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An N-Period Dynamic Programming Model for Constrained Multi-Item Economic Order Quantity (EOQ) Under Stochastic Demand

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ABSTRACT

This study considers an N-period dynamic programming model for constrained multi-item Economic Order Quantity (EOQ) under probabilistic demand. The objective is to obtain a generalized formulation for the finite planning horizon, in order for profits to be maximized for each state of demands for the constrained multiple items. A decision is made on whether additional units will be ordered or not, for each item which depends on the demand for the individual item and each planning horizon. The Linear programming technique is used to incorporate and handle the constraints. The application of the mode was demonstrated with a numerical example.

Keywords: Economic Order Quantity (EOQ), Dynamic Programming (DP), Inventory Management, Optimization, Markov chain, Stochastic Demand, Linear Programming, Multi-item.

1 INTRODUCTION

Inventory can be defined as the quantity of goods or materials that a company has stored for the purpose of resale or use in its operations. It includes both finished products ready for sale and raw materials used in manufacturing. Inventory is an important asset for businesses across various industries, and managing it effectively is essential for maintaining smooth operations and meeting customer demand.

Holding and maintaining inventories is necessary for any company dealing with physical products, including manufacturers, wholesalers and retailers. It is important for the manufacturers to keep inventories of raw materials needed for production and also inventories of the finished goods. Also, both wholesalers and retailers need to keep and maintain inventories of products for customers at the right time.

With a good and strong inventory management system, one can monitor the goods in the warehouse in order to avoid overstocking, stocking of obsolete items, under-stocking and then focus on making the business successful in the market space. In summary, the objectives of inventory management include profit maximization, inventory cost minimization and maximization of service level. Moreover, in order to meet the future demand, it is important to determine the optimal stock and optimal time of inventory replenishment.

The EOQ model can be used to calculate the optimal quantity of items to be purchased in order to minimize the costs of carrying inventory and the processing of purchased orders. The assumptions of EOQ includes constant and known demand, fixed ordering costs per order, and the holding costs are proportional to the amount of inventory to be held. With the help of these assumptions, the calculations are simplified, but practically, adjustments and changes may be necessary to reflect the variability in the real-world.

The multi-item inventory model refers to managing different types of products or items within a single inventory system. It involves monitoring stock levels, order fulfilment and replenishment of the items simultaneously. Multi-item EOQ models deals with multiple items, each with its own demand, ordering cost and holding cost. The objective is to find an order quantity which is optimal for each of the items by considering the given constraints with the available limited resources which includes the budget, warehouse space and production or ordering capacity [1] and [2].

In this study, we developed an N-period dynamic programming model for constrained multi-item EOQ under stochastic demand. The constraints were handled using the linear LP techniques to obtain an optimal result.

2 LITERATURE REVIEW

Purohit and Rathore [3] developed a perishable inventory control model for multiple items. The model assumptions are instantaneous production or supply with zero lead time, uniform and deterministic demand with no shortages allowed. The model was solved analytically in order to minimize the total inventory cost using the Kuhn-Tucker theorem. Numerical examples were used to demonstrate the application of the model.

Panda et al. [4] developed a single period multi-item model for a manufacturing system with stochastically imperfect items under continuous probabilistic demand with constraints on budget and shortage. The expected profit in general form in terms of density functions of the demand and percentage was imperfect.

Zhang and Du [5] considered a multi-item newsboy model that produces multiple items for fulfilling stochastic demand with the same production capacity. To avoid possible shortage for the limited capacity, productions could be outsourced. The model considered two outsourcing strategies which is zero lead time and nonzero lead time outsourcing. The structural properties and solution procedures for the profit-maximization models were developed. Numerical results were provided to show the application of the model.

Zhang and Wang [6] considered multiple-item EOQ model for deteriorating items in order to obtain an optimal policy for the inventory problem, they analyzed the structural properties of the model. They introduced an algorithm for generating an optimal solution to the problem. The efficacy of the proposed model was demonstrated with numerical example.

Mubiru [7] formulated a periodic inventory model for determining the EOQ of multiple items in order to minimize the total inventory costs of the multiple items under a periodic review system with probabilistic demand. With Markov decision process (MDP) approach, the states of demand were represented by Markov chain. The decision of whether to order additional items or not to order were made with dynamic programming (DP) technique. The model shows an optimal state-dependent EOQ with optimal ordering policies as well as total inventory costs for the items.

Fergany [8] developed a stochastic multi-item model for single-source with varying shortage cost under two restrictions. The model analyzed how a firm can find the optimal order quantity and reorder point for each item with the goal of minimizing the expected total cost.

Khalilpourazari and Pasandideh [9] proposed a multiple-objective model for multi-item EOQ model in which they considered partial backordering with defective supply batches. Considering practical situations, they considered different stochastic operational constraints and partial backordering as decision variable under uncertainty. The aim was to determine the time interval between supply deliveries, backordering rate and stock replenishment in order to minimize total inventory costs while minimizing the required warehouse space.

