

Numerical Analysis of Non-Darcian Forchheimer Free Convective Flow in a Vertical Channel with Thermal Radiation

Yusuf A.B¹ and Umar A.^{2*}

^{1,2}Department of Mathematics, Federal University Dutsin-Ma, Katsina State, Nigeria.

*Corresponding author : aliyuumar2029@gmail.com

Received: 19 January 2025

Revised: 17 June 2025

Accepted: 28 June 2025

ABSTRACT

This article investigates the influence of temperature-dependent viscosity and thermal radiation on free convective flow in a vertical channel, Incorporating a Forchheimer equation in the momentum equation. The governing equations are formulated and solved numerically using the finite difference method (FDM). The effects of various parameters, such as the Forchheimer number (Fr), radiation parameter(R), viscosity variation index(λ), and Grashof number (Gr), on the velocity and temperature profiles are analyzed. The results indicate that the inclusion of the Forchheimer equation significantly affects the thermal and flow fields, offering enhanced understanding of heat transfer behavior in porous media.

Keywords: Temperature-dependent viscosity, thermal radiation, free convective flow, suction/injection, finite-difference method.

1 INTRODUCTION

Free convective flows in vertical channels are of significant interest due to their applications in engineering and industrial processes, such as cooling of electronic equipment, solar collectors, and heat exchangers. The inclusion of thermal radiation and porous media effects adds complexity to the heat transfer mechanisms, warranting comprehensive analysis for accurate modelling. Thermal radiation plays a vital role in high-temperature environments, significantly affecting the temperature and velocity profiles in convective flows. In addition, the Darcy dissipation term, accounting for viscous dissipation within porous media, is crucial in modelling energy transfer. Despite its importance, several previous studies have either neglected this term or assumed steady-state conditions. The current study addresses these gaps by exploring the combined effects of thermal radiation and Darcy dissipation on unsteady free convective flow in a vertical channel.

Earlier works provide a strong foundation. Raptis and Perdikis [1] studied the influence of thermal radiation on viscous fluid flow past a semi-infinite vertical plate, emphasizing its impact on temperature distribution. Similarly, Molla et al. [2] analysed radiation effects on natural convection along a vertical wavy surface and found substantial changes in the heat transfer rate. Das et al. [3] extended this research to MHD convective flow in a vertical channel, showing that radiation enhances

both heat transfer and velocity profiles. Zaheer and Tasawar [4] noted that thermal radiation reduces the thickness of the velocity and thermal boundary layers while increasing surface temperature.

The effects of radiation have also been examined in porous media contexts. Hossain et al. [5] analysed free convection from a porous vertical plate, introducing the radiation parameters R_d and θ_w . Their results showed that increasing R_d thins the boundary layer, while θ_w thickens it. Suction helps stabilize the layer at a constant thickness as emissivity increases. Ferdousi et al. [6] investigated variable viscosity effects in a porous medium with heat generation using the finite difference method and the Keller–Box scheme. They concluded that variable viscosity significantly influences the surface shear stress, rate of heat transfer, and the velocity and temperature profiles. Likewise, Makinde and Ogulu [7] explored thermal radiation effects on the heat and mass transfer of a variable viscosity fluid in the presence of a transverse magnetic field, underscoring the combined influence of radiation and magnetic fields on transport phenomena.

The concept of viscous dissipation in porous media was introduced by Vafai and Tien [8], who showed that neglecting this term can lead to substantial errors in predicting temperature fields. Nield and Bejan [9] provided an in-depth analysis of convective heat transfer in porous media, highlighting the importance of Darcy dissipation. Mahdy and Chamkha [10] expanded this understanding by examining mixed convection flows, emphasizing Darcy dissipation's role in enhancing the heat transfer. Unsteady free convective flows have also attracted considerable attention. Gebhart and Pera [11] conducted seminal work on the stability of natural convection, offering insights into transient flow behaviour. Hossain and Wilson [12] studied unsteady convection past an impulsively started vertical plate, revealing significant transient effects on velocity and temperature fields. Roy et al. [13] examined unsteady MHD convective flow in a vertical channel, highlighting complex interactions between magnetic fields and convection. Ahmed et al. [14] applied the finite element method to simulate unsteady flow in a curved pipe, while Li et al. [15] employed the lattice Boltzmann method to explore wall roughness effects in unsteady flow.

LeVeque [16], in his book *Numerical Methods for Conservation Laws*, outlines various numerical techniques including finite difference and finite volume methods, offering foundational approaches for solving channel flow problems. Patankar [17] introduced the SIMPLE algorithm for solving the Navier–Stokes equations, now widely used in computational fluid dynamics. Zienkiewicz and Taylor [18] provided a comprehensive study of the finite element method, while recent advancements by Kumar and Singh [19] have improved the accuracy and efficiency of these methods.

Further notable studies include Takhar et al. [20], who investigated unsteady free convection over an infinite vertical porous plate under the influence of a magnetic field and Hall current. Minto et al. [21] analyzed free convection over a vertical surface embedded in a porous medium. Yih [22] examined transpiration effects over a vertical cone in a porous medium, and Sinha et al. [23] explored MHD free convective flow past a vertical plate with a ramped wall temperature, using the Laplace transform to obtain analytical solutions. These studies consistently show the importance of porosity, magnetic fields, and time dependence in shaping flow and thermal characteristics.

In summary, while many studies have addressed individual aspects of radiation, variable viscosity, or unsteady convection, this study contributes by incorporating the combined effects of thermal radiation, temperature-dependent viscosity, and Darcy dissipation into the governing equations of unsteady free convective flow in a vertical channel. The problem is solved numerically using the finite

difference method, and the influence of key parameters on the thermal and velocity profiles is thoroughly analyzed.

2 PHYSICAL AND MATHEMATICAL PROBLEM DESCRIPTION

Physical Problem Description

The physical problem under consideration involves a vertical channel formed by two infinite parallel plates separated by a distance h . The channel is filled with an optically thick, incompressible, viscous fluid subjected to an incident radiative heat flux q_r , which is absorbed by the plates and transferred to the fluid (see Figure 1).

