Tx_{2n+1},t) \Diamond N(Ax_{2n},Sx_{2n},t) \Diamond N(Bx_{2n+1},Tx_{2n+1},t) \Diamond N(Ax_{2n},Tx_{2n+1},t) \}$

that is

$N(y_{2n+1}, y_{2n+2},kt) \leq \{N(y_{2n}, y_{2n+1},t) \Diamond N(y_{2n+1}, y_{2n},t) \Diamond N(y_{2n+2}, y_{2n+1},t) \Diamond N(y_{2n+1}, y_{2n+1},t)\}$

$N(y_{2n+1}, y_{2n+2},kt) \leq \{N(y_{2n}, y_{2n+1},t) \Diamond N(y_{2n+1}, y_{2n+2},t)\}$

$N(y_{2n+1}, y_{2n+2},kt) \leq N(y_{2n}, y_{2n+1},t)$

Similarly,

$M(y_{2n+2}, y_{2n+3},kt) \geq M(y_{2n+1}, y_{2n+2}, t)$ and $N(y2n+2,y2n+3,kt) \leq N(y2n+1,y2n+2, t)$.

Thus, we have

$M(y_{n+1}, y_{n+2},kt) \geq M(y_n, y_{n+1}, t)$ and $N(y_{n+1}, y_{n+2},kt) \leq N(y_n, y_{n+1}, t)$ for n = 1,2,3,....

Therefore, we have

$M(y_n, y_{n+1}, t) \geq M (y_n, y_{n+1},\frac{t}{q}) \geq M(y_{n-1}, y_n , \frac{t}{q^2}) \geq \ldots \geq M(y_1, y_2, \frac{t}{q^n}) \to 1,$

and $N(y_n, y_{n+1}, t) \le N(y_n, y_{n+1}, \frac{t}{q}) \le N(y_{n-1}, y_n, \frac{t}{q^2}) \le \dots \le N(y_1, y_2, \frac{t}{q^n}) \to 0$ when $n \to \infty$.
For each $\varepsilon > 0$ and $t > 0$, we can choose $n_0 \in$ N
such that $M(y_n, y_{n+1}, t) > 1 - \varepsilon$ and $N(y_n, y_{n+1}, t) < \varepsilon$ for each $n \ge n_0$
For $m,n \in$ N, we suppose $m \ge n$. Then, we have

$$M(y_n, y_m, t) \ge M(y_n, y_{n+1}, \frac{t}{m-n}) * M(y_{n+1}, y_{n+2}, \frac{t}{m-n}) * \dots * M(y_{m-1}, y_m, \frac{t}{m-n})$$
$$> ((1-\varepsilon) * (1-\varepsilon) * \dots (m-n) times\dots * (1-\varepsilon))$$
$$\ge (1-\varepsilon),$$

and $N(y_n, y_m, t) \le N(y_n, y_{n+1}, \frac{t}{m-n}) \lozenge N(y_{n+1}, y_{n+2}, \frac{t}{m-n}) \lozenge \dots \lozenge N(y_{m-1}, y_m, \frac{t}{m-n})$

$$< ((\varepsilon) \lozenge (\varepsilon) \lozenge \dots (m-n) times \dots \lozenge (\varepsilon))$$
$$\le (\varepsilon).$$

$M(y_n, y_m, t) > (1-\varepsilon)$, $N(y_n, y_m, t) < \varepsilon$.
Hence $\{y_n\}$ is a Cauchy sequence in X. As X is complete, $\{y_n\}$ converges to some point $z \in X$. Also, its subsequences converges to this point $z \in X$, i.e. $\{Bx_{2n+1}\} \to z$, $\{Sx_{2n}\} \to z$, $\{Ax_{2n}\} \to z$, $\{Tx_{2n+1}\} \to z$.
Since the pair (A,S) and (B,T) are compatible mappings of type (A-1), then from proposition 2, we have $AAx_{2n} \to Sz$ and $BBx_{2n+1} \to Tz$
(3)

By (1) for x = Ax_{2n} and $y = Bx_{2n+1}$, we have
$M(AAx_{2n},BBx_{2n+1},kt) \ge \{M(SAx_{2n},TBx_{2n+1},t) * M(AAx_{2n},SAx_{2n},t) * M(BBx_{2n+1},TBx_{2n+1},t) * M(AAx_{2n},TBx_{2n+1},t)\}$
Taking limit $n \to \infty$, using (3) and proposition 1 we get
$M(Sz,Tz,kt) \ge \{M(Sz,Tz,t) * M(Sz,Sz,t) * M(Tz,Tz,t) * M(Sz,Tz,t)\}$
$M(Sz,Tz,kt) \ge M(Sz,Tz,t)$
By (2) for x = Ax_{2n} and $y = Bx_{2n+1}$, we have
$N(AAx_{2n},BBx_{2n+1},kt) \le \{N(SAx_{2n},TBx_{2n+1},t) \lozenge N(AAx_{2n},SAx_{2n},t) \lozenge N(BBx_{2n+1},TBx_{2n+1},t) \lozenge N(AAx_{2n},TBx_{2n+1},t)\}$
Taking limit $n \to \infty$, using (3) and proposition 1 we get
$N(Sz,Tz,kt) \le \{N(Sz,Tz,t) \lozenge N(Sz,Sz,t) \lozenge N(Tz,Tz,t) \lozenge N(Sz,Tz,t)\}$
$N(Sz,Tz,kt) \le N(Sz,Tz,t)$
By lemma 1, Sz = Tz.
(4)