Balkhi [10] considered an inventory model with infinite time planning horizon for multiple items under resource constraints. To determine an optimal production rate for each item, the author used Linear Programming to handle the constraints. The production, demand and deterioration rates in any cycle including the cost parameters were assumed as known and arbitrary functions of time where shortages were allowed but partially backlogged. The objective is to minimize the total inventory cost and to find the optimal production and restarting times for each of the items.

Kotb and Fergany [11] studied a multi-item inventory model to obtain an analytical solution for the EOQ model. They considered demand-dependent unit cost and lead time using geometric programming approach. The varying purchase and lead time crashing costs were considered to be continuous functions of demand rate and lead time, respectively. The authors obtained an optimal order quantity for the items.

Ghafour et al. [12] developed a multi-item model in which investment and shortage space constraints are active. They formulated four models using the Lagrange method to find the new multi-item EOQ for each of the four models considered with each of constraint thereby achieving a new formulation of EOQ.

Trinh [13] developed an inventory control system for multiple items with time varying demand using dynamic programming techniques. They proposed a policy in terms of the number of replenishments, EOQ and reorder point in order to minimize the total inventory cost. The result showed that the dynamic programming model performs better than the classical lot sizing models.

Gupta and Mishra [14] considered a multiple-item inventory model under probabilistic demand following a power demand pattern. A cumulative cycle demand was identified for each product and then the demand was gradually released to the inventory system by the power patterns within a cycle. The replenishment schedule was known, shortages were allowed but partially backlogged with others taken as lost sales.

Fallahi et al. [15] developed a multiple-product EOQ model for reusable items. The objective is to determine the optimal replenishment quantity and reuse quantity of each item in order to minimize the total cost of the system. They considered constraints on total budget, warehouse space and holding cost in order to obtain a realistic model.

Tan et al. [16] considered a multiple-period inventory model for profit optimization and shortage space under probabilistic demand. They developed a non-linear programming model to simulate the inventory operation. An effective inventory management system was realized using a multi-objective grey wolf optimization (MOGWO) method to reduce storage space while maximizing profit.

Mubiru [17] analyzed a multiple-product inventory model to optimize the EOQ for the multiple items using a multi-objective goal programming under probabilistic demand. Using Markov chain method, the author defines the demand transition matrix, inventory cost matrix, objective function and goal constraints for the model. The objective of the model is to minimize the deviation variables of the EOQ and total inventory costs of items over a finite period planning horizon.

3 MODEL DEVELOPMENT

This model considers constrained multiple items for N-period planning horizon. The model consists of a generalized formulation for solving a constrained multi-item EOQ problem using dynamic programming technique over N-period planning horizon. A decision is made during each period and for each item, which is whether to order additional units of item or not. The decision to order additional units is denoted by (z=1) and if no additional units are ordered, (z=0). The constraints are handled using linear programming approach.

3.1 Assumptions and Notations

Symbols	Meaning
i_n, j_n	The States of demand
n, N	The Stages going from n=1 to N
f	The Favorable state
u	The Unfavorable state
z	The Ordering policy
m	Number designating the mth item for m = 1, 2…M.
$D^z(\mathrm{m})$	Demand matrix for the mth item
$I^z(\mathrm{m})$	Inventory matrix for the mth item
$P^z(\mathrm{m})$	Profit matrix for the mth item
$P_s(\mathrm{m})$	Selling price per unit for the mth item
$P^z_{i_n,j_n}(\mathrm{m})$	Profit in state i, j for the mth item
$Q^z(\mathrm{m})$	The Demand transition matrix for the mth item
$Q^z_{i_n,j_n}(\mathrm{m})$	The Demand transition probability in state i, j for z policy in respect of the mth item
$I^z_{i_n,j_n}(\mathrm{m})$	Quantity of inventory in state i, j for z policy in respect of the mth item
$D^z_{i_n,j_n}(\mathrm{m})$	Quantity demanded in state i, j for z policy in respect of the mth item
$O^z_{i_n}(\mathrm{m})$	Economic order quantity for the mth item
$a^z_n(\mathrm{m})$	Expected total profits for the mth item

$a^{z}_{n,i_n}(\mathrm{m})$	Accumulated total profits at the end of the period for the mth item
$g_n(i,\mathrm{m})$	Optimal expected total profit for the mth item
$C_h(\mathrm{m})$	Holding cost per unit for the mth item
$C_o(\mathrm{m})$	Ordering cost per unit for the mth item
$C_s(\mathrm{m})$	Shortage cost per unit for the mth item
$C_p(\mathrm{m})$	Cost price per unit for the mth item