Assumptions:

- All fluid properties are constant except for viscosity, which is temperature-dependent.
- The fluid is optically thick, so radiative heat flux is modeled using non-linear Rosseland diffusion.
- Initially (at time $t = 0$), both the fluid and the plates are at rest with a uniform temperature T_0 .
- At $t > 0$, the plate at $y' = 0$ is suddenly heated to a temperature T_w , while the plate at $y' = h$ is kept at the initial temperature T_0 .
- The flow is fully developed, implying the axial velocity component u' is a function of transverse coordinate y' and time t only.
- Viscous dissipation is included.
- The radiative heat flux in the x' -direction is negligible compared to that in the y' -direction.

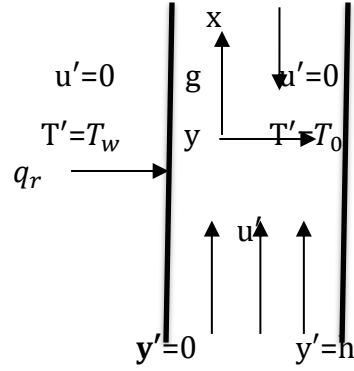


Figure 1: Schematic of the vertical channel with imposed temperature conditions.

Under these assumptions, the appropriate governing equations for the present problem in dimensional form are:

Governing Equations (Dimensional Form)

The system is governed by the following coupled partial differential equations:

Momentum Equation (x' -direction):

$$\frac{\partial u'}{\partial t} + V_0 \frac{\partial u'}{\partial y'} = \frac{1}{\rho} \frac{\partial}{\partial y'} \left(\mu \frac{\partial u'}{\partial y'} \right) + g\beta (T' - T_0) - \left(\frac{\mu}{\rho K} u' + \beta u'^2 \right) \quad (1)$$

Energy Equation (non-linear Rosseland approximation):

$$\frac{\partial T'}{\partial t} + V_0 \frac{\partial T'}{\partial y'} = \alpha \left(\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{k} \frac{\partial q_r}{\partial y'} \right) + \frac{\mu}{\rho C_p} \left(\frac{\partial u'}{\partial y'} \right)^2 \quad (2)$$

Boundary and Initial Conditions

At $t = 0$:

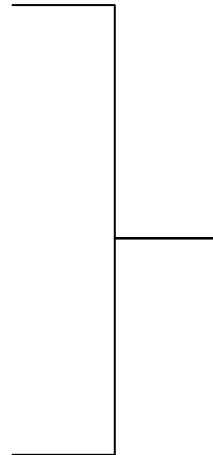
$$u' = 0, \quad T' = T_0 \text{ for } 0 \leq y' \leq h$$

At $y' = 0$ (heated plate):

$$u' = 0, \quad T' = T_w$$

At $y' = h$ (cooling plate):

$$u' = 0, \quad T' = T_0$$



(3)

where $u'(m/s)$ is the Velocity in x' -direction, $y'(m)$ Transverse coordinate, $t(s)$ is Time, $T'(K)$ is the Temperature, $T_0(K)$ is the Initial/ambient temperature, $T_w(K)$ is Wall temperature at $y' = 0$, $\mu(T')$ (pa.s) is the Temperature-dependent viscosity, $\rho(kg/m^3)$ is the Density of fluid, $C_p(J/kg \cdot K)$ is the Specific heat at constant pressure, $k(W/m \cdot K)$ is the Thermal conductivity, $g(m/s^2)$ is the Acceleration due to gravity, $q_r(W/m^2)$ is Radiative heat flux.

Eq. (1) represents the momentum equation with the first term on the LHS of the equation representing the rate of change of velocity (u) with respect to time (t). it describes how the velocity of the fluid changes over a time. the second term is representing convection term. It describes the transport of fluid velocity (u) in the y direction due to the mean flow velocity (V_0). While in the RHS of the equation we have the third term representing diffusion term. It describes the diffusion of fluid velocity (u) in the y direction due to the viscosity of the fluid (μ). And the fourth term is representing gravitational force term. It describes the effect of the temperature difference ($T' - T_0$) on the fluid flow. The term is proportional to the acceleration due to gravity (g), the coefficient of the thermal expansion (β) and the temperature difference between the fluid (T') and the reference temperature (T_0). It accounts for the density variation in the fluid caused by temperature differences, which in turn affect the fluid flow. The fifth term is representing Forchhermer term. Similarly, in the energy equation (Eq. (2)), in the L.H.S of the equation the first term represents the rate of change of temperature (T) with respect to time (t). it describes how temperature of the system changes over time. And the second term represents the convective heat transfer term. It describes the heat transfer due to the velocity (V_0) of the fluid in the y -direction and the temperature gradient in the same direction. In the R.H.S of the equation we have the third term represents conductive heat transfer term. It accounts for the heat transfer due to the temperature gradient and the radiative heat flux (q_r) in the y -direction. The term is scaled by the thermal diffusivity (α) and the thermal conductivity (K). The fourth term represents darcy dissipation term. Here μ is the dynamic viscosity of the fluid, u is the velocity of the fluid, K is the thermal conductivity, ρ is the density of the fluid and C_p is the specific heat capacity. This term accounts for the conversion of mechanical energy into heat due to fluid viscosity.

