

Mapping the Research Landscape of Ordinary Differential Equations through Bibliometric Analysis

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ABSTRACT

Ordinary Differential Equations (ODEs) are a fundamental field used for mathematical modelling in wide-ranging applications. This research visually represents the research landscape of ODEs by utilizing bibliometric analysis and social network analysis. We analysed 1,849 documents related to topic developments, patterns in publications, and collaborative networks spanning the years 2019 to 2024. Consequently, numerous papers have been published by authors from the United States, China, and Russia, where research activity is significantly focused. The analysis identifies the articles with the highest citation rates, as well as the important authors and universities that are at the centre of publishing on ODEs. Furthermore, we display the collaborative network of the research group and countries using the VOSviewer application. In conclusion, the study highlights the current state and progress of ODE research by identifying recent developments and intriguing topics for future exploration. This paper serves as a valuable reference for researchers seeking to understand the impact and progress of ODE investigations within the broader mathematical domain.

Keywords: Bibliometric Analysis, Ordinary Differential Equation, Collaboration Network, VOSviewer, social network analysis, centrality analysis

1 INTRODUCTION

Ordinary Differential Equations (ODEs) play a fundamental role in mathematical modelling and are essential tools in a wide range of scientific fields. These equations are crucial for articulating the rate of change of quantities and the interconnections between variables across several disciplines such as physics, biology, economics, education and engineering [1]. ODEs are used in physics to mathematically describe various phenomena such as planetary motion [2] and electromagnetic [3]. In biology, ODEs play a vital role in analyzing population dynamics [4] and studying the transmission

of diseases [3] [5]. Economists employ differential equations to predict market trends [6] [7] and assess risks [8], demonstrating their wide-ranging relevance and significance.

The complexities of ODE investigations are intensified by their multidisciplinary nature since they cross several mathematical domains and scientific disciplines. Interconnection fosters collaborative and cross-disciplinary study, but it also introduces complexities in synthesizing research trends and contributions. Given this context, there is an urgent want for systematic methods and research that can effectively arrange, examine, and condense these vast datasets. Bibliometric and network analysis offer reliable approaches for tackling these difficulties [9], [10]. They assist not only in measuring the influence of research [11] but also in identifying primary contributors and comprehending the patterns of collaboration among scholars and institutions globally [12].

The progress in computing approaches has greatly advanced research on ODE, enabling the creation of advanced algorithms and software that enhance the ability to solve and analyze complex systems [2], [13]. Bibliometric analyses have employed publishing and citation metrics to track the development of the discipline, emphasizing key scholars and pivotal studies that have influenced contemporary practices and theoretical knowledge [11], [14].

Network analysis has been used to investigate the collaboration networks within the ODE research community. The studies emphasize the significance of joint endeavors in promoting research, demonstrating how collaborations across different disciplines and countries improve the progress of mathematical models and their practical uses [15],[16].

This study utilizes bibliometric and social network analysis to thoroughly map the existing research environment in the dynamic and expanding field of ODE research. This research aims to gain valuable insights into the evolution of the field of ODE by analyzing publication trends, citation patterns, and collaborative networks from 2019 to 2024. The focus is on identifying influential research and emerging trends that can guide future studies. This technique not only improves the accessibility of the field for beginners but also assists experienced researchers in identifying promising areas for further exploration.

2 METHODOLOGY

This study utilizes a thorough bibliometric and network analysis methodology to investigate the research landscape of Ordinary Differential Equations (ODEs) from 2019 to 2024. The methodology comprises three components: data collecting, data analysis, and network visualization. Each component aims to provide insights into various elements of scholarly communications within the subject.

2.1 Data collecting

The data was retrieved on October 20, 2023, utilizing a Scopus database. Scopus was chosen as the preferred data source due to several key advantages over other databases like Web of Science or Google Scholar. Scopus is recognized for its stringent journal indexing standards, which guarantee that the included articles are peer-reviewed and meet high scientific quality criteria. This aspect is essential for bibliometric studies where data accuracy and source credibility are paramount.

Additionally, Scopus offers detailed bibliometric indicators such as citation counts and the h-index, which are instrumental for our analysis.

The core subject of this study centred on all research publications using the author's keyword "ordinary differential equation." The publishing year is limited from 2019 to 2024. This search utilized the query strings "AUTHKEY (ordinary AND differential AND equation) AND PUBYEAR > 2019 AND PUBYEAR 2024." With this filter in the search query, the total number of documents obtained is 1849.

2.2 Bibliometric Analysis

The data was exported from the Scopus database to Microsoft Excel 365 with CSV formatted. Data that has been obtained from Scopus will be aggregated and analyzed using Microsoft Excel 365 through related figures and tables.