The following possibilities represents the changes in demand as follows:

$$Q^{z}_{i_n,j_n}(\mathrm{m}) \Leftrightarrow \begin{bmatrix} Q^{z}_{fff\ldots f}(\mathrm{m}) & Q^{z}_{fff\ldots u}(\mathrm{m}) & Q^{z}_{ffu\ldots f}(\mathrm{m}) & \ldots & \ldots & \ldots & Q^{z}_{fuu\ldots f}(\mathrm{m}) & Q^{z}_{fuu\ldots u}(\mathrm{m}) \\ Q^{z}_{uff\ldots f}(\mathrm{m}) & Q^{z}_{uff\ldots u}(\mathrm{m}) & Q^{z}_{ufu\ldots f}(\mathrm{m}) & \ldots & \ldots & \ldots & Q^{z}_{uuu\ldots f}(\mathrm{m}) & Q^{z}_{uuu\ldots u}(\mathrm{m}) \end{bmatrix}$$

$$I^{z}_{i_n,j_n}(\mathrm{m}) \Leftrightarrow \begin{bmatrix} I^{z}_{fff\ldots f}(\mathrm{m}) & I^{z}_{fff\ldots u}(\mathrm{m}) & I^{z}_{ffu\ldots f}(\mathrm{m}) & \ldots & \ldots & \ldots & I^{z}_{fuu\ldots f}(\mathrm{m}) & I^{z}_{fuu\ldots u}(\mathrm{m}) \\ I^{z}_{uff\ldots f}(\mathrm{m}) & I^{z}_{uff\ldots u}(\mathrm{m}) & I^{z}_{ufu\ldots f}(\mathrm{m}) & \ldots & \ldots & \ldots & I^{z}_{uuu\ldots f}(\mathrm{m}) & I^{z}_{uuu\ldots u}(\mathrm{m}) \end{bmatrix}$$

$$P^{z}_{i_n,j_n}(\mathrm{m}) \Leftrightarrow \begin{bmatrix} P^{z}_{fff\ldots f}(\mathrm{m}) & P^{z}_{fff\ldots u}(\mathrm{m}) & P^{z}_{ffu\ldots f}(\mathrm{m}) & \ldots & \ldots & \ldots & P^{z}_{fuu\ldots f}(\mathrm{m}) & P^{z}_{fuu\ldots u}(\mathrm{m}) \\ P^{z}_{uff\ldots f}(\mathrm{m}) & P^{z}_{uff\ldots u}(\mathrm{m}) & P^{z}_{ufu\ldots f}(\mathrm{m}) & \ldots & \ldots & \ldots & P^{z}_{uuu\ldots f}(\mathrm{m}) & P^{z}_{uuu\ldots u}(\mathrm{m}) \end{bmatrix}$$

$$D^{z}_{i_n,j_n}(\mathrm{m}) \Leftrightarrow \begin{bmatrix} D^{z}_{fff\ldots f}(\mathrm{m}) & D^{z}_{fff\ldots u}(\mathrm{m}) & D^{z}_{ffu\ldots f}(\mathrm{m}) & \ldots & \ldots & \ldots & D^{z}_{fuu\ldots f}(\mathrm{m}) & D^{z}_{fuu\ldots u}(\mathrm{m}) \\ D^{z}_{uff\ldots f}(\mathrm{m}) & D^{z}_{uff\ldots u}(\mathrm{m}) & D^{z}_{ufu\ldots f}(\mathrm{m}) & \ldots & \ldots & \ldots & D^{z}_{uuu\ldots f}(\mathrm{m}) & D^{z}_{uuu\ldots u}(\mathrm{m}) \end{bmatrix}$$

The above representations can be converted to multiple transitions as follows:

$$Q^{z}_{i_n,j_n}(\mathrm{m}) = \begin{bmatrix} Q^{z}_{ff}(\mathrm{m}) & Q^{z}_{fu}(\mathrm{m}) \\ Q^{z}_{uf}(\mathrm{m}) & Q^{z}_{uu}(\mathrm{m}) \end{bmatrix} \begin{bmatrix} Q^{z}_{ff}(\mathrm{m}) & Q^{z}_{fu}(\mathrm{m}) \\ Q^{z}_{uf}(\mathrm{m}) & Q^{z}_{uu}(\mathrm{m}) \end{bmatrix} \begin{bmatrix} Q^{z}_{ff}(\mathrm{m}) & Q^{z}_{fu}(\mathrm{m}) \\ Q^{z}_{uf}(\mathrm{m}) & Q^{z}_{uu}(\mathrm{m}) \end{bmatrix} \ldots\ldots \begin{bmatrix} Q^{z}_{ff}(\mathrm{m}) & Q^{z}_{fu}(\mathrm{m}) \\ Q^{z}_{uf}(\mathrm{m}) & Q^{z}_{uu}(\mathrm{m}) \end{bmatrix}$$