3 VARIABLE PROPERTIES AND RADIATIVE HEAT FLUX

The fluid dynamic viscosity (μ) is assumed to vary with temperature difference as:

$$\mu = \mu_0(1 - \lambda\theta(y)), \text{ where } \lambda \in \mathbb{R} \quad (4)$$

The radiative heat flux q_r , according to Sparrow and Cess [24], is given by:

$$q_r = - (4\sigma\partial T^4) / (3\delta\partial y) \quad (5)$$

4 NON-DIMENSIONALISATION OF THE EQUATIONS

To transform the governing equations into dimensionless form, we introduce the following variables:

$$\begin{aligned} \mu &= \mu_0(1 - \lambda((T - T_0)/(T_w - T_0))), \quad u = U_0 f(y), \quad y = y/\sqrt{(vt)}, \\ \theta(y) &= (T - T_0)/(T_w - T_0), \quad \delta = 2\sqrt{(vt)} \end{aligned} \quad (6)$$

Using Eq. (6) in the momentum equation, the dimensionless momentum equation becomes:

$$f''(y) = \frac{-1}{2}(y+c)f'(y)(1+\lambda\theta(y))(1-\lambda\theta(y)) + \lambda\theta'(y)f'(y)(1+\lambda\theta(y)) - Gr\theta(y)(1+\lambda\theta(y))(1-\lambda\theta(y)) + \frac{1}{Re}(1-\lambda\theta(y))f(y) + F_0(f(y))^2 \quad (7)$$

Using Eqs. (5) and (7) in the energy equation:

$$\frac{-y\theta'(y)(T_w - T_0)}{2} + \frac{V_0 t^{1/2}}{\sqrt{g}}\theta'(y)(T_w - T_0) = \frac{\alpha}{g}\left(\theta''(y)(T_w - T_0) + \frac{4\sigma}{3k\delta}\frac{\partial^2 T^4}{\partial y^2}\right) + \frac{\mu U_0^2 (f'(y))^2}{9\rho C_p} \quad (8)$$

But,

$$\frac{\partial^2 T^4}{\partial y^2} = \frac{\partial^2}{\partial y^2}(\theta(y)(T_w - T_0) + T_0)^4 = \frac{\partial}{\partial y}(4(\theta(y)(T_w - T_0) + T_0)^3 \theta'(y)(T_w - T_0)) \quad (9)$$

After simplification:

$$\frac{\partial^2 T^4}{\partial y^2} = 4 \bullet 3(T_w - T_0)^4 (\theta(y) + \phi)^2 (\theta'(y))^2 + 4(T_w - T_0)^4 (\theta(y) + \phi)^3 \theta''(y) \quad (10)$$

Substituting Eq. (10) into Eq. (8):

$$\begin{aligned} \theta''(y) = & \frac{-1}{2}Pr(y+c)\theta'(y)\left(1 - \frac{4R}{3}(\theta(y) + \phi)^3\right) \\ & - 4R(\theta(y) + \phi)^2 (\theta'(y))^2 \left(1 - \frac{4R}{3}(\theta(y) + \phi)^3\right) \\ & - Br(1-\lambda\theta(y))(1+\lambda\theta(y))(f'(y))^2 \left(1 - \frac{4R}{3}(\theta(y) + \phi)^3\right) \end{aligned} \quad (11)$$

Boundary Conditions:

$$u = U_0, T = T_w \text{ at } y = 0 \rightarrow f(0) = 0, \theta(0) = 0$$

$$u = 0, T = T_0 \text{ at } y = h \rightarrow \theta(y) = f(y) = 1 \quad (12)$$

Dimensionless Parameters:

$$Gr = \frac{g\beta(T_w - T_0)\delta^2}{4gU_0}, c = \frac{V_0 t^{1/2}}{\sqrt{g}}, Re = \frac{k}{V_0 t}, \frac{\mu_0}{\rho} = V_0, F_0 = \frac{\rho\beta U_0 \delta^2}{4\mu}$$

$$\phi = \frac{T_0}{T_w - T_0}, R = \frac{4\sigma}{k\delta}(T_w - T_0)^3, Ec = \frac{U_0^2}{Cp(T_w - T_0)}, Pr = \mu Cp/k, Pr \cdot Ec = Br \quad (13)$$

5 METHOD OF SOLUTION

5.1 Mathematical Description of FDM

The Finite Difference Method (FDM) is a numerical technique used to solve differential equations by approximating derivatives with finite differences. This method involves discretizing the continuous domain (spatial or temporal) into a finite set of points and then approximating the derivatives at these points using difference equations. Common references for FDM techniques include: Smith [25], 'Numerical Solution of Partial Differential Equations', and Chapra and Canale [26], 'Numerical Methods for Engineers'.

The following steps are important in solving ODEs using the finite difference method:

- i. Discretization: The continuous domain is divided into a grid of discrete points. For example, if we have a domain $[a, b]$, we can divide it into (N) intervals of equal length $(h = (b-a)/N)$.
- ii. Finite Difference Approximation: Derivatives in the differential equation are replaced with finite difference approximations. For instance, the first derivative of a function $u(x)$ at a point (x_i) can be approximated using the central difference method:

$$du/dx \approx (u_{i+1} - u_{i-1}) / (2h). \quad (14)$$

The second derivative is approximated as:

$$d^2u/dx^2 \approx (u_{i+1} - 2u_i + u_{i-1}) / h^2. \quad (15)$$

- iii. Formulating the Difference Equations: Substitute the finite difference approximations into the original differential equation to obtain a system of algebraic equations.

5.2 FDM Solution of the Problem

Equations (5) and (6) under the boundary conditions (3) are solved using FDM as follows:

Discretized form of the dimensionless momentum equation:

$$\begin{aligned} \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2} &= \frac{-1}{2}(y_i + c) \left(\frac{f_{i+1} - f_{i-1}}{2h} \right) (1 + \lambda \theta(y))(1 - \lambda \theta(y)) \\ &+ \lambda \left(\frac{\theta_{i+1} - \theta_{i-1}}{2h} \right) \left(\frac{f_{i+1} - f_{i-1}}{2h} \right) (1 + \lambda \theta(y)) \\ &- Gr \theta(y)(1 + \lambda \theta(y))(1 - \lambda \theta(y)) + \frac{1}{Re} (1 - \lambda \theta(y)) f(y_i) + F_0 f(y_i)^2 \end{aligned} \quad (16)$$