Again by inequality (1), for $x = z$ and $y = Bx_{2n+1}$, we have
$M(Az,BBx_{2n+1},kt) \ge \{M(Sz,TBx_{2n+1},t) * M(Az,Sz,t) * M(BBx_{2n+1},TBx_{2n+1},t) * M(Az,TBx_{2n+1},t)\}$
Taking limit $n \to \infty$, using (3) , (4)we get
$M(Az,Tz,kt) \ge \{M(Sz,Tz,t) * M(Az,Sz,t) * M(Tz,Tz,t) * M(Az,Tz,t)\}$
$M(Az,Sz,kt) \ge \{M(Sz,Sz,t) * M(Az,Sz,t) * M(Tz,Tz,t) * M(Az,Sz,t)\}$
$M(Az,Sz,kt) \ge M(Az,Sz,t)$.

Again by inequality (2), for x = z and $y = Bx_{2n+1}$, we have
$N(Az,BBx_{2n+1},kt) \le \{N(Sz,TBx_{2n+1},t) \lozenge N(Az,Sz,t) \lozenge N(BBx_{2n+1},TBx_{2n+1},t) \lozenge N(Az,TBx_{2n+1},t)\}$
Taking limit $n \to \infty$, using (3) , (4)we get
$N(Az,Tz,kt) \le \{N(Sz,Tz,t) \lozenge N(Az,Sz,t) \lozenge N(Tz,Tz,t) \lozenge N(Az,Tz,t)\}$
$N(Az,Sz,kt) \le \{N(Sz,Sz,t) \lozenge N(Az,Sz,t) \lozenge N(Tz,Tz,t) \lozenge N(Az,Sz,t)\}$
$N(Az,Sz,kt) \le N(Az,Sz,t)$.
By lemma 1, $Az = Sz$.
(5)

Again by inequality (1), for $x = z$ and $y = z$, we have
$M(Az,Bz,kt) \ge \{M(Sz,Tz,t) * M(Az,Sz,t) * M(Bz,Tz,t) * M(Az,Tz,t)\}$

Using (4) and (5)
$M(Az,Bz,kt) \geq \{M(Sz,Sz,t) * M(Sz,Sz,t) * M(Bz,Az,t) * M(Tz,Tz,t)\}$
$M(Az,Bz,kt) \geq M(Bz,Az,t)$
Again by inequality (2), for $x = z$ and $y = z$, we have
$N(Az,Bz,kt) \leq \{N(Sz,Tz,t) \Diamond N(Az,Sz,t) \Diamond N(Bz,Tz,t) \Diamond N(Az,Tz,t)\}$
Using (4) and (5)
$N(Az,Bz,kt) \leq \{N(Sz,Sz,t) \Diamond N(Sz,Sz,t) \Diamond N(Bz,Az,t) \Diamond N(Tz,Tz,t)\}$
$N(Az,Bz,kt) \leq N(Bz,Az,t)$
By lemma 1, $Az = Bz$
(6)

Thus from (4), (5) and (6) we get $Az = Bz = Sz = Tz$
(7)
Now we will prove that $Az = z$
By inequality (1), putting $x = z$ and $y = x_{2n+1}$,
$M(Az,Bx_{2n+1},kt) \geq \{M(Sz,Tx_{2n+1},t) * M(Az,Sz,t) * M(Bx_{2n+1},Tx_{2n+1},t) * M(Az,Tx_{2n+1},t)\}$
Taking limit $n \to \infty$, using (7) we get
$M(Az,z,kt) \geq \{M(Sz,z,t) * M(Az,Sz,t) * M(z,z,t) * M(Az,z,t)\}$
$M(Az,z,kt) \geq M(Az,z,t)$
By inequality (2), putting $x = z$ and $y = x_{2n+1}$,
$N(Az,Bx_{2n+1},kt) \leq \{N(Sz,Tx_{2n+1},t) \Diamond N(Az,Sz,t) \Diamond N(Bx_{2n+1},Tx_{2n+1},t) \Diamond N(Az,Tx_{2n+1},t)\}$
Taking limit $n \to \infty$, using (7) we get
$N(Az,z,kt) \leq \{N(Sz,z,t) \Diamond N(Az,Sz,t) \Diamond N(z,z,t) \Diamond N(Az,z,t)\}$
$N(Az,z,kt) \leq N(Az,z,t)$
By lemma 1, $Az = z$.

