Applied Mathematics and Computational Intelligence
Volume 14, No. 2, 2025 [156-175]

Optimizing the Number of Hospital's Emergency Department in Petaling Jaya Using Set Covering Problem Analysis

Muhammad Alif Safwan Mohd Radzi[1*], Siti Suzlin Supadi[2]

[1,2]Institute of Mathematical Sciences, Faculty of Science, Universiti Malaya, 50603, Kuala Lumpur, Malaysia

[*]Corresponding author: muhammadalifsaf@gmail.com

Received: 10 December 2024
Revised: 27 February 2025
Accepted: 14 March 2025

ABSTRACT

The emergency department (ED) is a critical frontline service for unscheduled patients requiring urgent care. Its primary role is to diagnose, stabilize, and treat patients with life-threatening illness, critical injuries, or other emergencies. EDs operate 24/7 to provide continuous service. However, ensuring adequate coverage remains a challenge. Research shows that greater distance from an ED increases mortality rates. This study aims to determine the minimum number of EDs needed as ambulance locations and emergency providers to ensure fast response times for priority case one patients in Petaling Jaya. The Maximal Covering Location Problem (MCLP) and Maximum Expected Covering Location Problem (MEXCLP) are used to evaluate the efficiency of the existing EDs in covering demand. Results show that eight EDs are sufficient to meet all demand, optimizing emergency coverage. These findings can help improve emergency response strategies and resource allocation in Petaling Jaya.

Keywords: emergency department of hospital, mathematical programming, set covering location problem

1 INTRODUCTION

Every country has its own emergency medical service (EMS), which is part of the pre-hospital care system requiring a fast response to emergencies. This involves providing initial treatment and transportation by a trained emergency team, equipped with the necessary tools, to identify and transport patients to the ED of a hospital. When an emergency call is received, trained personnel determine whether the patient requires emergency care. They then dispatch the appropriate air or ground ambulances and other EMS responders to the scene. The team triages, treats, and transports the patient(s) to the appropriate medical facility for final care.

Based on [1], the EMS system in Malaysia begins with an individual's call via Malaysia Emergency Response Services 999 (MERS999) or the emergency number of a hospital, connecting to the Medical Emergency Coordinating Centre (MECC) in the Emergency Trauma Department of the hospital for priority decisions. Subsequently, an ambulance is directed to the patient's location and transports them to the ED.

The efficiency of the pre-hospital system is crucial for saving patients' lives, especially those in unstable conditions. Several factors need emphasis to increase the efficiency of pre-hospital care, such as the transportation of patients to the ED. According to [1], the Ministry of Health Malaysia has not achieved the key performance index (KPI) for the arrival time of ambulances to patient locations in under 15 minutes for priority case one, spanning from 2017 to 2021. The audit report cites management weaknesses in activating response teams at the nearest hospitals to the patient's location and the considerable distance between hospitals, ranging from 60 km to 443 km. Therefore, this scenario underscores the two importance of covering hotspot areas or demand points with ambulance services for emergency cases.

Understanding the pre-hospital system in Malaysia and its challenges, this research aims to investigate the optimum number of ED in Petaling Jaya based on travel duration from EDs to patient locations for priority case one using mathematical optimization methods.

Given that the objective of this optimization is related to location services, this research will utilize the MCLP model and MEXCLP model under mathematical optimization: set covering problem analysis.

1.1 Background of Study Area

Petaling Jaya, commonly known as PJ City, is one of the sub-districts with the highest population density in Malaysia. Together with Shah Alam, Subang Jaya, and Sungai Buloh, Petaling constitutes a district with a population of 2.3 million in 2023. Petaling Jaya covers an area of approximately 97.2 square kilometres, with over 520,000 residents and four main sections: Petaling Jaya Utara (PJU), Petaling Jaya Selatan (PJS), Sungai Way-Subang (SS), and Seksyen. These four main sections are further subdivided into smaller neighbourhoods.

There are 14 existing EDs that have the potential to cover specific demands in Petaling Jaya. Six of them are within Petaling Jaya, eight are around Petaling Jaya, and only Sungai Buloh Hospital is part of the government sector.

Table 1 : List of ED Hospitals

Label	Hospital	City Address
H-1	Assunta Hospital	Petaling Jaya
H-2	KPJ Damansara Specialist Hospital	Petaling Jaya
H-3	Columbia Asia Hospital	Petaling Jaya
H-4	Thomson Hospital	Petaling Jaya
H-5	Beacon International Specialist Centre	Petaling Jaya
H-6	KMI Kelana Jaya Medical Centre	Petaling Jaya
H-7	Park City Medical Centre	Kuala Lumpur
H-8	University Malaya Medical Centre	Kuala Lumpur
H-9	KPJ Damansara Specialist Hospital 2	Kuala Lumpur
H-10	MSU Medical Centre	Shah Alam
H-11	Ara Damansara Medical Centre	Shah Alam
H-12	Subang Jaya Medical Centre	Subang Jaya

H-13	Sunway Medical Centre	Subang Jaya
H-14	Sungai Buloh Hospital	Sungai Buloh

Petaling Jaya is an urban area with a high population, indicating the necessity for an efficient pre-hospital system capable of addressing potential demands. According to [2], cities have special vulnerabilities that should be considered when preparing for health emergencies. Unprepared urban environments are frequently the front lines of response efforts, making them more susceptible to the devastating effects of health emergencies and the escalation of disease outbreaks. Therefore, Petaling Jaya city has been chosen as the study area, with several potential demand points scattered around Petaling Jaya, including residential, industrial, medical, and education areas for medical priority case 1.

2 LITERATURE REVIEW

2.1 Pre-hospital in Malaysia

Prehospital care is defined by [3] as the period of patient care that begins at the scene of a sickness or accident and ends at the location of final treatment. [4] showed in their study prehospital emergency health services are increasingly important because they have been shown to decrease morbidity and mortality in common situations, improving recovery and survival rates, and fostering the growth of these services. To guarantee that emergency health services are supplied in a fair, efficient, and equal manner across the nation, it is crucial to assess the organizational structure and service delivery.

According to [3], the hospital-based system is the oldest and primary source of prehospital care in Malaysia, viewed as a complex system. Public or government hospitals provide most of these services, with the Civil Defence Department being the second-largest organization, offering prehospital care around the clock in most Malaysian cities. Private medical centres, although having a small role, typically charge for these services.

2.2 The Important of Ambulance and ED Location

Several recent review papers demonstrate the extensive examination of locating EMS vehicles [5]. In life-threatening situations, patients should be transported to nearest ED, highlighting the critical importance of ED distribution and accessibility [6]. A study by [7] found a correlation between longer travel times to hospital and higher mortality rates. Their findings indicate a 1% absolute increase in mortality for every 10 km increase in straight-line distance.