Then, the data was converted from the CSV formatted to RIS formatted to apply it on Publish or Perish (PoP) software. PoP software will calculate total publication (TP), number of cited papers (NCP), total citation (TC), citation per paper (C/P), citation per cited paper (C/CP), h-index and g-index.

2.3 Network Visualization

Network visualization is a pivotal aspect of this bibliometric study, enabling the graphical representation of relationships and patterns among researchers, institutions, and countries involved in the field of ODEs. The study utilized VOSviewer, a software tool specifically designed for constructing and viewing bibliometric networks [17]. VOSviewer offers capabilities to create various types of network maps, including co-authorship, co-citation, and bibliographic coupling networks. These maps visually depict how authors, articles, and institutions are interconnected based on shared authorship, citations, or keywords.

For the weight of each edge, this study used the full counting approach instead of the fractional counting method. The reason for this is that the full counting approach is more straightforward to comprehend in comparison to the fractional counting method [18]. For instance, the full counting technique assigns whole numbers to represent the weight of all collaborations, while the fractional counting method assigns fractional numbers to represent the weight of collaborations. This is because each co-authorship relationship has a strength of $1/n$ when one author co-authors a document with n additional authors as shown in Figure 1.

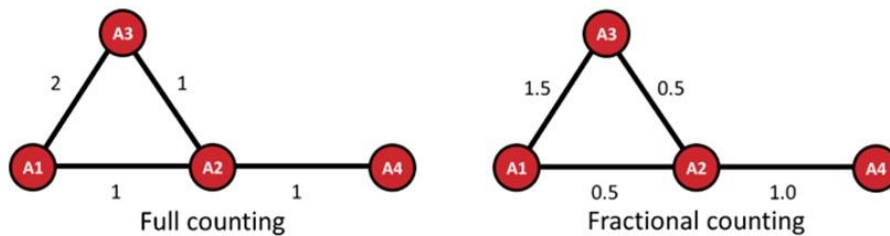


Figure 1: Difference between full counting and fractional counting (Source: [19])

The network visualization was complemented by the analysis of network metrics such as degree centrality [20]. Degree centrality (DC) measures the number of direct connections a node has, indicating its activity level within the network. DC of the node in a network is the total number of edges connected to it [10]. It can define as Equation (1) where $\text{deg}(v)$ is the number of the link or edge that connects node v .

$$DC(v) = \text{deg}(v) \tag{1}$$

3 RESULTS AND DISCUSSIONS

Analyzed data was used to ascertain the annual publication count, the subject areas, and the author's keywords associated with each article. The study compiled a comprehensive list of the leading countries, authors, institutes, and articles about the ODE publications. The results were presented in terms of frequency and percentage.

3.1 Number of Publications by Year

The investigation of publications on ODE from 2019 to 2024 provides valuable insights into the research activity trends in this discipline. Figure 2 demonstrates a rise in the number of publications from 298 in 2019 to 338 in 2020. The emergence of the COVID-19 pandemic at the beginning of the year undoubtedly prompted a surge in research focused on the mathematical modelling of infectious diseases, which is a crucial field for the use of ODE.