Discretized form of the dimensionless energy equation:

$$\begin{aligned} \left(\frac{\theta_{i-1} - 2\theta_i + \theta_{i+1}}{h^2} \right) &= \frac{-1}{2} Pr(y_i + c) \left(\frac{\theta_{i+1} - \theta_{i-1}}{2h} \right) \left(1 - \frac{4R}{3} (\theta(y) + \phi)^3 \right) \\ &- 4R(\theta(y) + \phi)^2 \left(\frac{\theta_{i+1} - \theta_{i-1}}{2h} \right)^2 \left(1 - \frac{4R}{3} (\theta(y) + \phi)^3 \right) \\ &- Br(1 - \lambda \theta(y))(1 + \lambda \theta(y)) \left(\frac{f_{i+1} - f_{i-1}}{2h} \right)^2 \left(1 - \frac{4R}{3} (\theta(y) + \phi)^3 \right) \end{aligned} \quad (17)$$

Boundary conditions:

At $y = 0$: $f(0) = 0$, $\theta(0) = 0$

At $y = h$: $f(h) = 1$, $\theta(h) = 1$

5.3 Nusselt Number and Skin Friction on the Channel Plates

In this study, the Nusselt number and skin friction are evaluated to characterize the thermal and flow behavior on the channel plates. Due to the assumed variable viscosity of the fluid, the skin friction coefficient is particularly important in understanding the shear stress distribution near the walls.

Skin friction (τ) at the wall is defined as:

$$\tau_0 = (1 - \lambda \theta) \frac{df}{dy} \text{ at } y = 0 \text{ and } \tau_1 = (1 - \lambda \theta) \frac{df}{dy} \text{ at } y = 1.$$

- μ : dynamic viscosity of the fluid, λ : viscosity variation parameter, θ : dimensionless temperature, f : dimensionless stream function, df/dy : velocity gradient

Nusselt number (Nu) is used to quantify the rate of heat transfer at the wall and is defined as:

$$NU_0 = -\frac{d\theta}{dy} \text{ at } y = 0 \text{ and } NU_1 = -\frac{d\theta}{dy} \text{ at } y = 1.$$

- θ : dimensionless temperature, $d\theta/dy$: temperature gradient normal to the surface

These expressions are evaluated at the channel walls ($y = 1$) because that is where both skin friction and heat transfer are most significant due to the no-slip boundary condition and thermal interaction with the plates. In internal flows, these wall interactions critically influence the overall flow and heat transfer characteristics.

6 RESULTS AND DISCUSSION

The governing equations (10) and (11) were solved numerically using **MATLAB** with the following parameters:

- **Prandtl number (Pr)** = 0.71 (representing air and R-12 refrigerant).
- **Subintervals (N)** = 100, step size **h** = 0.01 on the interval **[1, 0]**, where **a = 1** and **b = 0**.
- **Radiation (R)**, **suction (S)**, and **viscosity variation (λ)** parameters were varied between **0 and 3**.
- **Grashof number (Gr)** = 10, 12, 14 ($Gr > 0$, indicating cooling of the channel by free convection).

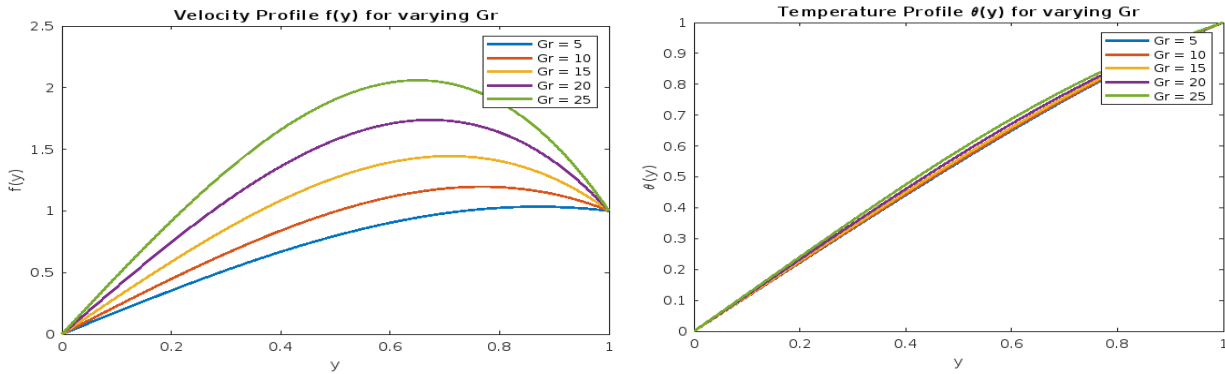


Figure 2: velocity and temperature profile for different Gr. ($\phi=0.2$, $Pr = 0.71$, $R = 0.1$, $c = 0.1$, $\lambda = 0.1$, $Br = 0.5$ $Gr = 5$, $Re = 0.04$, $Fr = 0.002$)

Figure 2 illustrates that higher Grashof number (Gr) enhance buoyancy-driven convection, increasing both fluid velocity and temperature due to improved heat transfer efficiency.

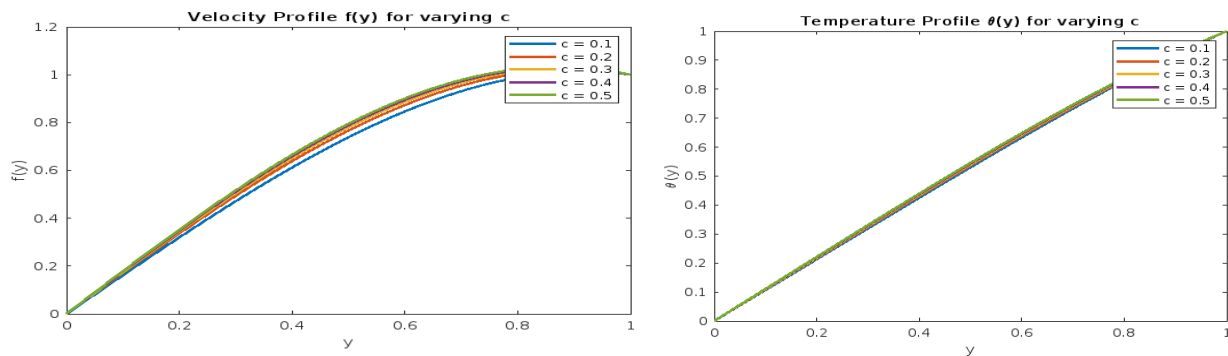


Figure 3: velocity and temperature profile for different C. ($\phi=0.2$, $Pr = 0.71$, $R = 0.1$, $c = 0.1$, $\lambda = 0.1$, $Br = 0.5$ $Gr = 5$, $Re = 0.04$, $Fr = 0.002$)

Figure 3 shows that higher suction improves fluid flow and heat transfer by thinning the boundary layers and intensifying thermal gradients.