2.3 The Situation Emergency Case in Urban Area

EMS planning should be tailored to regional features due to differences between urban areas and other places [8]. A study by [9][9] showed that pre-hospital duration in urban areas is noticeably shorter than in rural areas. However, the study reported that urban areas have a higher EMS demand compared to suburban and rural areas [10]. According [2], the cooperation of several partners and sectors outside of health at all levels is necessary to improve health emergency preparation at the urban level. To maintain coherence in preparedness efforts and boost resilience, coordination across partners and sectors is essential. This coordination should involve all stakeholders, including the private sector and civil society.

2.4 Maximum Covering Location Model in Solving Location Problem

The difficulty of determining the best location for a given number of facilities inside a network is known as the coverage location problem. The goal is to optimize the overall demands of the covered population within certain bounds [11]. [12] presented MCLP for the first time to find an optimal location pattern within the desired service distance by locating a fixed number of facilities. [13] classified The Location Set Covering Problem (LSCP) and MCLP as two common classifications for the covering location problem. In a traditional MCLP, the goal is to maximize the covered population by strategically placing several facilities on a network. If a facility is located less than the threshold away from a demand node, it covers that node. This predetermined cutoff point, also known as the coverage radius, has an immediate impact on the problem's solution.

However, [14] suggested that solving MCLP alone may not provide a sufficiently robust location plan to solve the ambulance location problem. The MEXCLP proposed by [15] assumes that facilities covering demands may not be able to respond to demand due to a congested system.

In conclusion, this literature review provides a strong relationship between the importance of ambulance and ED locations and the increase in the mortality rate. Even though in urban areas such as Petaling Jaya, the traveling distance of ambulances to patient locations is shorter, the demand for pre-hospital care is much higher than in rural areas. Therefore, public or government hospitals may face challenges handling more cases. Collaboration between public and private hospitals is crucial to accepting critical patients nearest to their location and covering each other in case the EMS of the nearest hospital is overwhelmed due to high demand.

Additionally, this literature review highlights the MCLP model for solving location problems. However, due to high demand in urban areas, there is a probability that ambulance services or EDs may not support more patients. Therefore, the MEXCLP model used to find a robust model. Hence, in this research, MCLP and MEXCLP models were used to find the optimum number of hospitals as ambulance and ED providers.

3 METHODOLOGY

3.1 ED Location Information

In this research, hospitals that provide EMS within Petaling Jaya and its vicinity used as ambulance provider and ED centre. The data collected by using R software package: osmdata for retrieving and downloading data from OpenStreetMap (OSM). This package provides the information about name, availability of ED services, address, and the coordinates of hospitals. The coordinate location of hospitals was rechecked by using Google Map and adjusted from the centre of the building to the ED entrance. Afterward, the availability of ED services was checked on their respective websites, and hospitals that did not provide these services were removed. Also, EDs not included in OSM database were added to the list, bringing the total number of ED locations to 14. All the hospitals in the list are categorized by city address and labelled as Table 1.

3.2 Potential Demand Points Location Information

In this research, several types of areas were focused on as demand points including residential areas, medical areas, education areas, and industry areas. The data was also collected by using the osmdata package in R for retrieving information about the name of the area, type of area, and coordinate. All the coordinates were checked with Google Map and added information about address city, and sections of the coordinate. All points outside Petaling Jaya have been removed from the list, then total of demand point is 470 consist of 308 residentials, 45 industry area, 110 education area and 7 medical area.

3.3 Travel Distance and Duration Information

Another important parameter in MCLP and MEXCLP is the travel distance or duration between EDs and potential demands. The matrix distance and duration from each ED to each potential demand generated from osrm package in R. This package run on OSM and found the shortest route between two points in road network.

3.4 Set Covering Problem Analysis

The set covering location problem (SCLP) is set covering problem that related to location problem with distance or duration become cost of covering set. SCLP was used in this research to define the relationship between the set of ED and set of potential demand that has been defined in section 3.1 and 3.2.

Let $H = \{H_1, \dots, H_i, \dots, H_{14}\}$ and $D = \{D_1, \dots, D_j, \dots, D_{470}\}$ become collection's set element of ED and potential demands respectively, $R = \{R_1, \dots, R_a, \dots, R_{308}\}$, $I = \{I_1, \dots, I_b, \dots, I_{45}\}$, $E = \{E_1, \dots, E_c, \dots, E_{110}\}$, and $M = \{M_1, \dots, M_d, \dots, M_7\}$ become collection's set element of residential, industry area, education area and medical area respectively in Petaling Jaya where $R \cup I \cup E \cup M = D$ and $T_j = \{T_{j,1}, \dots, T_{j,14}\}$ become collection's set element of travel distance or duration from each element in H to potential demand D_j. The objective in this problem is to find what is the maximum element of D can be covered by set $N_m = \{n_1, \dots, n_m\}$ where is m is the number of ED to be located and N_m is a subset of H, $N_m \subset H$ if $T_{j,i} \leq l$, where l is limitation travel duration from H_i to D_j.

3.5 MCLP Formulation

The SCLP defines in section 3.4 can be formulated into MLCP integer linear programming as follow:

Sets,

H - Set of ED within and around Petaling Jaya.
i - Index of set H, $i = 1, \dots, 14$.
H_i - The i-th element of H
D - Set of potential demand in Petaling Jaya.
j - Index of set D, $j = 1, \dots, 470$.
D_j - The j-th element of D

Parameters,

d_j	-	The associated population demand value at D_j.
m	-	The number of ED to be located.
l	-	The associated travel duration.
$T_{j,i}$	-	Duration taken from H_i node to D_j node.
$t_{j,i}$	=	$\begin{cases} 1, & T_{j,i} \leq l \\ 0, & otherwise \end{cases}$

Decision Variables,

$$D_j = \begin{cases} 1, if\ at\ least\ one\ emergency\ can\ cover\ demand\ j \\ 0, otherwise \end{cases}$$

$$H_i = \begin{cases} 1, if\ the\ emergency\ department\ i\ is\ chose\ as\ demand\ coverage \\ 0, otherwise \end{cases}$$

Objective Function,

$$Max\ F = \sum_{j=1}^{470} d_j D_j \qquad (1)$$

Constraint,

$$\sum_{i=1}^{14} t_{j,i} H_i \geq D_j, \forall j = 1, \dots, 470 \qquad (2)$$

$$\sum_{i=1}^{14} H_i = m \qquad (3)$$

Equation (1) is our main objective which is to maximize demand population in Petaling Jaya that can be covered by at least one ED. In this research, all demand population values for each point assumed to have equal demand and associate value for $d_j = 1$ for all $j = 1, \dots, 470$. While for constraint in equation (2) suggest that at least one ED, H_i or none covering demand point, D_j. An additional constraint has been introduced in (3) to restrict the number of EDs that can be utilized to cover potential demand.