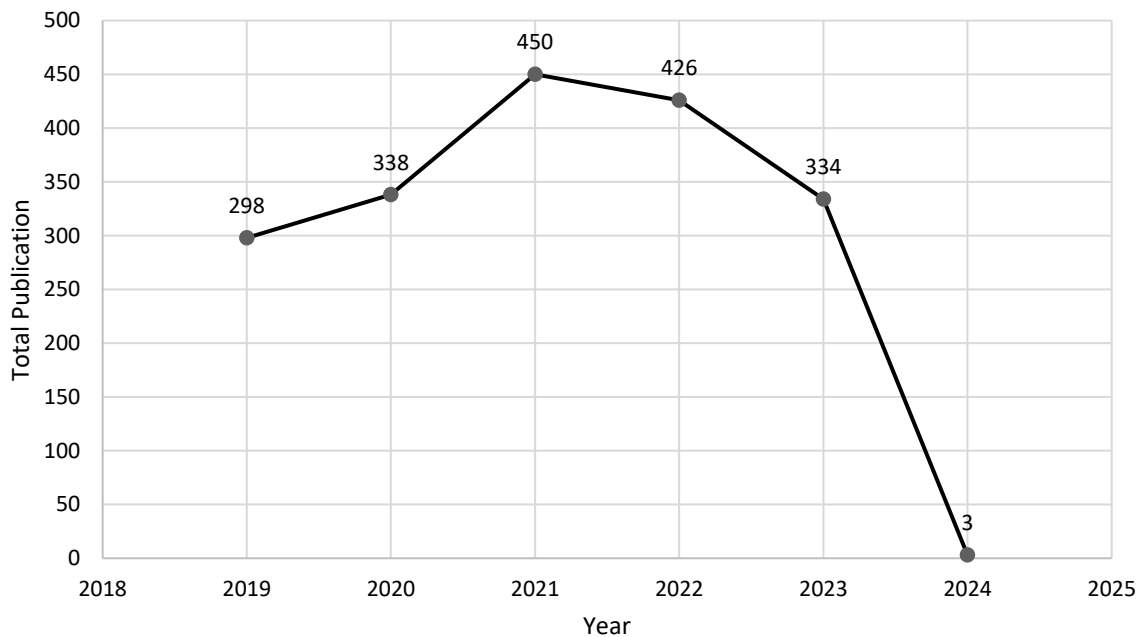


Figure 2. Documents publication by year

Despite a small decline to 426 articles in 2022, the number remained notably high, suggesting that the research community continued to heavily utilize ODEs. At this point, the attention may have shifted from just dealing with the epidemic to also considering the lasting effects of COVID-19 on healthcare systems, economies, and societal institutions. The significant decline to just 3 publications in 2024 is noteworthy; nevertheless, this number probably reflects an incomplete dataset for the year, as data gathering for 2024 would not have been completed at the time of this analysis.

3.2 Subject Area

The distribution of subject areas in documents on ODE highlights the multidisciplinary character and extensive use of ODEs in several scientific domains. The findings indicate that Mathematics had the highest representation, with notable contributions also observed in Computer Science, Engineering, and several other fields, as represented in Table 1. The field of mathematics has the highest number of publications, with 1,248 papers, making up 67.50% of the total. This importance is anticipated because ODEs are essentially mathematical entities utilized to articulate and resolve difficulties that pertain to rates of change and dynamic systems.

Table 1: Subject Area

| Subject Area | Total Publication | Percentage (%) |
|--|-------------------|----------------|
| Mathematics | 1248 | 67.50 |
| Computer Science | 569 | 30.77 |
| Engineering | 507 | 27.42 |
| Physics and Astronomy | 213 | 11.52 |
| Biochemistry, Genetics and Molecular Biology | 127 | 6.87 |
| Agricultural and Biological Sciences | 97 | 5.25 |
| Decision Sciences | 86 | 4.65 |
| Materials Science | 70 | 3.79 |
| Chemistry | 69 | 3.73 |
| Environmental Science | 65 | 3.52 |

The field of Computer Science accounts for 30.77% of the articles, while Engineering accounts for 27.42%. ODEs are frequently used in these domains to simulate, optimize, and model processes that are crucial in software development, hardware design, and numerous engineering applications. The significant convergence in these domains demonstrates the pivotal function of computational methodologies and engineering concepts in broadening the pragmatic utilization of ODEs. Upon further analysis, it becomes evident that several domains such as Material, Chemistry and Environmental Science have made significant contributions to the study of ODEs. This demonstrates

the wide range of applications of ODEs, which include ecological modelling, medical research, economic dynamics, and social behaviour studies.

3.3 Authors Keyword

The keywords highlight the wide range of study topics within the area, encompassing both theoretical elements and practical applications. Table 2 reveals that the terms "Ordinary Differential Equations" and its variant "Ordinary Differential Equation" are the most used keywords, appearing in 1202 and 334 publications respectively. This highlights a significant emphasis on fundamental elements of ODE, with more than 65% of the literature devoted to exploring different aspects of these equations.

Table 2: Top 20 author's keywords in ODE

| Keyword | Frequency | Degree Centrality | Keyword | Frequency | Degree Centrality |
|--|-----------|-------------------|--|-----------|-------------------|
| Ordinary Differential Equations | 1202 | 19 | Partial Differential Equations | 64 | 16 |
| Ordinary Differential Equation | 334 | 19 | Humans | 63 | 16 |
| Numerical Methods | 191 | 19 | Neural Ordinary Differential Equations | 61 | 14 |
| Nonlinear Equations | 138 | 19 | Stability | 57 | 14 |
| Human | 91 | 18 | Initial Value Problems | 56 | 14 |
| Article | 88 | 17 | Ordinary Differential Equation (ODE) | 50 | 13 |
| Nonlinear Ordinary Differential Equation | 69 | 17 | Deep Learning | 49 | 13 |
| Neural Ordinary Differential Equation | 68 | 17 | Boundary Value Problems | 48 | 13 |
| Dynamical Systems | 66 | 16 | Stochastic Systems | 48 | 12 |
| Mathematical Model | 64 | 16 | Neural-networks | 47 | 12 |

The topics of "Numerical Methods" and "Nonlinear Equations" are widely recognized and have been mentioned in 191 and 138 publications, respectively. The keywords emphasize the importance of creating and using numerical methods to solve ODEs, especially when dealing with nonlinear systems that are common in various real-world applications. The terms "Human," "Humans," and "Neural Ordinary Differential Equation" are mentioned 91, 63, and 68 times respectively. These keywords

demonstrate the wide-ranging uses of ODE, particularly in the fields of biology, medicine, and new areas such as machine learning.