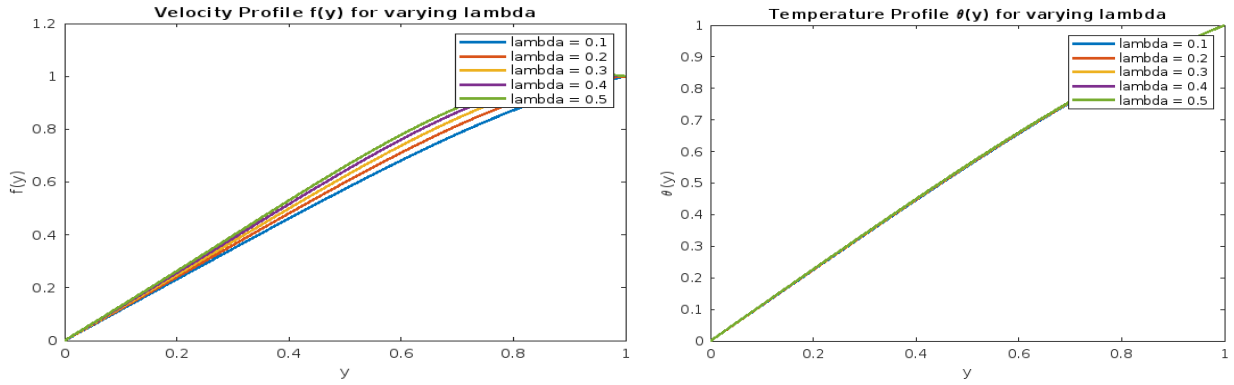


Figure 4: velocity and temperature profile for different λ . ($\phi=0.2$, $Pr = 0.71$, $R = 0.1$, $c = 0.1$, $\lambda = 0.1$, $Br = 0.5$ $Gr = 5$, $Re = 0.04$, $Fr = 0.002$)

Figure 4 shows that as λ increases, fluid velocity and temperature profiles rise due to reduced viscosity, which enhances flow but diminishes convective heat transfer, thickening the thermal boundary layer and increasing temperature.

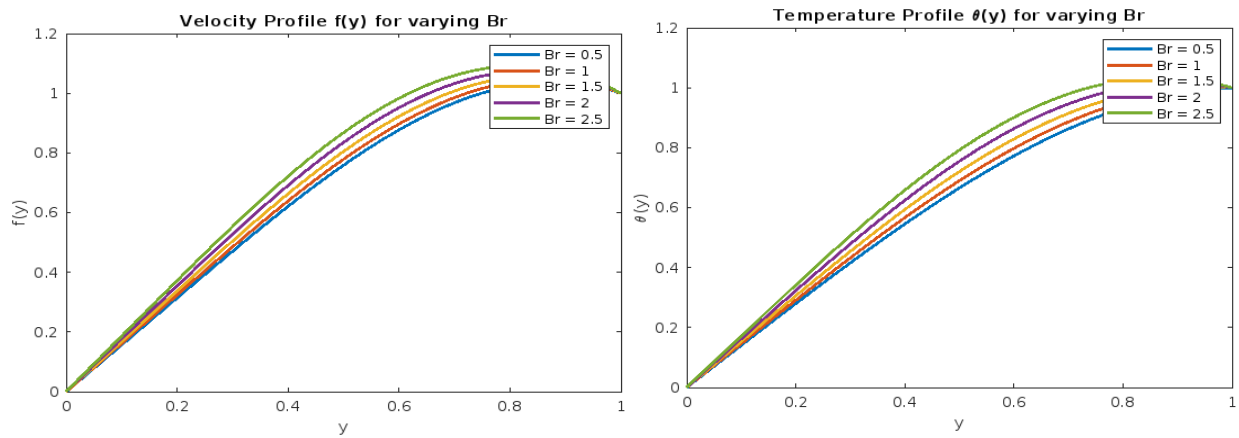


Figure 5: velocity and temperature profile for different Br . ($\phi=0.2$, $Pr = 0.71$, $R = 0.1$, $c = 0.1$, $\lambda = 0.1$, $Br = 0.5$ $Gr = 5$, $Re = 0.04$, $Fr = 0.002$)

Figure 5 above demonstrates that the velocity and temperature profiles increase with increase in Brinkman number (Br), this is due to the fact that, when Br increases, the effects of viscous heating become more significant, leading to increased velocity and temperature profiles.