3.6 MEXCLP Formulation

The MEXCLP model incorporates an additional parameter p denoted as the probability of the EDs not working. In this model, it is assumed that all EDs share the same probability of not functioning simultaneously. The formulation of MEXCLP model as below:

Decision Variables,

$$D_{ij} = \begin{cases} 1 \text{ , } if\ at\ least\ i\ ED\ cover\ demand\ at\ node\ j \\ 0 \text{ , } if\ less\ i\ ED\ cover\ demand\ at\ node\ j \end{cases}$$

$$H_i = \begin{cases} 1 \text{ , } if\ the\ ED\ i\ is\ chose\ as\ demand\ coverage \\ 0 \text{ , } otherwise \end{cases}$$

Objective Function,

$$Max\ \mathrm{G} = \sum_{j=1}^{470}\sum_{i=1}^{14}(1-p)p^{i-1}d_jD_{ij} \quad (4)$$

Constraint,

$$\sum_{i=1}^{14} t_{j,i}H_i \geq \sum_{i=1}^{14} D_{ij}\ , \forall j = 1, \dots , 470 \quad (5)$$

$$\sum_{i=1}^{14} H_i = m \quad (6)$$

The objective of this model, given by (4) is to find the maximum expected coverage model value where D_j can be covered by more than one ED by satisfying constraint (5) and (6). The probability of p, can be due to various reasons such as the ED currently responding another emergency, a crowded environment in the ED, and maintenance of the ED, making it difficult to assign a specific p to each ED. Therefore, in this research, p is considered as a control parameter, ranging from zero to one in increments of 0.1.

3.7 Solving MCLP and MEXCLP

1-0 linear programming is used to solve both the MCLP and MEXCLP models by using the R package: lpsolver. All variable D_j, D_{ij} and H_i declared as binary numbers that can only have 1 or 0 value. Each model is optimal at a certain value of parameter d, t and p, if the inequality of (2) and (5) be satisfied by equality, $\sum_{i=1}^{14} t_{j,i}H_i = D_j$ and $\sum_{i=1}^{14} t_{j,i}H_i = \sum_{i=1}^{14} D_{ij}$ respectively. Based on [16], the algorithm is used in lpsolver for solving integer or binary linear programming is the Branch and Bound Algorithm (B&B).

4 RESULTS AND DISCUSSION

4.1 Result of MCLP Model for Travelling from ED to Potential Demand Point by Controlling Parameters

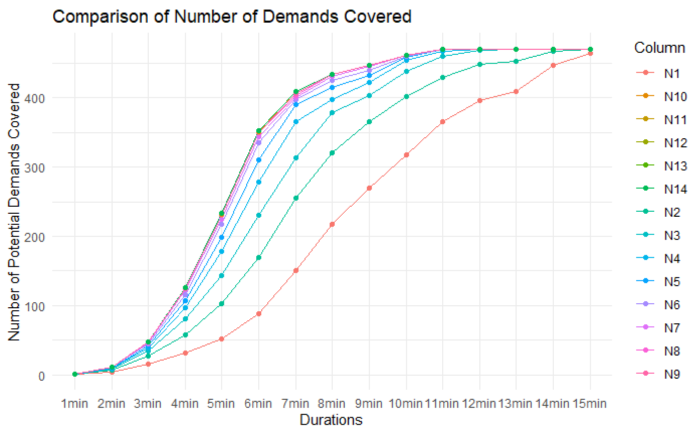

Figure 1 : The number of potential demands covered according to its travelling duration from EDs to the demand points.

Figure 1 shows the result of the objective value (1) for each limited number of EDs for different travelling durations from EDs to potential demand points by using the MCLP model. The labels N1, N2, ..., N14 represent the set of solutions N_m mentioned in section 3.4, where m refers to the limited number of EDs controlled in the MCLP model. For example, each point in red line labelled by N1 indicates the objective value (1) by limiting constraint (3), m = 1, for different travel duration, in minutes from EDs to the potential demand point covered by one ED. As the increase of limited number of EDs, the number of demands covered, N_m for each associated travel duration become more alike each other's. For the number of emergencies, N11, N12, N13 and N14, the number of potential demands covered are the same for each minute reduced. Also, the earliest duration to cover all the demand is 11 minutes where the limited number of ED starting from 5 until 14. Hence in this model, the travel duration of ambulances from the ED to the potential demands can be reduced, emphasizing a focus on demands below 11 minutes. Therefore, in Petaling Jaya, there will be no issue where the ambulance cannot reach its destination within 15 minutes.

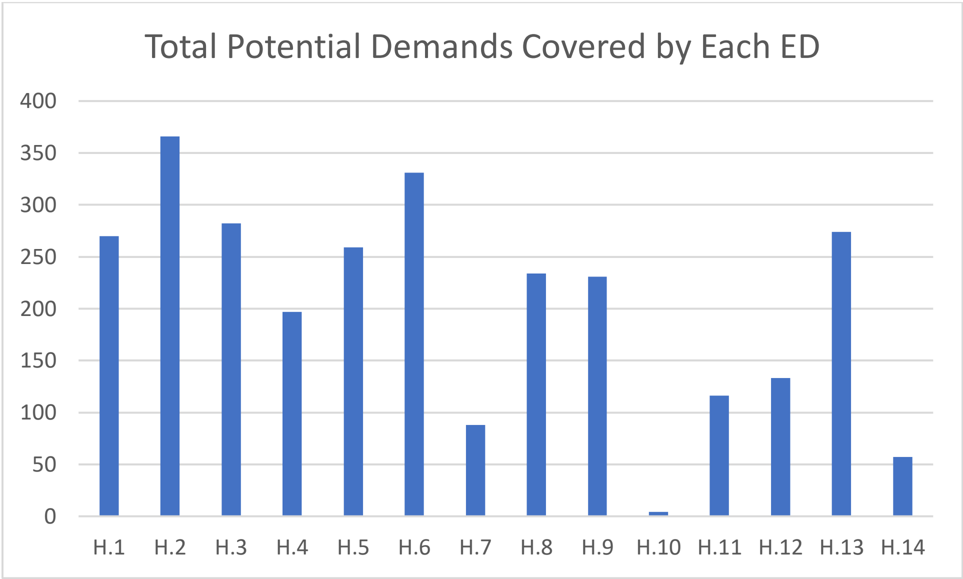

Figure 2 : The total potential demands covered by each ED.

Figure 2 shows the total demand of each ED covered in less than 11 minutes. H-2 covers the highest demand in total 366 points followed by H-6 by 331 demand points meanwhile, H-10 is the lowest only cover 4 demand points and followed by second lowest, H-14 cover 57 demand points.

4.2 The Removed ED on 11 minutes.

Based on the result in section 4.1, we are focusing on the removed ED in MCLP model when duration travel from EDs to demand point less or equal to 11 minutes.

Table 2 : The list of removal EDs from selection as a number of ED is limited at travel duration of 11 minutes.