The author's keywords were analyzed and shown in Figure 3 using VOSviewer software, a tool for constructing and visualizing bibliometric networks. In this visualization, nodes represent keywords, the thickness of the edges depicts the links between keywords (co-occurrence), and the colour indicates the cluster to which each term belongs. Keywords of identical colour were grouped in the same cluster.

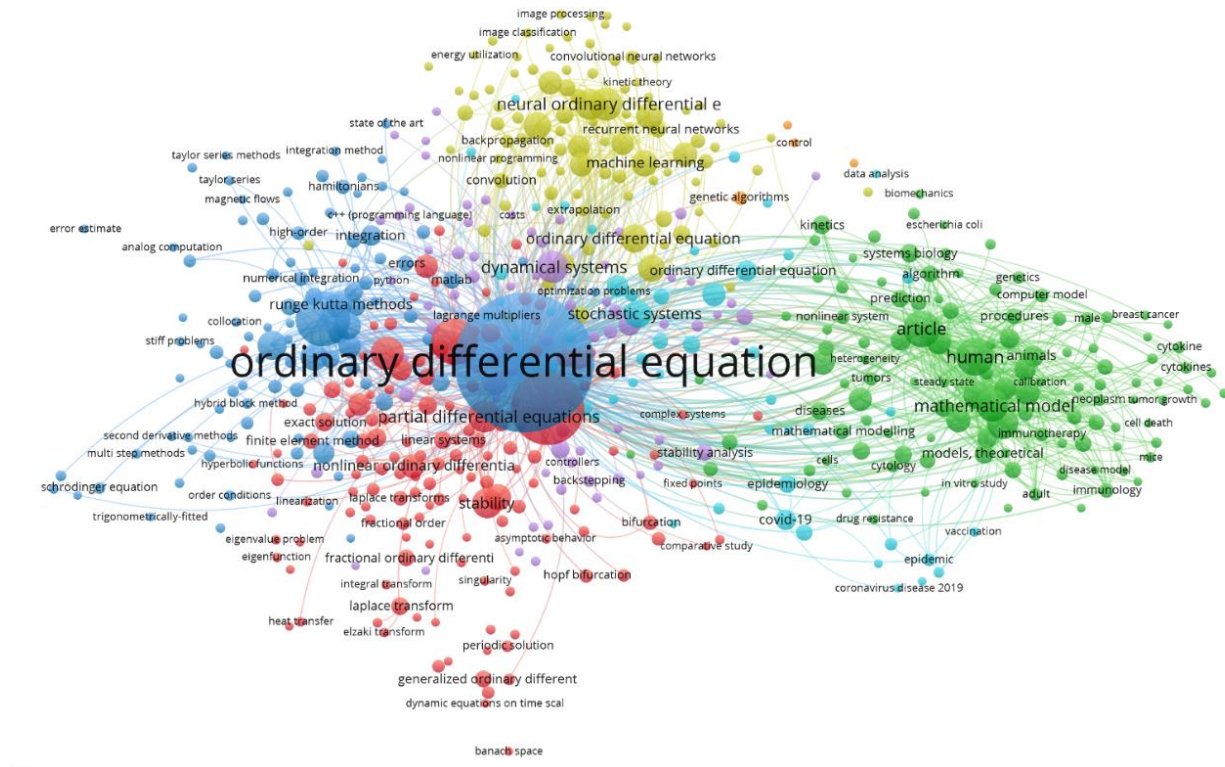


Figure 3: Network Visualization for author keywords

There are five primary clusters in this network. The blue clusters represent fundamental and advanced numerical methods employed in solving ODEs. The yellow clusters signify a strong focus on utilizing machine learning algorithms and computational techniques in complex systems, such as image processing and neural networks. The green clusters represent Biomedical Modeling and Research. The cyan cluster reflects the significant contribution of mathematical models in comprehending the dynamics of infectious diseases, especially during the COVID-19 pandemic. Lastly, the red clusters are associated with analytical and semi-analytical techniques for solving ODEs and investigating their properties, particularly in terms of stability and solution behaviours.

3.4 Country Distribution

The global distribution of publications in the field of ODEs demonstrates the worldwide scope of this scientific pursuit, emphasizing significant contributions from both well-established and rising

research institutes across countries. Table 3 reveals that the United States and China are at the forefront, with 283 and 271 publications, respectively. This demonstrates their strong research frameworks and significant investment in scientific and technical innovation. Germany, Italy, Spain, France, and the United Kingdom, which are European countries, have significant productivity. This reflects the continent's well-developed educational system and the cooperative research climate fostered by university networks and European Union programs. The geographical diversity in research on ODE not only highlights the universal significance and practicality of these mathematical models in addressing various scientific problems but also demonstrates a transition towards a more inclusive global research environment where knowledge and innovation are widely shared.