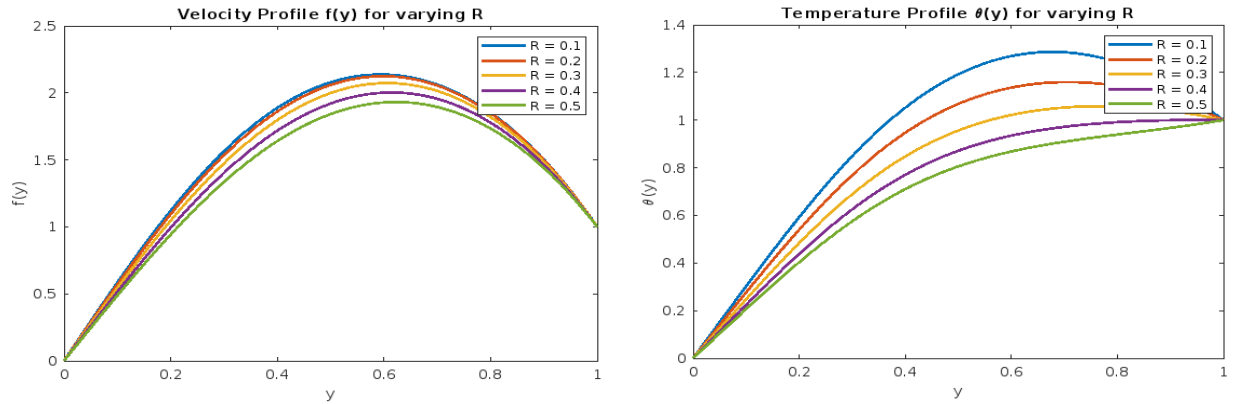


Figure 6: velocity and temperature profile for different R . ($\phi=0.2$, $Pr = 0.71$, $R = 0.1$, $c = 0.1$, $\lambda = 0.1$, $Br = 0.5$ $Gr = 5$, $Re = 0.04$, $Fr = 0.002$)

Figure 6 above shows that; decrease in Radiation Parameter R lead to the increase in temperature and velocity profiles.

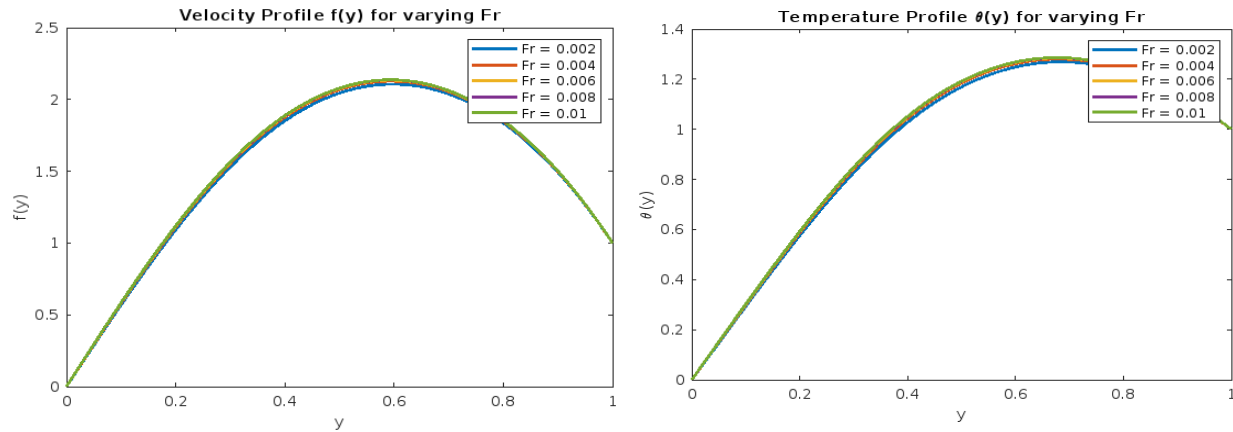


Figure 7: velocity and temperature profile for different Fr . ($\phi=0.2$, $Pr = 0.71$, $R = 0.1$, $c = 0.1$, $\lambda = 0.1$, $Br = 0.5$ $Gr = 5$, $Re = 0.04$, $Fr = 0.002$)

Figure 7 above depicted that: increase in Forchheimer parameter (Fr) led to increase in velocity and temperature profiles, this is obviously true because an increase in the Forchheimer parameter leads to more pronounced porous media effects, non-Darcy behavior, and enhanced convective heat transfer, resulting in higher velocity and temperature profiles.

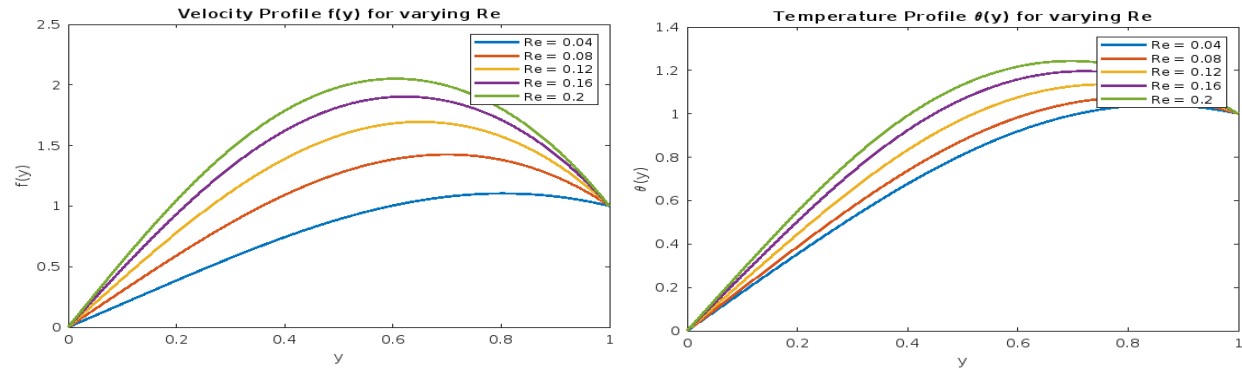


Figure 8: velocity and temperature profile for different Re. ($\phi=0.2$, $Pr = 0.71$, $R = 0.1$, $c = 0.1$, $\lambda = 0.1$, $Br = 0.5$
 $Gr = 5$, $Re = 0.04$, $Fr = 0.002$)

Figure 8 demonstrates that higher Reynolds numbers intensify fluid motion and heat transfer, resulting in increased velocity and temperature profiles due to stronger inertial effects and enhanced convection.