Number of ED	Removed ED	Total Demand Covered
N13	H-13	470
N12	H-12, H-13	470
N11	H-11, H-12, H-13	470
N10	H-10, H-11, H-12, H-13	470
N9	H-1, H-10, H-11, H-12, H-13	470
N8	H-1, H-8, H-10, H-11, H-12, H-13	470
N7	H-1, H-7, H-8, H-10, H-11, H-12, H-13	470
N6	H-1, H-2, H-7, H-8, H-10, H-11, H-12, H-13	470
N5	H-2, H-3, H-5, H-7, H-8, H-10, H-11, H-12, H-13	470
N4	H-3, H-5, H-6, H-7, H-8, H-9, H-10, H-11, H-12, H-13	467
N3	H-2, H-3, H-5, H-6, H-7, H-8, H-9, H-10, H-11, H-12, H-13	460
N2	H-1, H-3, H-4, H-5, H-6, H-7, H-8, H-9, H-10, H-11, H-12, H-14	429
N1	H-1, H-3, H-4, H-5, H-6, H-7, H-8, H-9, H-10, H-11, H-12, H-13, H-14	366

Table 2 displays the list of EDs removed from the selection in the MCLP model at travel duration of 11 minutes. When reducing the number of EDs from 14 to 13, the removed ED in the MCLP model is H-13. However, this outcome does not imply that H-13 is unimportant in covering the demand, as the total covered demand still encompasses all demand points. The reason for the removal of H-13 lies in the behaviour of the B&B algorithm work in the lpsolver. The selection of EDs to satisfy constraint (3) allows the removal of the last variables if the total demand remains the same. Therefore, H-14 would have been removed instead of H-13 if the total demand had remained the same. However, H-13 was chosen because removing H-14 would have meant that the objective function (1) would not be maximized. This implies that demand covered by H-14 would only be covered once.

When limiting the number of EDs to nine, H-9 cannot be removed otherwise, the objective function (1) will not reach its maximum value. Consequently, H-1 is removed first instead of choosing H-8. Subsequently, the algorithm removes H-8 when total EDs are reduced to eight, but the total demand does not change. The selection process repeats when the number of EDs is reduced to six, where H-6 cannot be removed, prompting the algorithm to remove H-2 to maintain the objective value remain the same.

For N5, H-1 is reintroduced as an ED to cover demand. Despite removing two EDs, H-3 and H-5, the total objective value still reaches its maximum. Therefore, there exists a set of demands covered at least twice by H-1 and either H-3 or H-5. Therefore, to maximize the demand and satisfied constraint (3) when m = 5, H-3 and H-5 are removed from the model, while H-1 is reintroduced as a selected ED. Additionally, in N2, H-13 is reintroduced as an ED to cover demand, replacing H-6, even though H-6 is second highest covering demand. This implies that the set of demands covered by H-1 is more similar to H-6 compared to H-13, making the combination of H-1 and H-13 cover most of the demand area when number of EDs is limited to two.

Therefore, in this result, the universal set for covering all demands in MCLP model is H-1 ∪ H-4 ∪ H-6 ∪ H-9 ∪ H-14, where the other EDs are subsets of this union. Furthermore, H-14 cannot be removed to satisfy all demands, even though this ED only covers 57 demands, as there are elements or demand points that can only be covered by H-14. While all demands covered by H-1 can also be covered by other EDs, since H-1 is removed when N9, and yet all demands in Petaling Jaya are covered. Regarding H-4, H-6, and H-9, it is still unknown whether they have demands that can be covered once or more. However, it is certain that H-9 has demands that can covered once if H-9 is not a subset of H-10 ∪ H-11 ∪ H-12 ∪ H-13. Moreover, the removal of EDs while maintaining the maximum total demand does not provide strongly justify that EDs are not important in providing ambulances for demand, as all EDs have the same weightage in constraint (3).

4.3 EDs Covering Only Once Potential Demand

In the previous discussion, when excluding H-1, H-2, H-3, H-5, H-7, H-8, H-10, H-11, H-12, or H-13 from the selection of ED to cover potential demand above N5, the objective (1) reaches its maximum value. This indicates that each of these EDs is a subset of H-1 ∪ H-4 ∪ H-6 ∪ H-9 ∪ H-14. Among them, only H-14 is confirmed to have potential demand that can be covered once, while H-4, H-6 and H-9 remain unknown. Consequently, in this section, the order of variables in constraint (3) in the lpsolver is changed to prioritize H-4, H-6, and H-14 as the last variables to assess their selection in the MCLP model for N13.

```
> result$objval
[1] 470
> result$solution[471:484]
 [1] 0 1 1 1 1 1 1 1 1 1 1 1 1 1
```

Figure 3 : Results with H-4 as the last variables in lpsolver.

Figure 3 displays the outcomes of rearranging constraint (3) in the lpsolver with H-4 as the last variable. The objective value reaches its maximum, yet the removed ED is H-1 even though H-4 is the last variable. This suggests that H-4 covers demand points that are unique and only covered once by an ED.

```
> result$objval
[1] 470
> result$solution[471:484]
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 0
```

Figure 4 : Results with H-6 as the last variable in lpsolver.

When H-6 is placed as the last variable in the lpsolver, the objective value remains at 470, and the solution removes H-6 to satisfy constraint (3) as shown in Figure 4 above. This implies that all demands covered by H-6 can also be covered by another ED.

```
> result$objval
[1] 470
> result$solution[471:484]
 [1] 0 1 1 1 1 1 1 1 1 1 1 1 1 1
```

Figure 5 : Results with H-9 as the last variable in lpsolver.

Finally, when H-9 is placed as the last variable in the lpsolver as shown in Figure 5, then the B&B algorithm removes H-1 from the selection, indicating that H-9 also covers demand points that are unique and only covered once. Hence, from this section, it can be concluded that H-4, H-9, and H-14 are crucial EDs to cover the demand, as none of these EDs can be substituted for their unique demand coverage, particularly when the association duration is below 11 minutes.

4.4 Alternative Solution for N5

In integer linear programming, multiple optimal solutions may exist. Also, considering that all demand in H-1 and H-6 within the universal set H-1 ∪ H-4 ∪ H-6 ∪ H-9 ∪ H-14 can be covered by other ED, a question arises: does limiting the number of EDs to 5 provide an alternative optimum set of EDs? An alternative solution can be found by removing either variable H-1 or H-6 from constraint (3) while still maintaining the maximum objective value for N5.

When variable H-1 is removed from constraint (3), the result yields an alternative solution: H-4 ∪ H-5 ∪ H-6 ∪ H-9 ∪ H-14, with the objective value still reaching its maximum. However, removing both variables H-1 and H-5 from the model leads to a suboptimal objective value for N5. In contrast, the objective is maximized for N12, suggesting that there are demands from H-1 and H-5 covered by at least two EDs.

Similarly, when variable H-6 is removed, another solution emerges: H-1 ∪ H-4 ∪ H-9 ∪ H-11 ∪ H-14 with a same maximum objective value. Yet, removing both H-6 and H-11 from the model results in suboptimal objective values for both N5 and N12, indicating that there are demand points in H-6 ∪ H-11 can only be covered twice.