Table 3: The top 14 countries contributed to ODE

| Country | Frequency | Degree Centrality |
|--------------------|-----------|-------------------|
| United States | 283 | 12 |
| China | 271 | 9 |
| Russian Federation | 188 | 7 |
| Germany | 107 | 11 |
| India | 102 | 8 |
| Italy | 99 | 12 |
| Spain | 83 | 10 |
| France | 82 | 11 |
| Iran | 73 | 10 |
| United Kingdom | 68 | 11 |
| Canada | 63 | 11 |
| Malaysia | 58 | 7 |
| Brazil | 56 | 9 |
| Saudi Arabia | 55 | 10 |

The countries' co-occurrence networks were analyzed and shown using VOSviewer software in Figure 4. In this visualization, nodes represent countries, the thickness of the edges depicts the links between countries (co-occurrence), and the colour indicates the cluster to which each country belongs. Countries of identical colour were grouped in the same cluster. This network consists of four main clusters. The green cluster consists of the United States, China, Japan, Australia, and Canada, which are the foremost countries in advancing state-of-the-art mathematical theories and

applications. The red clusters primarily originate from Europe, while the blue clusters correspond to regions experiencing an expansion in their academic sectors. The yellow clusters may indicate smaller or more specialized research communities within the topic of ODEs. Although these countries have modest outputs, they play a significant role in contributing distinct perspectives and enhancing the diversity of research in ODEs. Their involvement underscores the extensive interest and utilization of ODEs in many regional contexts.

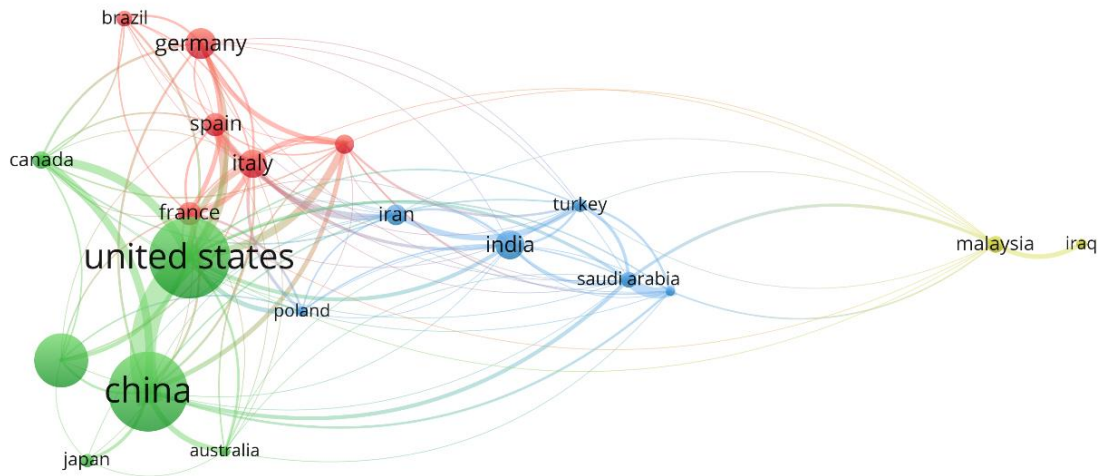


Figure 4: Network Visualization for Countries Distribution

3.5 Authorship Analysis

Table 4 illustrates the various contributions made to research on ODE, which is a dynamic topic that has been affected by a diverse collection of academics from different parts of the world. Federson, M. is the frontrunner, ranking first with 20 publications and a total of 39 citations. These metrics suggest that the individual has made consistent contributions, albeit the influence of their work is moderate.

Ramos, H. has made a notable contribution with 13 published works and an impressive citation count of 98, resulting in a citation per publication ratio of 7.54. Ramos has the greatest h-index of 7 and a g-index of 9 among the group, which indicates that he is a significant player in ODE research. Similarly, Rufai, M.A. has significant influence with a C/P (Citation per Publication) ratio of 8.13 derived from only 8 publications, indicating pioneering research in the field of ODE.