$$= \begin{bmatrix} Q^{z}_{FF}(\mathrm{m}) & Q^{z}_{FU}(\mathrm{m}) \\ Q^{z}_{UF}(\mathrm{m}) & Q^{z}_{UU}(\mathrm{m}) \end{bmatrix}$$

$$D^{z}_{i_n,j_n}(\mathrm{m}) = \begin{bmatrix} D^{z}_{ff}(\mathrm{m}) & D^{z}_{fu}(\mathrm{m}) \\ D^{z}_{uf}(\mathrm{m}) & D^{z}_{uu}(\mathrm{m}) \end{bmatrix} \begin{bmatrix} D^{z}_{ff}(\mathrm{m}) & D^{z}_{fu}(\mathrm{m}) \\ D^{z}_{uf}(\mathrm{m}) & D^{z}_{uu}(\mathrm{m}) \end{bmatrix} \begin{bmatrix} D^{z}_{ff}(\mathrm{m}) & D^{z}_{fu}(\mathrm{m}) \\ D^{z}_{uf}(\mathrm{m}) & D^{z}_{uu}(\mathrm{m}) \end{bmatrix} \ldots\ldots \begin{bmatrix} D^{z}_{ff}(\mathrm{m}) & D^{z}_{fu}(\mathrm{m}) \\ D^{z}_{uf}(\mathrm{m}) & D^{z}_{uu}(\mathrm{m}) \end{bmatrix}$$

$$= \begin{bmatrix} Q^{z}_{FF}(\mathrm{m}) & Q^{z}_{FU}(\mathrm{m}) \\ Q^{z}_{UF}(\mathrm{m}) & Q^{z}_{UU}(\mathrm{m}) \end{bmatrix}$$

$$I^z_{i_n,j_n}(\mathrm{m}) = \begin{bmatrix} I^z_{ff}(\mathrm{m}) & I^z_{fu}(\mathrm{m}) \\ I^z_{uf}(\mathrm{m}) & I^z_{uu}(\mathrm{m}) \end{bmatrix} \begin{bmatrix} I^z_{ff}(\mathrm{m}) & I^z_{fu}(\mathrm{m}) \\ I^z_{uf}(\mathrm{m}) & I^z_{uu}(\mathrm{m}) \end{bmatrix} \begin{bmatrix} I^z_{ff}(\mathrm{m}) & D^z_{fu}(\mathrm{m}) \\ I^z_{uf}(\mathrm{m}) & D^z_{uu}(\mathrm{m}) \end{bmatrix} \cdots\cdots \begin{bmatrix} I^z_{ff}(\mathrm{m}) & I^z_{fu}(\mathrm{m}) \\ I^z_{uf}(\mathrm{m}) & I^z_{uu}(\mathrm{m}) \end{bmatrix}$$

$$= \begin{bmatrix} I^z_{FF}(\mathrm{m}) & I^z_{FU}(\mathrm{m}) \\ I^z_{UF}(\mathrm{m}) & I^z_{UU}(\mathrm{m}) \end{bmatrix}$$

$$P^z_{i_n,j_n}(\mathrm{m}) = \begin{bmatrix} P^z_{ff}(\mathrm{m}) & P^z_{fu}(\mathrm{m}) \\ P^z_{uf}(\mathrm{m}) & P^z_{uu}(\mathrm{m}) \end{bmatrix} \begin{bmatrix} P^z_{ff}(\mathrm{m}) & P^z_{fu}(\mathrm{m}) \\ P^z_{uf}(\mathrm{m}) & P^z_{uu}(\mathrm{m}) \end{bmatrix} \begin{bmatrix} P^z_{ff}(\mathrm{m}) & P^z_{fu}(\mathrm{m}) \\ Q^z_{uf}(\mathrm{m}) & P^z_{uu}(\mathrm{m}) \end{bmatrix} \cdots\cdots \begin{bmatrix} P^z_{ff}(\mathrm{m}) & P^z_{fu}(\mathrm{m}) \\ P^z_{uf}(\mathrm{m}) & P^z_{uu}(\mathrm{m}) \end{bmatrix}$$

$$= \begin{bmatrix} P^z_{FF}(\mathrm{m}) & P^z_{FU}(\mathrm{m}) \\ P^z_{UF}(\mathrm{m}) & P^z_{UU}(\mathrm{m}) \end{bmatrix}$$

From the above matrices, it is noted that the number of matrices that make up the right-hand matrix at stage N is calculated from the formula below:

$$I_N(\mathrm{m}) = \sum_{n=0}^{N-2} 2^n = 2^{N-1} - 1; N \geq 2 \tag{1}$$

N is the number of periods which is the planning horizon, the demand transition has to start from 2. Then we have the following formulations:

$$I_2(\mathrm{m}) = \sum_{n=0}^{0} 2^n = 2^{2-1} - 1 = 2 - 1 = 1 \tag{2}$$

$$I_3(\mathrm{m}) = \sum_{n=0}^{1} 2^n = 2^{3-1} - 1 = 2^2 - 1 = 3 \tag{3}$$

$$I_4(\mathrm{m}) = \sum_{n=0}^{2} 2^n = 2^{4-1} - 1 = 2^3 - 1 = 7 \tag{4}$$

and so on.