Therefore, from these new alternative solutions, it is evident that there exists another viable set of solutions: H-4 ∪ H-5 ∪ H-9 ∪ H-11 ∪ H-14, covering all demand in Petaling Jaya. Consequently, there are four different sets of solutions to cover all demand for N5. Additionally, it is discovered that there are demand points covered only twice by H-6 and H-11.

4.5 Result of MEXCLP Model for Traveling from ED to Potential Demand Point for Duration 11 Minutes

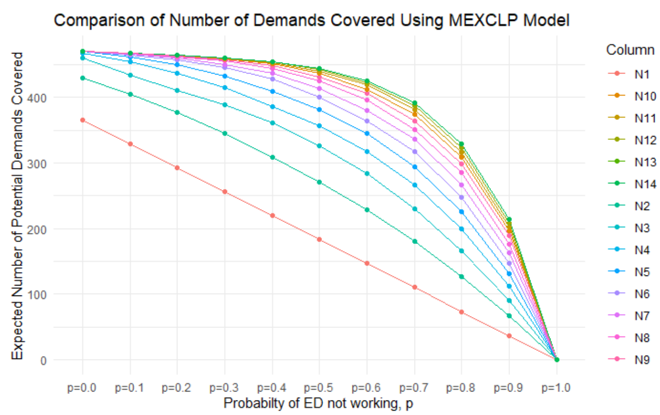

Figure 6 : The expected number of potential demands covered based on the probability of all EDs not working for each limited number of EDs.

Figure 6 illustrates the expected number of potential demands covered when the probability of all EDs not working, p is assumed to be the same and increases for specific limited numbers of EDs at the travel duration of 11 minutes. In the scenario where all EDs are operational (p = 0), the expected number of potential demands covered by the selected ED in MEXCLP model is identical as in the MCLP model for the same travel duration. The concavity of each line for the various limited numbers of EDs indicates the existence of demand points covered by more than one ED, while a straight line signifies that all demands are covered only once.

Each coloured line represents a different limited number of EDs based on constraint (6). For instance, the red line labelled N1 depicts the expected number of potential demands covered based on objective (4) by a limited number of ED, in constraint (6), m = 1 for different probabilities of all EDs not working. As per Figure 2, H-2 is assigned as the ED to cover the demand for each probability, as it covers most demands in Petaling Jaya. The straight line of N1 indicates that all demands covered by H-2 are covered only once.

As the limited number of EDs increases, the expected number of potential demands covered becomes more similar. For N14 and N13, the expected number of potential demands covered remain the same for each probability of EDs not working, indicating that one ED not significantly covers the demand.

4.6 The Removal EDs from N13 until N5 for Specific Probability of EDs Not Working

Table 3 : The list of removed EDs based on the limited number of EDs for specific probability of EDs not working.

Number of ED	Probability ED not working, p	Removed ED
N13	0	H-13
	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9	H-10
N12	0	H-12, H-13
	0.1, 0.2	H-3, H-10
	0.3, 0.4, 0.5, 0.6	H-10, H-12
	0.7, 0.8	H-10, H-11
	0.9	H-10, H-14
N11	0	H-11, H-12, H-13
	0.1, 0.2, 0.3, 0.4	H-3, H-10, H-12
	0.5, 0.6, 0.7, 0.8	H-10, H-11, H-12
	0.9	H-7, H-10, H-14
N10	0	H-10, H-11, H-12, H-13
	0.1, 0.2	H-3, H-8, H-10, H-12
	0.3, 0.4, 0.5	H-3, H-10, H-11, H-12
	0.6, 0.7, 0.8	H-7, H-10, H-11, H-12
	0.9	H-7, H-10, H-11, H-14
N9	0	H-1, H-10, H-11, H-12, H-13
	0.1, 0.2, 0.3, 0.4	H-3, H-8, H-10, H-11, H-12
	0.5, 0.6, 0.7	H-7, H-8, H-10, H-11, H-12
	0.8, 0.9	H-7, H-10, H-11, H-12, H-14
N8	0	H-1, H-8, H-10, H-11, H-12, H-13
	0.1	H-3, H-8, H-10, H-11, H-12, H-13
	0.2, 0.3, 0.4, 0.5, 0.6	H-3, H-7, H-8, H-10, H-11, H-12
	0.7, 0.8, 0.9	H-7, H-8, H-10, H-11, H-12, H-14
N7	0	H-1, H-7, H-8, H-10, H-11, H-12, H-13
	0.1, 0.2	H-3, H-7, H-8, H-10, H-11, H-12, H-13
	0.3	H-3, H-5, H-7, H-8, H-10, H-11, H-12
	0.4, 0.5	H-3, H-7, H-8, H-9, H-10, H-11, H-12
	0.6, 0.7, 0.8 0.9	H-3, H-7, H-8, H-10, H-11, H-12, H-14
N6	0	H-1, H-2, H-7, H-8, H-10, H-11, H-12, H-13
	0.1	H-3, H-7, H-8, H-9, H-10, H-11, H-12, H-13
	0.2, 0.3, 0.4, 0.5	H-3, H-5, H-7, H-8, H-9, H-10, H-11, H-12
	0.6, 0.7, 0.8, 0.9	H-3, H-5, H-7, H-8, H-10, H-11, H-12, H-14
N5	0	H-2, H-3, H-5, H-7, H-8, H-10, H-11, H-12, H-13
	0.1, 0.2, 0.3	H-3, H-5, H-6, H-7, H-8, H-9, H-10, H-11, H-12
	0.4	H-3, H-4, H-5, H-7, H-8, H-9, H-10, H-11, H-12
	0.5, 0.6	H-3, H-5, H-7, H-8, H-9, H-10, H-11, H-12, H-14
	0.7	H-3, H-4, H-5, H-7, H-8, H-10, H-11, H-12, H-14
	0.8, 0.9	H-4, H-5, H-7, H-8, H-9, H-10, H-11, H-12, H-14

Table 3 displays the list of EDs removed from the model selection when limiting the number of EDs from 13 to 5, where all these configurations cover all demands when the probability of all EDs not working, p = 0. The table demonstrates that the EDs removed can vary as p increases for each model.

For N13, the consistently removed ED is H-10 for all probabilities of all EDs not working. This is attributed to the fact that H-10 only covers 4 demand points, as mentioned in section 4.1. Additionally, the total expected demand covered for N13 and N14 remains the same for each probability of EDs not working, suggesting that H-10 can be permanently excluded from the EDs that cover demand in Petaling Jaya as emergency ambulance provider.