Table 4: The top ten authors contributed to ODE

| Author Name | Affiliation | Country | Total Publication (P) | Total Citation (C) | C/P | H Index | G Index |
|--------------------|---|----------------|------------------------------|---------------------------|------------|----------------|----------------|
| Federson, M. | Universidade de São Paulo, Sao Paulo | Brazil | 20 | 39 | 1.95 | 3 | 6 |
| Mesquita, J.G. | Universidade de Brasília, Brasilia | Brazil | 15 | 39 | 2.6 | 3 | 6 |
| Ramos, H. | Universidad de Salamanca, Salamanca | Spain | 13 | 98 | 7.54 | 7 | 9 |
| Bonotto, E.M. | Universidade de São Paulo, Sao Paulo | Brazil | 12 | 3 | 0.25 | 1 | 1 |
| Senu, N. | Universiti Putra Malaysia, Serdang | Malaysia | 12 | 21 | 1.75 | 2 | 4 |
| Ibrahim, Z.B. | Universiti Putra Malaysia, Serdang | Malaysia | 11 | 13 | 1.18 | 2 | 3 |
| Hojjati, G. | University of Tabriz, Tabriz | Iran | 8 | 10 | 1.25 | 2 | 2 |
| Rufai, M.A. | Free University of Bozen-Bolzano, Bolzano | Italy | 8 | 65 | 8.13 | 6 | 8 |
| Toon, E. | Universidade Federal de Juiz de Fora, Juiz de For a | Brazil | 8 | 19 | 2.38 | 2 | 4 |
| Abdi, A. | University of Tabriz, Tabriz | Iran | 7 | 7 | 1 | 2 | 2 |

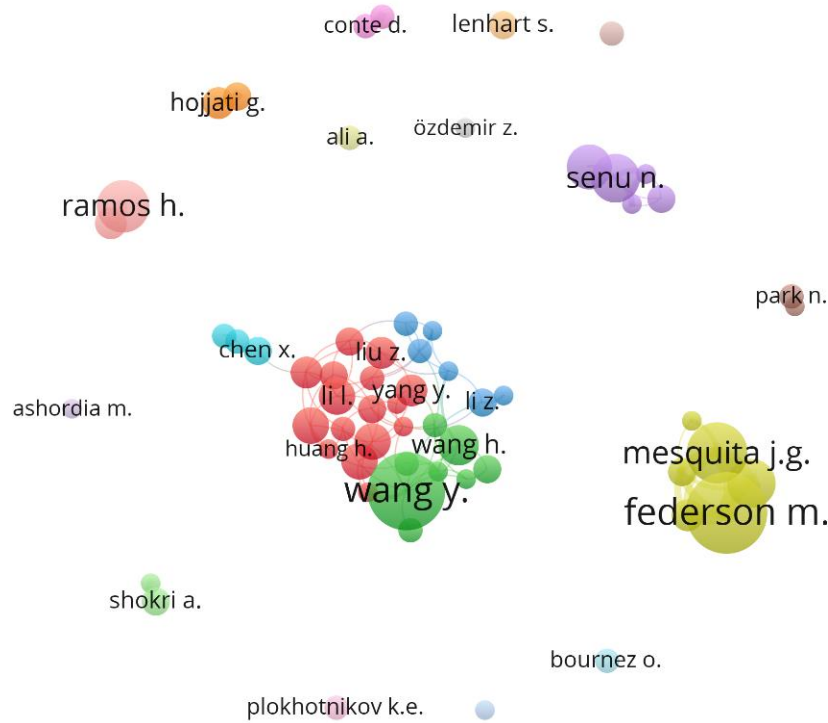


Figure 5 demonstrates the network analysis carried out using VOSViewer for authors specializing in ODE. The analysis revealed three main clusters that were considerably more populated than the rest, suggesting the presence of strong collaboration networks and areas of concentrated research activity. The red and green clusters primarily consist of authors from China, emphasizing the country's significant collective contributions to ODE research. This indicates a significant degree of cooperation among Chinese academics, which could be indicative of national research priorities or established research teams that concentrate on particular facets of ODEs. The yellow cluster comprises esteemed authors, namely Federson, M. and Mesquita, J.G., who have been recognized for their prolific contributions.