Combining all results, we get $z = Az = Bz = Sz = Tz$.
From this we conclude that z is a common fixed point of A, B, S and T.

Uniquess: Let z_1 be another common fixed point of A, B, S and T. Then
$z_1 = Az_1 = Bz_1 = Sz_1 = Tz_1$
and $z = Az = Bz = Sz = Tz$
then by inequality (1), putting $x = z$ and $y = z_1$, we get
$M(Az,Bz_1,kt) \geq \{M(Sz,Tz_1,t) * M(Az,Sz,t) * M(Bz_1,Tz_1,t) * M(Az,Tz_1,t)\}$
$M(z,z_1,kt) \geq \{M(z,z_1,t) * M(z,z,t) * M(z_1,z_1,t) * M(z,z_1,t)\}$
$M(z,z_1,kt) \geq M(z,z_1,t)$
and by inequality (2), putting $x = z$ and $y = z_1$, we get
$N(Az,Bz_1,kt) \leq \{N(Sz,Tz_1,t) \Diamond N(Az,Sz,t) \Diamond N(Bz_1,Tz_1,t) \Diamond N(Az,Tz_1,t)\}$
$N(z,z_1,kt) \leq \{N(z,z_1,t) \Diamond N(z,z,t) \Diamond N(z_1,z_1,t) \Diamond N(z,z_1,t)\}$
$N(z,z_1,kt) \leq N(z,z_1,t)$
By lemma 1, $z = z_1$.
Thus, z is the unique common fixed point of A, B, S and T.

If we increase the number of self maps from four to six, then we have the following.

Corollary 1. Let $(X, M, N, *, \Diamond)$ be a complete intuitionistic fuzzy metric space with $t * t \geq t$ and $(1-t) \Diamond (1-t) \leq (1-t)$. Let A, B, S, T, I and J be self mappings of X such that the following conditions are satisfied :

(i) $AB(X) \subseteq J(X)$ and $ST(X) \subseteq I(X)$,
(ii) I and J are continuous,
(iii) There exists $k \in (0,1)$ such that for every $x, y \in X$, and $t > 0$
$M(ABx,STy,kt) \geq \{M(Ix,Jy,t) * M(ABx,Ix,t) * M(STy,Jy,t) * M(ABx,Jy,t)\}$ (8)

$N(ABx,STy,kt) \leq \{ N(Ix,Jy,t) \lozenge N(ABx,Ix,t) \lozenge N(STy,Jy,t) \lozenge N(ABx,Jy,t)\}$ (9)

If the pair *(AB, I)* and *(ST, J)* are compatible mappings of type (A-1), then AB*, ST, I* and *J* have a unique common fixed point. Furthermore, if the pairs *(A, B), (A, I), (B, I), (S, T), (S, J)* and *(T, J)* are commuting mapping then *A, B, S, T, I* and *J* have a unique common fixed point.

Proof. From theorem 1, *z* is the unique common fixed point of AB*, ST, I* and *J*.
Finally, we need to show that *z* is also a common fixed point of *A, B, S, T, I*, and *J*. For this, let *z* be the unique common fixed point of both the pairs *(AB, I)* and *(ST, J)*. Then, by using commutativity of the pair *(A, B), (A, I),* and *(B, I)*, we obtain

$$Az = A(ABz) = A(BAz) = AB(Az),\ Az = A(Iz) = I(Az), \quad (10)$$
$$Bz = B(ABz) = B(A(Bz)) = BA(Bz) = AB(Bz),\ Bz = B(Iz) = I(Bz),$$

which shows that *Az* and *Bz* are common fixed point of *(AB, I)*, yielding thereby

$$Az = z = Bz = Iz = ABz \quad (11)$$

in the view of uniqueness of the common fixed point of the pair *(AB, I)*. Similarly, using the commutativity of *(S, T), (S, J), (T, J)* it can be shown that

$$Sz = Tz = Jz = STz = z. \quad (12)$$

Now, we need to show that *Az = Sz (Bz = Tz)* also remains a common fixed point of both the pairs *(AB, I)* and *(ST, J)*. For this, put *x = z* and *y = z* in (8) and using (11) and (12), we get

$M(ABz,STz,kt) \geq \{M(Iz,Jz,t) * M(ABz,Iz,t) * M(STz,Jz,t) * M(ABz,Jz,t)\}$
$M(Az,Sz,kt) \geq M(Az,Sz,t)$
and by (2)
$N(ABz,STz,kt) \leq \{ N(Iz,Jz,t) \lozenge N(ABz,Iz,t) \lozenge N(STz,Jz,t) \lozenge N(ABz,Jz,t)\}$
$N(Az,Sz,kt) \leq N(Az,Sz,t)$
By lemma 1, we get
Az = Sz. Similarly, it can be shown that *Bz = Tz*. Thus, *z* is the unique common fixed point of *A, B, S, T, I* and *J*.

4. CONCLUSION

The established results generalize some results of [11] in the setting of Intuitionistic Fuzzy Metric Space.

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