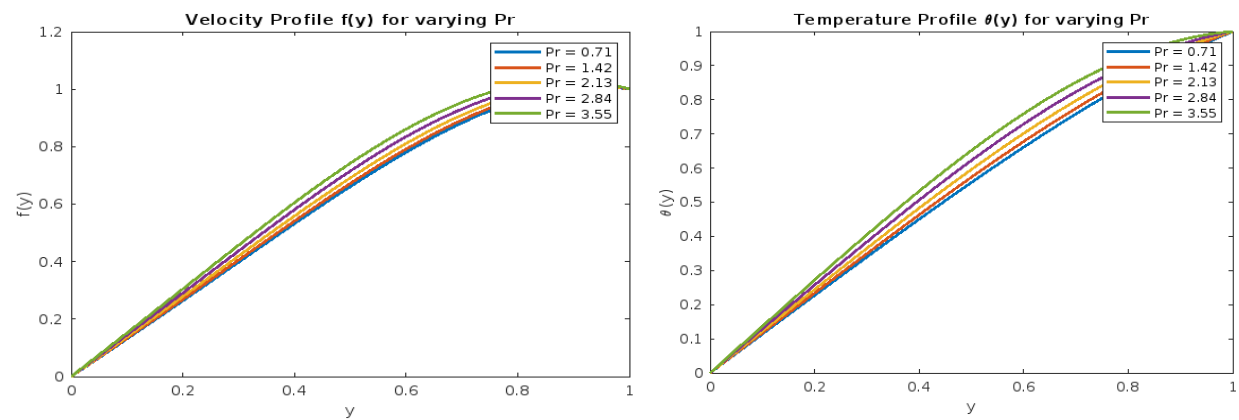


Figure 9: velocity and temperature profile for different Pr. ($\phi=0.2$, $Pr = 0.71$, $R = 0.1$, $c = 0.1$, $\lambda = 0.1$, $Br = 0.5$
 $Gr = 5$, $Re = 0.04$, $Fr = 0.002$)

Figure 9 shows that decreasing the prandtl number enhances thermal diffusion, resulting in reduced velocity and temperature profiles due to weakened convection and lower thermal gradients.

Table 1: Numerical values of skin friction on the walls.

λ	R = 0.1, $\varphi=0.2$, Gr = 5, C = 0.1, Br = 0.5, Fr=0.04		R = 0.2, $\varphi = 0.2$, Gr = 5, C = 0.1, Br = 0.5, Fr=0.04		R=0.1, $\varphi=0.5$, Gr=5, c=0.1, Br=0.5, Fr=0.04	
	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
0.1	0.282510	3.768100	0.283230	3.765500	0.282870	3.767500
0.2	0.263200	3.538800	0.263920	3.535700	0.263600	3.538100
0.3	0.241290	3.314300	0.241920	3.310700	0.241640	3.313600
0.4	0.216540	3.094900	0.217080	3.090800	0.216840	3.09400
0.5	0.188800	2.880700	0.189300	2.876200	0.189050	2.87970

Table 1 shows the effects of varying parameter(λ) on the skin friction. It reveals that the skin friction between the channel walls and the working fluids decreases on all the walls with increase in λ . Similarly, the skin friction is seen to decrease with increase in R and φ for some fixed values of Pr, c, and Gr.

Table 2: Nusselt Number on the channel plates.

Pr	R=0.1, $\varphi=0.2$, Gr=5, c=0.1, Br=0.5, Fr=0.04		R=0.3, $\varphi=0.2$, Gr=5, c=0.1, Br=0.5, Fr=0.04		R=0.1, $\varphi=0.5$, Gr=5, c=0.1, Br=0.5, Fr=0.04	
	Nu ₀	Nu ₁	Nu ₀	Nu ₁	Nu ₀	Nu ₁
0.016	1.01550	0.879000	1.026100	0.839300	1.024000	1.011600
0.44	1.02510	0.837200	1.034500	0.812800	1.029400	1.010100
0.71	1.03130	0.810500	1.039900	0.796000	1.029900	1.009200
1	1.03800	0.781700	1.045700	0.778100	1.033600	1.008200
2	1.06613	0.681700	1.066000	0.716500	1.046500	1.005100

The effect of Prandlt number (Pr) on the rate of heat transfer between the walls and the working fluid is displayed in Table II. The table displayed that; the Nusselt number on the wall stationed at $y = 0$ increases with increase in Pr while it is seen to decrease with increase in Pr on the wall stationed at $y = 1$. Furthermore, the Nusselt number on the wall kept at $y = 0$ is observed to increase with increase in R where as it is noticed to increase with increase in R on the wall at $y = 1$. Also, the Nusselt number on the wall positioned at $y = 0$ is found to decrease with increase in φ while it is viewed to increase on the wall at $y = 1$ with increase in φ .

7 CONCLUSION

This study investigates the unsteady free convective flow of a viscous, radiating fluid in a vertical channel using the finite difference method. The governing equations were numerically solved with MATLAB, and the results were presented through graphs and tables for analysis. The study confirms that fluid velocity increases with viscosity, indicating enhanced momentum diffusion. Additionally, an increase in thermal radiation reduces both temperature and velocity, demonstrating its damping effect on heat and momentum transfer. Furthermore, higher values of the Brinkman number, Reynolds number, and Forchheimer number were found to significantly enhance both fluid velocity and temperature, highlighting the roles of viscous dissipation, inertial effects, and porous media resistance in heat transfer enhancement. These findings contribute to a deeper understanding of thermal-fluid dynamics in radiating and porous media environments. They may be useful in optimizing industrial processes such as cooling systems, thermal insulation, and energy transport in porous structures. Future studies may explore non-linear effects or turbulence models for more complex flow behaviours.

8 ACKNOWLEDGEMENT

The researchers wish to acknowledge the effort of Prof. Ismail Baoku of the department of mathematics, federal university, dutsin-Ma katsina state; for his supports and guidance received. Similarly, the Author acknowledge the efforts of Dr Auwal Yusuf Bichi of the department of Mathematics, federal University Dutsin-Ma Katsina state Nigeria.