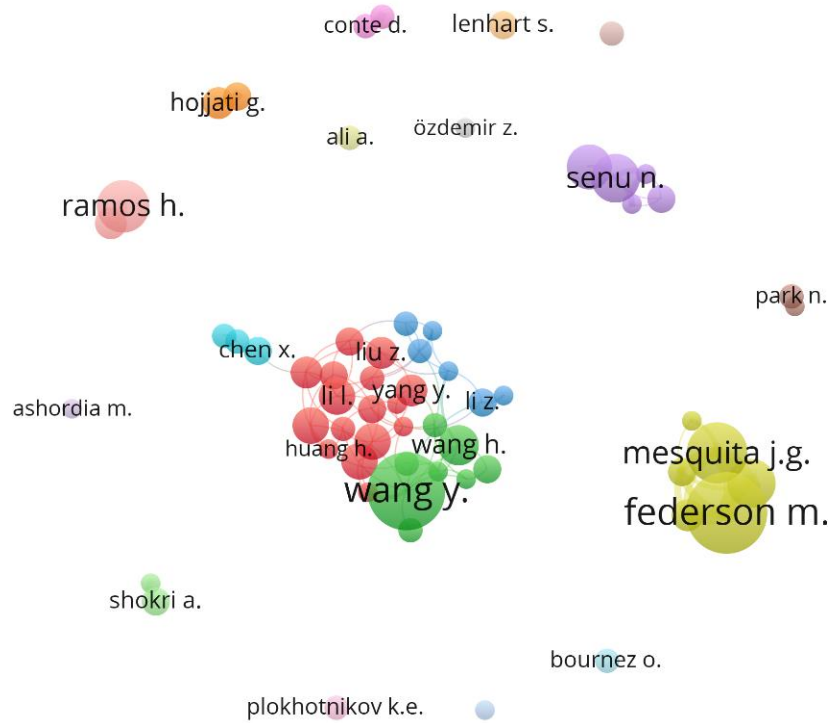


Figure 5: Network Visualization for author contributes to ODE

3.6 Citation Analysis

An examination of highly cited publications in the field of ODE provides valuable insights into the areas that have a significant influence and are now of great interest among researchers in the community. The paper by [21] is the most cited paper, with a total of 430 citations and an average of 215 citations per year. This indicates its significant impact and relevance in applications related to thermal processes and nanotechnology, as shown in Table 5. Subsequently, the paper by [22] and the book by [23] demonstrate substantial annual citations. This underscores the continued interest in fractional calculus and peridynamics, which expand the conventional limits of differential equations to encompass more intricate and practical models.

Lower on the list, the paper by [24] demonstrates the immediate use of ODE in the field of health science. These papers specifically address global challenges like the COVID-19 pandemic, highlighting the importance of mathematical modelling in the development of public health strategies.

Table 5: The top 20 articles related to ODE

| Article | Cites | Cites Per Year | Article | Cites | Cites Per Year |
|---------|-------|----------------|---------|-------|----------------|
| [21] | 430 | 215 | [25] | 45 | 22.5 |
| [22] | 143 | 35.75 | [26] | 44 | 11 |
| [23] | 134 | 33.5 | [27] | 38 | 19 |
| [24] | 79 | 26.33 | [28] | 35 | 11.67 |
| [29] | 72 | 18 | [30] | 34 | 8.5 |
| [31] | 63 | 31.5 | [32] | 33 | 8.25 |
| [33] | 58 | 14.5 | [34] | 32 | 16 |
| [35] | 47 | 23.5 | [36] | 30 | 10 |
| [37] | 47 | 15.67 | [38] | 30 | 10 |
| [39] | 46 | 15.33 | [40] | 28 | 14 |

3.7 Affiliation Analysis

The examination of affiliations in the field of ODE is displayed in Table 6. The CNRS Centre National de la Recherche Scientifique in France is at the top of the list in terms of total publications (TP), with 39 publications. The Universidade de São Paulo in Brazil and the Russian Academy of Sciences have 32 and 29 publications, respectively, which closely follow each other. These universities are well-known for their strict research standards and have historically played a significant role in advancing the field of mathematical sciences, namely in the practical and theoretical progress of ODEs.

Additional significant contributions are made by institutions such as Lomonosov Moscow State University and Universiti Putra Malaysia, demonstrating the worldwide impact and cooperative essence of ODE research. The participation of universities from Spain, Italy, the UK, and the US highlights the global collaboration in tackling intricate mathematical problems through ODEs. In addition, the participation of institutions from rising research hubs such as Malaysia and Taiwan underscores the growing scope of ODE research beyond conventional Western centres.

Table 6: The top 20 affiliations contribute to ODE

| Affiliation | Country | Total Publication | Affiliation | Country | Total Publication |
|---|----------|-------------------|--|---------|-------------------|
| CNRS Centre National de la Recherche Scientifique | France | 39 | University of Oxford | Uk | 14 |
| Universidade de São Paulo | Brazil | 32 | INRIA Institut National de Recherche en Informatique et en Automatique | France | 13 |
| Russian Academy of Sciences | Russian | 29 | Università degli studi di Bari Aldo Moro | Italy | 13 |
| Lomonosov Moscow State University | Russian | 25 | Moscow Aviation Institute National Research University | Russian | 13 |
| Universiti Putra Malaysia | Malaysia | 24 | Ministry of Education of the People's Republic of China | China | 12 |
| Universidad de Salamanca | Spain | 18 | Universidade Estadual Paulista Júlio de Mesquita Filho | Brazil | 12 |
| Universidade de Brasília | Brazil | 17 | The University of Tennessee, Knoxville | Us | 12 |
| Universiti Kebangsaan Malaysia | Malaysia | 15 | University of Kufa | Iraq | 12 |
| China Medical University | Taiwan | 15 | Universidad de Santiago de Compostela | Spain | 11 |
| RUDN University | Russian | 15 | China Medical University Hospital | Taiwan | 11 |

3.8 Source Title

The analysis of source titles from which research on ODE is published provides a window into the dissemination channels and impact of this field within the academic community. From Table 7, leading the list in terms of total publications is "Mathematics" by the MDPI, with 53 publications and 155 citations, highlighting its role as a significant platform for the latest research in mathematical sciences.