3.2 Multi-Item Dynamic Programming Formulation For n-th Period

The dynamic programming formulation for nth-period multi-item inventory system considered two states of demand which is the favorable state and the unfavorable state. We expressed the problem as a dynamic programming model with finite period in order to obtain an optimal EOQ. The expected total profit accumulated during the n periods is denoted with $g_n(\mathrm{m})$ given $n \le N$. The state of the system is given as $i_n \in \{f, u\}$ at the start of period n.

$g_n(\mathrm{m})$ and $g_{n+1}(\mathrm{m})$ are related by the following recursive equation:

$$g_n(\mathrm{m}) = \max_z \{Q^z_{i_n f}(\mathrm{m})[P^z_{i_n f}(\mathrm{m}) + g_{n+1}(f, \mathrm{m})], Q^z_{i_n u}(\mathrm{m})[P^z_{i_n u}(\mathrm{m}) + g_{n+1}(u, \mathrm{m})]\} \tag{5}$$

$$z \in \{0,1\},\ i_n \in \{f, u\},\ n = 1, 2, \dots, \mathrm{N}$$

for $$g_N(f, \mathrm{m}) = g_{N+1}(u, \mathrm{m}) = 0 \tag{6}$$

The cumulative total profit can be represented as follows:

$$P^z_{i_n, j_n}(\mathrm{m}) + g_{n+1}(j_n) \tag{7}$$

The above formulation results to reaching state $j_n \in \{f, u\}$ at the start of period n+1 from state $i_n \in \{f, u\}$ at the initial period n, occurring with $Q^z_{i_n, j_n}(\mathrm{m})$ as the probability.

The accumulated profit can be obtained using the formulation below:

$$a^z_{n, i_n}(\mathrm{m}) = Q^z(\mathrm{m})\{P^z(\mathrm{m})\}^T \tag{8}$$

The recursive equation is represented below:

$$g_n(i_n, \mathrm{m}) = \mathrm{m}ax_z \{a^z_{n, i_n}(\mathrm{m}) + Q^z_{i_n f}(\mathrm{m}) g_{n+1}(f, \mathrm{m}) + Q^z_{i_n u}(\mathrm{m}) g_{n+1}(u, \mathrm{m})\} \tag{9}$$

or $$g_N(i_n, \mathrm{m}) = \max_z \left[a^z_{n, i_n}(\mathrm{m})\right] \tag{10}$$

3.3 Demand Transition Matrix, Profit Matrix and EOQ Computation

The demand transition probability from state i_n to j_n could be expressed as the quantity demanded when the demand is at initial state i_n and then transiting to state j_n for ordering policy $z \in \{0,1\}$ divided by the total quantity demanded over all states. This can be represented in the equation below:

$$Q^z_{i_n,j_n}(\mathrm{m}) = \frac{D^z_{i_n,j_n}(\mathrm{m})}{[D^z_{i_n f}(\mathrm{m}) + D^z_{i_n u}(\mathrm{m})]} \tag{11}$$

$i_n, j_n \in \{f,u\}$; $z \in \{0,1\}$

The profit matrix P^z can be computed using equation (13) for the situation when the demand is greater than available inventory:

To obtain profit, cost price must be subtracted from the selling price as represented below:

Profit = selling price – cost price

$$P^z(\mathrm{m}) = P_s(\mathrm{m}) - C_p(\mathrm{m}) \tag{12}$$

Therefore,

$$P^z(\mathrm{m}) = P_{i_n,j_n}(\mathrm{m})D^z_{i_n,j_n}(\mathrm{m}) - \{C_o(\mathrm{m}) + C_h(\mathrm{m})\}I^z_{i_n,j_n}(\mathrm{m}) - \{C_o(\mathrm{m}) + C_s(\mathrm{m})\}[D^z_{i_n,j_n}(\mathrm{m}) - I^z_{i_n,j_n}(\mathrm{m})] \tag{13}$$

Also, if demand is less than or equal to the inventory at hand, then

$$P^z(\mathrm{m}) = P_{i_n,j_n}(\mathrm{m})D^z_{i_n,j_n}(\mathrm{m}) - \{C_o(\mathrm{m}) + C_h(\mathrm{m})\}D^z_{i_n,j_n}(\mathrm{m}) \tag{14}$$