Contrary to the results in section 4.3, where demand points covered once by H-4, H-9 or H-14 highlighted the importance of these three EDs as emergency ambulance providers, in MEXCLP, these three can also be removed to satisfy objective function (4). For instance, in N12 at p = 0.9, H-14 is removed from the model, and replaced by H-12 because covering additional demand as backup increases the objective (6) rather than covering specific demand with the increase of p. The removal of EDs that can cover demands that are not covered by others become more significant as the number of EDs is reduced.

Moreover, in N5 at p = 0.1, the removed EDs include H-6 and H-11, where, based on the results section 4.4 there is demand points that can only be covered twice by H-6 and H-11. Therefore, allocating ambulances to just five EDs may result in certain demand points not being covered within 11 minutes, even though the probability of all EDs not working is relatively small which is 0.1. This scenario also holds for N6 and N7, where at p = 0.1 and p = 0.4, respectively, there are demands that cannot be covered due to the removal of H-9.

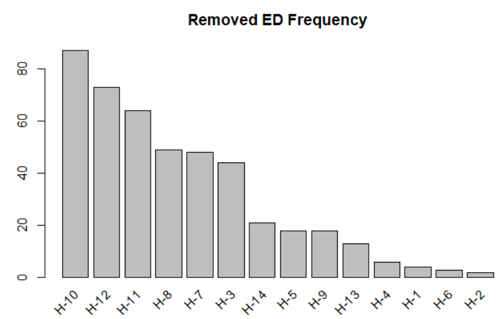

Figure 7 : The frequency of each ED removed from the selection of MEXCLP.

Figure 7 provides a visual representation of the frequency of each ED is removed, based on detailed in Table 3. Overall, H-10 is the most frequently removed, followed by H-12, while H-2 and H-6 are the least frequently removed from the MEXCLP model. This observation suggests that when an ED covers a greater number of demands, it is more likely to be selected to satisfy objective value (4).

4.7 Analysis for N8 in MEXCLP Model

Given that certain EDs were removed from N7 and below due to their coverage of demands that cannot be covered by others, an analysis of N8 becomes pertinent to determine the optimum number of EDs. This analysis is especially important as H-14 was identified as the optimum solution only after a probability within the range of 0.7.

Table 4 : The list of EDs in optimum solution for N8.

Probability EDs not working, p	**Optimum Solution**
0	H-1, H-2, H-4, H-5, H-6, H-7, H-9, H-14
0.1	H-1, H-2, H-4, H-5, H-6, H-7, H-9, H-14
0.2	H-1, H-2, H-4, H-5, H-6, H-9, H-13, H-14
0.3	H-1, H-2, H-4, H-5, H-6, H-9, H-13, H-14
0.4	H-1, H-2, H-4, H-5, H-6, H-9, H-13, H-14
0.5	H-1, H-2, H-4, H-5, H-6, H-9, H-13, H-14
0.6	H-1, H-2, H-4, H-5, H-6, H-9, H-13, H-14
0.7	H-1, H-2, H-3, H-4, H-5, H-6, H-9, H-13
0.8	H-1, H-2, H-3, H-4, H-5, H-6, H-9, H-13
0.9	H-1, H-2, H-3, H-4, H-5, H-6, H-9, H-13

Table 4 presents the list of EDs in the optimum solution for limiting the number of EDs to eight in the MEXCLP model. The table reveals three different sets of solutions based on the range of p. When the $0 \leq p < 0.2$, the set of optimum solution is H-1 ∪ H-2 ∪ H-4 ∪ H-5 ∪ H-6 ∪ H-7 ∪ H-9 ∪ H-14, while $0.2 \leq p < 0.7$, the set of the optimum solution is H-1 ∪ H-2 ∪ H-4 ∪ H-5 ∪ H-6 ∪ H-9 ∪ H-13 ∪ H-14, and $0.7 \leq p < 0$, the set of optimum solution is H-1 ∪ H-2 ∪ H-3 ∪ H-4 ∪ H-5 ∪ H-6 ∪ H-9 ∪ H-13. The EDs that change between these sets are H-3, H-7, H-13, and H-14, with H-13 replacing H-7 while H-3 replacing H-14.

When the probability of EDs not working is relatively small, the objective value (4) does not vary significantly between optimal and suboptimal solutions. Also, when p = 0, both H-7 and H-10 are subsets of the universal set solutions in the MCLP model, as discussed in section 4.4. Thus, H-7 in the list of optimum solution at p = 0 and p = 0.1 is replaced by H-13, even though the objective value (4) is not optimal. Additionally, as the objective of this research is to ensure that all demand points can be covered, H-3 is replaced by H-14 when the probability p less than 0.7 because all demand points covered by H-3 can be covered by others EDs especially H-1, as mentioned in section 4.2 and H-14 covers the unique demands that cannot be covered by other EDs. Therefore, the set of suboptimum solution H-1, H-2, H-4, H-5, H-6, H-9, H-13, H-14 will be utilized as emergency ambulance providers and EMS for all demands in Petaling Jaya for each probability of EDs not working.

Table 5 : The comparison between optimum and suboptimum solutions values.

Probability ED not working, p	**The Optimum Solution Value**	**The Suboptimum Solution Value**	**The Difference Percentage (%)**
0	470.0000	470.0000	0.0000000
0.1	466.9387	466.7292	0.0448667
0.2	462.1839	462.1839	0.0000000
0.3	455.1920	455.1920	0.0000000
0.4	443.9046	443.9046	0.0000000
0.5	425.5000	425.5000	0.0000000
0.6	395.4059	395.4059	0.0000000
0.7	351.4059	348.7140	0.7660372
0.8	285.3400	275.6914	3.3814397
0.9	176.3936	164.8371	6.5515416

Table 5 compares the objective (4) value between the actual optimum set of EDs solution and suboptimum solution. The table illustrates that when $p = 0.1$, the difference percentage is very small, as covering the demand as a backup ED when p is low does not significantly impact the MEXCLP model. However, when $p \geq 0.7$, the difference percentage become larger, highlighting the importance of demand points that do not have backup EDs. The expected coverage of that area during EDs not working is low, leading to a higher difference percentage.

Since the difference percentage of the suboptimal solution compared to the optimum solution is relatively low, H-1, H-2, H-4, H-5, H-6, H-9, H-13, H-14 become the EDs that cover demands in Petaling Jaya.

4.8 The Comparison of The Selected 8 EDS with Other Optimum Solution

As per the results presented in section 4.7, the EDs selected to cover potential demands within 11 minutes and below are H-1, H-2, H-4, H-5, H-6, H-9, H-13, and H-14. Notably, all EDs in Petaling Jaya are included in this list except H-3, while three EDs outside Petaling Jaya are part of the selection.