REFERENCES

- [1] A. Raptis and C. Perdikis, "Thermal radiation of an optically thin gray gas," *International Journal of Heat and Mass Transfer*, vol. 49, no. 1-2, pp. 225-230, 2006.
- [2] M. Molla, M. A. Hossain, and D. A. S. Rees, "Radiation effects on natural convection flow along a vertical wavy surface," *Acta Mechanica*, vol. 186, no. 1-4, pp. 75-86, 2006.
- [3] K. Das, R. N. Jana, and O. D. Makinde, "Thermal radiation and MHD effects on convective flow in a vertical channel," *Journal of Thermal Analysis and Calorimetry*, vol. 139, no. 2, pp. 1235-1248, 2020.
- [4] A. Zaheer and H. Tasawar, "Radiation effects on MHD flow in a porous space," *International Journal of Heat and Mass Transfer*, vol. 51, pp. 1024-1033, 2008.
- [5] M. A. Hossain, M. A. Alim, and D. A. S. Rees, "The effect of radiation on free convection from a porous vertical plate," *International Journal of Heat and Mass Transfer*, vol. 42, pp. 181-191, 1999.
- [6] A. Ferdousi, M. M. Rahman, M. S. Parvez, and M. A. Alim, "The effect of radiation on natural convection flow of fluid with variable viscosity from a porous vertical plate in presence of heat generation," *Applied and Computational Mathematics*, vol. 2, no. 2, pp. 54-63, 2013.

- [7] O. D. Makinde and A. Ogulu, "The effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field," *Chemical Engineering Communications*, vol. 195, no. 12, pp. 1575–1584, 2008.
- [8] K. Vafai and C. L. Tien, "Boundary and inertia effects on flow and heat transfer in porous media," *International Journal of Heat and Mass Transfer*, vol. 24, no. 2, pp. 195–203, 1981.
- [9] D. A. Nield and A. Bejan, *Convection in Porous Media*. Springer, 2006.
- [10] A. Mahdy and A. J. Chamkha, "Effects of Darcy dissipation on mixed convection flow in a porous medium," *Journal of Porous Media*, vol. 22, no. 5, pp. 451–462, 2019.
- [11] B. Gebhart and L. Pera, "The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion," *International Journal of Heat and Mass Transfer*, vol. 14, no. 12, pp. 2025–2050, 1971.
- [12] M. A. Hossain and M. Wilson, "Natural convection flow past an impulsively started vertical plate with Newtonian heating," *International Journal of Thermal Sciences*, vol. 42, no. 9, pp. 889–896, 2003.
- [13] S. Roy, S. Das, and R. N. Jana, "Unsteady MHD convective flow in a vertical channel with thermal radiation," *Journal of Applied Fluid Mechanics*, vol. 14, no. 1, pp. 123–134, 2021.
- [14] S. Ahmed, M. I. Khan, and M. Ali, "Numerical simulation of unsteady channel flow in a curved pipe," *Journal of Fluid Mechanics*, vol. 907, p. 1, 2021.
- [15] X. Li, L. Wang, and H. Zhang, "Effects of wall roughness on unsteady channel flow using the lattice Boltzmann method," *International Journal of Heat and Fluid Flow*, vol. 84, pp. 102945, 2020.
- [16] R. J. LeVeque, *Numerical Methods for Conservation Laws: Time-Dependent Problems*. Birkhäuser, 1992.
- [17] S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*. Hemisphere Publishing Corporation, 1980.
- [18] O. C. Zienkiewicz and R. L. Taylor, *The Finite Element Method: Its Basis and Fundamentals*. Butterworth-Heinemann, 2000.
- [19] A. Kumar and P. Singh, "Advances in numerical methods for fluid flow and heat transfer," *Computational Thermal Sciences*, vol. 14, no. 3, pp. 245–260, 2022.
- [20] H. S. Takhar, S. Roy, and G. Nath, "Unsteady free convection flow over an infinite vertical porous plate due to the combined effects of thermal and mass diffusion, magnetic field and Hall currents," *Heat and Mass Transfer*, vol. 39, pp. 825–834, 2003.
- [21] B. J. Minto, D. B. Ingham, and I. Pop, "Free convection driven by an exothermic reaction on a vertical surface embedded in porous media," *International Journal of Heat and Mass Transfer*,

vol. 41, pp. 11–24, 1998.

- [22] K. A. Yih, “Uniform transpiration effect in the combined heat and mass transfer by natural convection over a cone in saturated porous media: uniform wall temperature/concentration or heat/mass flux,” *International Journal of Heat and Mass Transfer*, vol. 42, pp. 3533–3537, 1999.
- [23] A. Sinha, N. Ahmed, and S. Agarwalla, “MHD free convective flow through a porous medium past a vertical plate with ramped wall temperature,” *Applied Mathematical Sciences*, vol. 11, no. 20, pp. 963–974, 2017.
- [24] E. M. Sparrow and R. D. Cess, *Radiation Heat Transfer*, augmented ed. Washington, D.C., USA: Hemisphere, 1962.
- [25] G. D. Smith, *Numerical Solution of Partial Differential Equations: Finite Difference Methods*, 3rd ed. Oxford, U.K.: Oxford University Press, 1985.
- [26] S. C. Chapra and R. P. Canale, *Numerical Methods for Engineers*, 7th ed. New York, NY, USA: McGraw-Hill Education, 2015.

NOMENCLATURE AND GREEK SYMBOLS

Symbol	Description	Unit
y'	Dimensional length	mm
g	Gravitational acceleration	m/s^2
k	Thermal conductivity	$W/m \cdot K$
T'	Dimensional temperature	K
H	Channel width (dimensional)	m
T_w	Wall temperature	K
T	Ambient temperature	K
v'	Dimensional velocity	m/s
μ	Dynamic viscosity	$kg/m \cdot s$
μ_0	Initial fluid viscosity	$kg/m \cdot s$
$\vartheta (\mu/\rho)$	Kinematic viscosity	m^2/s
α	Thermal diffusivity	m^2/s
β	Volumetric expansion coefficient	$1/K$
q_r	Radiative heat flux	W/m^2
φ	Temperature difference parameter	K
σ	Stefan–Boltzmann constant	$W/m^2 \cdot K^4$
V_0	Dimensional suction velocity	m/s