Combining equations (13) and (14), we obtain equation (15) below:

$$\Rightarrow$$

$$P^z(\mathrm{m}) = \left\{ \begin{array}{l} P_{i_n,j_n}(\mathrm{m})D^z_{i_n,j_n}(\mathrm{m}) - \{C_o(\mathrm{m}) + C_h(\mathrm{m})\}I^z_{i_n,j_n}(\mathrm{m}) - \{C_o(\mathrm{m}) + C_s(\mathrm{m})\}[D^z_{i_n,j_n}(\mathrm{m}) - I^z_{i_n,j_n}(\mathrm{m})] \\ if \quad D^z_{i_n,j_n}(\mathrm{m}) > I^z_{i_n,j_n}(\mathrm{m}) \\ P_{i_n,j_n}(\mathrm{m})D^z_{i_n,j_n}(\mathrm{m}) - \{C_o(\mathrm{m}) + C_h(\mathrm{m}\}D^z_{i_n,j_n}(\mathrm{m}) \qquad if \quad D^z_{i_n,j_n}(\mathrm{m}) \le I^z_{i_n,j_n}(\mathrm{m}) \end{array} \right\}$$

$$\forall \quad i_n, j_n \in \{f,u\} \quad z \in \{0,1\} \tag{15}$$

To obtain the EOQ when demand is in state $i_n \in \{f,u\}$ with ordering policy $z \in \{0,1\}$, the following equation can be applied.

$$O^z_{i_n}(\mathrm{m}) = \{D^z_{i_n f}(\mathrm{m}) - I^z_{i_n f}(\mathrm{m})\} + \{D^z_{i_n u}(\mathrm{m}) - I^z_{i_n u}(\mathrm{m})\};\ D^z_{i_n,j_n}(\mathrm{m}) > I^z_{i_n,j_n}(\mathrm{m}) \tag{16}$$

$$i_n, j_n \in \{f,u\},\ z \in \{0,1\}$$

Otherwise, $O^z_{i_n}(\mathrm{m}) = 0$.

3.4 Computation Of EOQ For the Planning Horizons

An N-period planning horizon was considered in this section and so the planning starts from $n = 1, 2, 3 \ldots\ldots\ldots\ldots\ldots N-1, N$, as follows:

First planning horizon

For the ordering policy during period 1 when the demand is favorable we have:

$$z = \begin{cases} 1;\ if\ a_{1,f}^{1}(\mathrm{m}) > a_{1,f}^{0}(\mathrm{m}) \\ 0;\ if\ \ a_{1,f}^{1}(\mathrm{m}) \le a_{1,f}^{0}(\mathrm{m}) \end{cases} \tag{17}$$

The associated total profit is represented as follows:

$$g_1(f, m) = \begin{cases} a_{1,f}^{1}(\mathrm{m});\ if\ z = 1 \\ a_{1,f}^{0}(\mathrm{m});\ if\ z = 0 \end{cases} \tag{18}$$

The EOQ for each of the planning horizons are is represented in (16).

The ordering policy during period 1 when the demand is unfavorable:

$$z = \begin{cases} 1,\ \ if\ a_{1,u}^{1}(\mathrm{m}) > a_{1,u}^{0}(\mathrm{m}) \\ 0,\ if\ a_{1,u}^{1}(\mathrm{m}) \le a_{1,u}^{0}(\mathrm{m}) \end{cases} \tag{19}$$

The associated total profit:

$$g_1(u, m) = \begin{cases} a_{1,u}^{1}(\mathrm{m});if\ z = 1 \\ a_{1,u}^{0}(\mathrm{m});\ if\ z = 0 \end{cases} \tag{20}$$

3.5 Second Planning Horizon

The following recursive equation represents the accumulated profit $a_{2,i_n}^{z}(\mathrm{m})$ at the end of the first period as a result of decisions made:

$$a_{2,i_n}^{z}(\mathrm{m}) = a_{1,i_n}^{z}(\mathrm{m}) + Q_{if}^{z}(\mathrm{m}) g_1(f, m) + Q_{iu}^{z}(\mathrm{m}) g_1(u, m) \tag{15}$$

The ordering policy when the demand is favorable is as follows:

$$z = \begin{cases} 1; \ if \ a_{2,f}^{1}(\mathrm{m}) > a_{2,f}^{0}(\mathrm{m}) \\ 0; \ if \ a_{2,f}^{1}(\mathrm{m}) \le a_{2,f}^{0}(\mathrm{m}) \end{cases} \tag{21}$$

The associated total profits is:

$$g_2(f,m) = \begin{cases} a_{2,f}^{1}(\mathrm{m}); \ if \ z = 1 \\ a_{2,f}^{0}(\mathrm{m}); \ if \ z = 0 \end{cases} \tag{22}$$

Similarly, when demand is unfavorable, the ordering policy is given below with the associated profit:

$$z = \begin{cases} 1; \ if \ a_{2,u}^{1}(\mathrm{m}) > a_{2,u}^{0}(\mathrm{m}) \\ 0; \ if \ a_{2,u}^{1}(\mathrm{m}) \le a_{2,u}^{0}(\mathrm{m}) \end{cases} \tag{23}$$

$$g_2(u,m) = \begin{cases} a_{2,u}^{1}(\mathrm{m}); \ if \ z = 1 \\ a_{2,u}^{0}(\mathrm{m}); \ if \ z = 0 \end{cases} \tag{24}$$