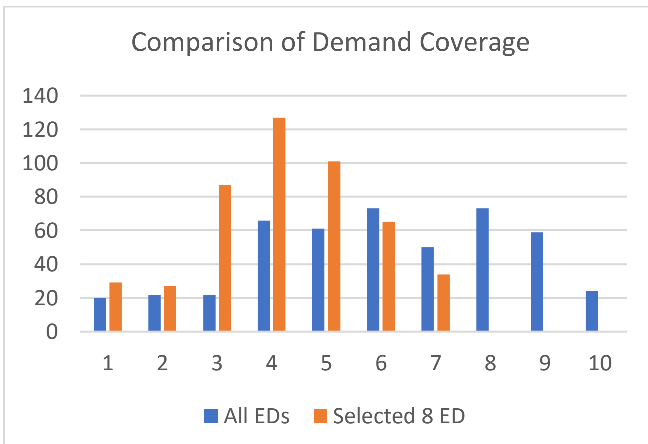

Figure 8 : Comparison of demand coverage between all EDs and selected 8 EDs.

Figure 8 illustrates a bar plot comparing demand coverage between all 14 EDs and the selected 8 EDs within 11 minutes. The x-axis represents the total number of EDs covering each demand, while the y-axis indicates the total demand covered by that specific number of EDs. For instance, when all EDs are used to cover Petaling Jaya demand, there are 24 demand points covered by 10 EDs. In comparison, when the selected eight EDs are used, the demand is covered by up to seven EDs, with the most of the demand covered by four EDs. Overall, this comparison reveals that the difference in demand coverage between demand covered by one and two EDs is not significant and most of the demand covered by the selected eight EDs has backup EDs.

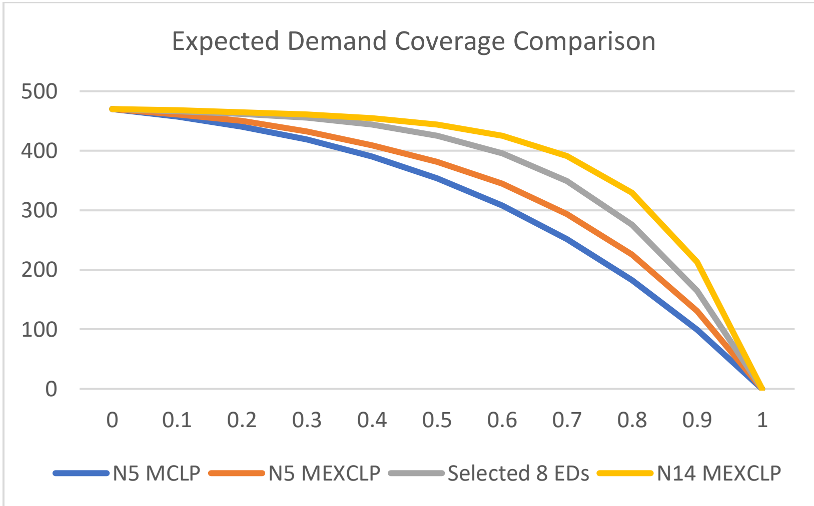

Figure 9 : The comparison of expected demand coverage between several solutions.

Figure 9 presents the expected demand coverage for optimal ED solutions N5 in the MCLP model, N5 in the MEXCLP model, the selected 8 EDs, and N14 in the MEXCLP model, as the probability of all EDs not working, p increases within 11 minutes. The expected demand coverage by N5 in the MCLP model, utilizing the set covering demand H-1 ∪ H-4 ∪ H-6 ∪ H-9 ∪ H-14, shows the lowest coverage across p, followed by N5 in the MEXCLP model. The highest expected value can be achieved for covering maximum demand across the probability as possible is N14 in MEXCLP model.

In comparison, the expected coverage by the selected eight EDs is closer to N14 in the MEXCLP model than N5 in the MEXCLP model, even though six EDs are removed from the total existing EDs compared to removing three EDs from N8. Consequently, the selected eight EDs provide a more robust model for covering demand in Petaling Jaya.

4.9 The Distribution Demand Coverage by The Selected 8 EDs

Figure 10 displays a frequency histogram of travel duration from the selected 8 EDs to all demands in Petaling Jaya within or below 11 minutes. The average travel duration from the selected ED to demands is 7.781662 minutes, with most demands covered within 9 minutes. The range of coverage spans from 0.9 to 11 minutes.

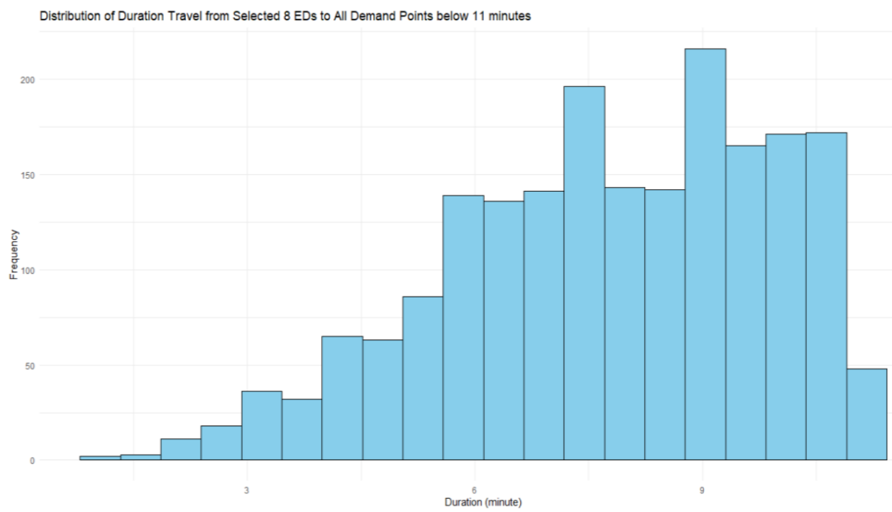

Figure 10 : The distribution of duration travel from selected 8 EDs to all demands below 11 minutes.

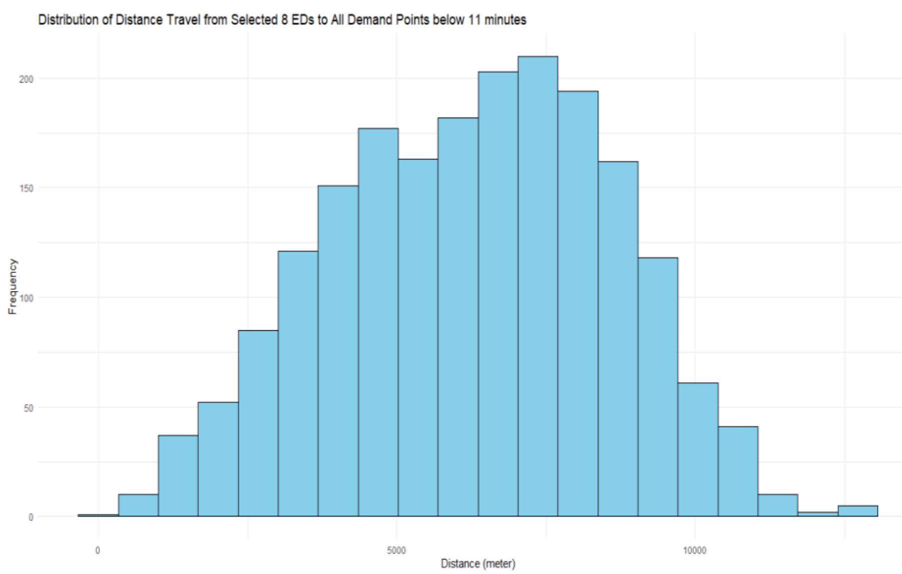

Figure 11 : The distribution of travel distance from selected 8 EDs to all demands below 11 minutes.