Table 7: Top 15 source title

| Source Title | Publisher | Total Publication | Total Citation |
|--|---|-------------------|----------------|
| Mathematics | Multidisciplinary Digital Publishing Institute (Mdpi) | 53 | 155 |
| Mathematical Methods in The Applied Sciences | Wiley-Blackwell | 30 | 118 |
| Lecture Notes in Computer Science Including Subseries Lecture Notes In Artificial Intelligence And Lecture Notes In Bioinformatics | Springer Nature | 26 | 31 |
| Symmetry | Multidisciplinary Digital Publishing Institute (Mdpi) | 23 | 81 |
| Aims Mathematics | Aims Press | 21 | 38 |
| Applied Numerical Mathematics | Elsevier | 18 | 58 |
| Journal Of Computational and Applied Mathematics | Elsevier | 17 | 60 |
| Bulletin Of Mathematical Biology | Springer Nature | 15 | 152 |
| Generalized Ordinary Differential Equations in Abstract Spaces and Applications | Wiley | 14 | 1 |
| Journal Of Interdisciplinary Mathematics | Taylor & Francis | 14 | 30 |
| Numerical Algorithms | Springer Nature | 14 | 37 |
| Applied Mathematics and Nonlinear Sciences | Walter De Gruyter | 13 | 35 |
| Lobachevskii Journal of Mathematics | Pleiades Publishing | 13 | 17 |
| Fractional Calculus and Applied Analysis | Springer Nature | 12 | 39 |
| Ieee Access | Ieee | 12 | 68 |

Further down the list, publications such as "Lecture Notes in Computer Science" including subseries in Artificial Intelligence and Bioinformatics from Springer Nature, and "Symmetry" also by MDPI, show that interdisciplinary applications of ODEs are well-represented in the literature. Notably, "Bulletin of Mathematical Biology" with only 15 publications, has a remarkably high citation count of

152, underscoring the crucial role of ODEs in modelling biological phenomena and demonstrating the high impact of research published in this journal. This pattern illustrates that while some journals may not lead in publication volume, their influence and the significance of the research they publish can be profound.

4 CONCLUSION

The analysis reveals significant geographical diversity in ODE research, with prominent contributions from countries such as the United States, China, and Russia, as well as strong representation from European and emerging countries. This geographical spread underscores the global importance and applicability of ODE research, with leading institutions like CNRS in France and Universidade de São Paulo in Brazil driving studies, indicating that established academic and research centres play crucial roles in advancing ODE knowledge.

Network visualizations using VOSViewer showcased distinct clusters of authors, notably dense networks in China and among top contributors like Federson and Mesquita from Brazil. These clusters represent collaborative hubs where research is vibrant and highly interconnected, often acting as influential communities that shape the direction and dissemination of ODE research globally. The presence of such robust networks is a marker of healthy and dynamic research environments that foster significant advancements in the field.

The analysis of top-cited papers indicates that highly impactful research often revolves around applications of ODEs to real-world problems, such as modelling thermal phenomena or addressing public health crises like COVID-19. These papers receive high citations and influence related disciplines, highlighting the practical relevance of ODE research. Publications in journals such as "Mathematics" by MDPI and "Mathematical Methods in the Applied Sciences" by Wiley-Blackwell, which lead to total publications and citations, reflect the interdisciplinary approach and applied research emphasis in ODEs. This suggests a field that values theoretical advancements as well as practical applications, keen on solving complex problems through mathematical modelling.

Emerging research themes, such as the integration of ODEs with cutting-edge computational technologies like machine learning, indicate evolving trends that enhance the utility and application scope of traditional mathematical models. This crossover marks a significant evolution in the field, where traditional methods are increasingly combined with modern technologies to tackle complex systems more effectively. The focus on public health modelling, especially during the COVID-19 pandemic, shows how ODE research directly impacts policy and decision-making, informing public health strategies and emergency responses, thus highlighting the societal value of these studies.

Considering current trends and highlighted challenges, future research in ODEs could explore further integration of machine learning and artificial intelligence to develop sophisticated models capable of handling large datasets and complex dynamic systems. Strengthening international collaborations and interdisciplinary projects could also enhance the development of innovative solutions to global challenges, ensuring the continued relevance and impact of ODE research. Through these discussions, we gain a comprehensive view of the current state and prospects of ODE research, illustrating its dynamic nature and pivotal role in advancing science and technology across various domains.

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