3.6 Third Planning Horizon

The accumulated profits at the end of the third period as a result of decisions made can be obtained using recursive equation below:

$$a_{3,i_n}^{z}(\mathrm{m}) = a_{2,i_n}^{z}(\mathrm{m}) + Q_{if}^{z}(\mathrm{m}) g_2(f,m) + Q_{iu}^{z}(\mathrm{m}) g_2(u,m) \tag{25}$$

The ordering policy when the demand is favorable with the associated profits are represented below:

$$z = \begin{cases} 1; \ if \ a_{3,f}^{1}(\mathrm{m}) > a_{3,f}^{0}(\mathrm{m}) \\ 0; \ if \ a_{3,f}^{1}(\mathrm{m}) \le a_{3,f}^{0}(\mathrm{m}) \end{cases} \tag{26}$$

$$g_3(f,m) = \begin{cases} a_{3,f}^{1}(\mathrm{m}); \ if \ z = 1 \\ a_{3,f}^{0}(\mathrm{m}); \ if \ z = 0 \end{cases} \tag{27}$$

Also, for unfavorable demand, the ordering policy and the associated profits are given below:

$$z = \begin{cases} 1; \ if \ a_{3,u}^{1}(\mathrm{m}) > a_{3,u}^{0}(\mathrm{m}) \\ 0; \ if \ a_{3,u}^{1}(\mathrm{m}) \le a_{3,u}^{0}(\mathrm{m}) \end{cases} \tag{28}$$

$$g_3(u,m) = \begin{cases} a_{2,u}^{1}(\mathrm{m}); \ if \ z = 1 \\ a_{2,u}^{0}(\mathrm{m}); \ if \ z = 0 \end{cases} \tag{29}$$

3.7 (N-1)th Planning Horizon

At the end of (N-1)-period, the profit accumulated as a result of decisions made using recursive equation below:

$$a_{N-1,i_n}^{z}(\mathrm{m}) = a_{N-2,i_n}^{z}(\mathrm{m}) + Q_{if}^{z}(\mathrm{m}) g_{N-2}(f,m) + Q_{iu}^{z}(\mathrm{m}) g_{N-2}(u,m) \tag{30}$$

The ordering policy when the demand is favorable with the associated total profits are as follows:

$$z = \begin{cases} 1; \ if \ a_{N-1,f}^{1}(\mathrm{m}) > a_{N-1,f}^{0}(\mathrm{m}) \\ 0; \ if \ a_{N-1,f}^{1}(\mathrm{m}) \le a_{N-1,f}^{0}(\mathrm{m}) \end{cases} \tag{31}$$

$$g_{N-1}(f,m) = \begin{cases} a_{N-1,f}^{1}(\mathrm{m}); \ if \ z = 1 \\ a_{N-1,f}^{0}(\mathrm{m}); \ if \ z = 0 \end{cases} \tag{32}$$

Similarly, the ordering policy and the associated profit for the unfavorable demand are as follows:

$$z = \begin{cases} 1; \ if \ a_{N-1,u}^{1}(\mathrm{m}) > a_{N-1,u}^{0}(\mathrm{m}) \\ 0; \ if \ a_{N-1,u}^{1}(\mathrm{m}) \le a_{N-1,u}^{0}(\mathrm{m}) \end{cases} \tag{33}$$

$$g_{N-1}(u,m) = \begin{cases} a_{N-1,u}^{1}(\mathrm{m}); \ if \ z = 1 \\ a_{N-1,u}^{0}(\mathrm{m}); \ if \ z = 0 \end{cases} \tag{34}$$

3.8 N^th^ Planning Horizon

At the end of period N, the profit accumulated as a result of decisions made can be obtained using the following recursive equation:

$$a^{z}_{N,i_n}(\mathrm{m}) = a^{z}_{N-1,i_n}(\mathrm{m}) + Q^{z}_{if}(\mathrm{m})g_{N-1}(f,m) + Q^{z}_{iu}(\mathrm{m})g_{N-1}(u,m) \tag{35}$$