Figure 11 illustrates the frequency distribution of travel distance from the selected 8 EDs to all demand points in Petaling Jaya within and below 11 minutes. The average travel distance from the selected 8 EDs to all demand points in Petaling Jaya below 11 minutes is 6266.065 meters, with the distance coverage ranging from 224 until 12942 meters.

5 CONCLUSION

In conclusion, this research successfully achieved its goal of determining the optimal number of EDs in Petaling Jaya through a set covering location problem analysis. All potential demands in Petaling Jaya can be met by at least one ED below 11 minutes by Assunta Hospital (H-1), KPJ Damansara

Specialist Hospital (H-2), Thomson Hospital (H-4), Beacon Hospital (H-5), KMI Kelana Jaya Medical Centre (H-6), KPJ Damansara Specialist Hospital 2 (H-9), Sunway Medical Centre (H-13), and Sungai Buloh Hospital (H-14). The average travel duration and distance for these EDs are approximately 7.781662 minutes and 6266.065 meters respectively. These strategically located EDs will serve as ambulance bases for responding to emergency priority cases one, ensuring immediate patient treatment. The findings are derived from the analysis of the set covering problem using the MCLP model and reinforced by the MEXCLP model to create a robust solution.

To enhance the research, several factors could be incorporated into the model. These include considering the maximum capacity each ED can handle in responding to emergency, the number of ambulances available at each ED, the demographic distribution of the population at each demand point, the reliability of EMS in each ED, and the well-defined of the probability of each ED not working. Incorporating of the emergency response system, improving the overall effectiveness of the proposed model.

ACKNOWLEDGEMENT

The authors would like to thank the Institute of Mathematical Sciences, Faculty of Science, Universiti Malaya, for its invaluable support in facilitating this research.

REFERENCES

[1] National Audit Department Malaysia, "Laporan Ketua Audit Negara 2021 Siri 2: Aktiviti Kementerian/Jabatan Kerajaan Persekutuan dan Badan-Badan Berkanun Persekutuan," 16 February 2023. [Online]. Available: https://lkan.audit.gov.my/laporan/manage/1474.

[2] World Health Organization, "Framework for Strengthening Health Emergency Preparedness in Cities and Urban Settings," 9 November 2021. [Online]. Available: https://www.who.int/publications/i/item/9789240037830.

[3] K. S. Chew & H. C. Chan, "Prehospital care in Malaysia: issues and challenges," *International Paramedic Practice,* vol. 1, no. 2, pp. 47-51, 2014. [Online]. Available: https://doi.org/10.12968/ippr.2011.1.2.47

[4] M. Değer & M. Atak, "Evaluation of Bingol 112 Emergency Health Services," *Karya Journal of Health Science,* vol. 4, no. 2, pp. 159-163, 2023. [Online]. Available: http://dx.doi.org/10.52831/kjhs.1341402

[5] R. Aringhieri, M. E. Bruni, S. Khodaparasti, & J. T. V. Essen, "Emergency medical services and beyond: Addressing new challenges through a wide literature review," *Computers & Operations Research,* vol. 78, pp. 349-368, 2017. [Online]. Available: https://doi.org/10.1016/j.cor.2016.09.016

[6] W. Kisiała, I. Rącka & K. Suszyńska, "Population Access to Hospital Emergency Departments: The Spatial Analysis in Public Health Research.," *International Journal of Environmental Research and Public Health,* vol. 19, no. 3, p. 1473, 2022. [Online]. Available:

https://doi.org/10.3390/ijerph19031437

[7] J. Nicholl, J. West, S. Goodacre, & J. Turner, "The relationship between distance to hospital and patient mortality in emergencies: an observational study," *Emergency Medicine Journal,* vol. 24, no. 9, pp. 665-668, 2007. [Online]. Available: https://doi.org/10.1136%2Femj.2007.047654

[8] K. Hegenberg, H. Trentzsch, S. Gross, & S. Prückner, "Use of pre-hospital emergency medical services in urban and rural municipalities over a 10 year period: an observational study based on routinely collected dispatch data.," *Scandinavian Journal of Trauma, Resuscitation and Emergency Medicine,* vol. 27, p. 35, 2019. [Online]. Available: https://doi.org/10.1186/s13049-019-0607-5

[9] A. Alruwaili & A. R. M. Alanazy, "Prehospital Time Interval for Urban and Rural Emergency Medical Services: A Systematic Literature Review. Healthcare (Basel, Switzerland)," *Healthcare (Basel, Switzerland),* pp. 10(12), 2391., 2022. [Online]. Available: https://doi.org/10.3390/healthcare10122391

[10] Y. Li, Y. et al., "Urban-sub-urban-rural variation in the supply and demand of emergency medical services.," *Frontiers in Public Health,* vol. 10, p. 2022, 2023. [Online]. Available: https://doi.org/10.3389/fpubh.2022.1064385

[11] M. S. Islam & M. R. Islam, "A solution method to maximal covering location problem based on chemical reaction optimization (CRO) algorithm.," *Soft Computing 27,* pp. 7337-7361, 2023. [Online]. Available: https://doi.org/10.1007/s00500-023-07972-w

[12] R. L. Church & C. S. Revelle, "The maximal covering location problem," *Papers of the Regional Science Association,* vol. 32, pp. 101-118, 1974. [Online]. Available: https://doi.org/10.1007/BF01942293

[13] M. H. F. Zarandi, S. Davari, & S. A. H. Sisakht, "The large scale maximal covering location problem," *Scientia Iranica,* vol. 18, no. 6, pp. 1564-1570, 2011. [Online]. Available: https://doi.org/10.1016/j.scient.2011.11.008

[14] L. Brotcorne, G. Laporte, & F. Semet, "Ambulance location and relocation models," *European Journal of Operational Research,* vol. 147, no. 3, pp. 451-463, 2003. [Online]. Available: https://doi.org/10.1016/S0377-2217(02)00364-8

[15] M. Daskin, "A Maximum Expected Covering Location Model: Formulation, Properties and Heuristic Solution," *Transportation Science,* pp. 48-70, 1983. [Online]. Available: https://doi.org/10.1287/trsc.17.1.48

[16] K. Konis, "lpSolveAPI Package Users Guide". Accessed: Jan. 16, 2024. [Online]. Available: https://civil.colorado.edu/~balajir/CVEN5393/R-sessions/sess1/lpSolveAPI-vignettes.pdf