When the demand is favorable, the ordering policy with the associated profit and EOQ are as follows:

$$z = \begin{cases} 1, & if\ a^{1}_{N,f}(\mathrm{m}) > a^{0}_{N,f}(\mathrm{m}) \\ 0, & if\ a^{1}_{N,f}(\mathrm{m}) \le a^{0}_{N,f}(\mathrm{m}) \end{cases} \tag{36}$$

$$g_N(f,\mathrm{m}) = \begin{cases} a^{1}_{N,f}(\mathrm{m});\ if\ z=1 \\ a^{0}_{N,f}(\mathrm{m});\ if\ z=0 \end{cases} \tag{37}$$

and

$$O^{z}_{f}(\mathrm{m}) = \begin{cases} \{D^{1}_{ff}(\mathrm{m}) - I^{1}_{ff}(\mathrm{m})\} + \{D^{1}_{fu}(\mathrm{m}) - I^{1}_{fu}(\mathrm{m})\}; & if\ z=1 \\ 0;\ if\ z=0 \end{cases} ; D^{z}_{ij}(\mathrm{m}) > I^{z}_{ij}(\mathrm{m}) \tag{38}$$

For the unfavorable demand, the ordering policy, associated profits and EOQ are given below:

$$z = \begin{cases} 1; & if\ a^{1}_{N,u}(\mathrm{m}) > a^{0}_{N,u}(\mathrm{m}) \\ 0; & if\ a^{1}_{N,u}(\mathrm{m}) \le a^{0}_{N,u}(\mathrm{m}) \end{cases} \tag{39}$$

$$g_N(u,\mathrm{m}) = \begin{cases} a^{1}_{N,u}(\mathrm{m});\ if\ z=1 \\ a^{0}_{N,u}(\mathrm{m});\ if\ z=0 \end{cases} \tag{40}$$

and

$$O^{z}_{u}(\mathrm{m}) = \begin{cases} \{D^{1}_{uf}(\mathrm{m}) - I^{1}_{uf}(\mathrm{m})\} + \{D^{1}_{uu}(\mathrm{m}) - I^{1}_{uu}(\mathrm{m})\}; & if\ z=1 \\ 0;\ if\ z=0; \end{cases} ; D^{z}_{ij}(\mathrm{m}) > I^{z}_{ij}(\mathrm{m}) \tag{41}$$

3.9 Numerical Example

Suppose a business sells five different items (1, 2, 3, 4 and 5) whose demands are stochastic. The goal is to determine the optimal order quantity for each of the items in order to maximize the total profit subject to order quantity constraint, warehouse constraint and capital constraint. The total available capital for purchasing inventory is ₦30,000 and the total warehouse space available for the units of the item are for 20 units, 30 units, 35units, 30 units and 40 units respectively which must not be more than 1000 units of the items altogether. The profits per unit of the items are ₦500, ₦500, ₦500, ₦600 and ₦500 respectively. Management requires minimum order quantities for the five items to be, 6 units, 8 units, 6 units, 7 units and 5 units respectively. The demand and inventory for the five items are given in the following tables for the favorable and the unfavorable states.

Table 1: For m = 1, when z = 1

i	Demand	Inventory
ff	07	05
fu	06	04
uf	05	04
uu	04	03

Table 2: For m = 1, when z = 0.

i	Demand	Inventory
ff	06	08
fu	07	05
uf	05	07
uu	03	05

Table 3: For m = 2, when z = 1

i	Demand	Inventory
ff	06	03
fu	05	01
uf	05	04
uu	04	02

Table 4: For m = 2, when z = 0.

i	Demand	Inventory
ff	03	05
fu	02	04
uf	02	03
uu	04	03

Table 5: For m = 3, when z = 1

i	Demand	Inventory
ff	06	03
fu	04	02
uf	05	05
uu	03	02

Table 6: For m = 3, when z = 0.

i	Demand	Inventory
ff	07	04
fu	05	06
uf	03	04
uu	02	04

Table 7: For m = 4, when z = 1

i	Demand	Inventory
ff	05	03
fu	06	02
uf	05	04
uu	03	01

Table 8: For m = 4, when z = 0.

i	Demand	Inventory
ff	04	06
fu	03	05
uf	05	05
uu	02	03

Table 9: For m = 5, when z = 1

i	Demand	Inventory
ff	05	03
fu	04	02
uf	03	02
uu	02	01

Table 10: For m = 5, when z = 0.

i	Demand	Inventory
ff	04	03
fu	03	02
uf	02	01
uu	03	02

For a 5-period planning horizon for each of the items, the model can be expressed as 15-multiple transition matrices because a 5-period planning horizon generates 15 transitions. The transition matrices for the 5-period for all the items can therefore be expressed below as obtained with the help of MATLAB computer package.

For item 1; m=1, z=1:

$$D_F^1(1) = \begin{bmatrix} 07 & 06 \\ 05 & 04 \end{bmatrix}^{15} = \begin{bmatrix} 3{,}363{,}213{,}869{,}197{,}363 & 2{,}810{,}912{,}501{,}340{,}018 \\ 3{,}342{,}427{,}084{,}450{,}015 & 1{,}957{,}757{,}618{,}527{,}354 \end{bmatrix}$$

$$= \begin{bmatrix} 3.3632 & 2.8109 \\ 3.3424 & 1.9578 \end{bmatrix} \times 10^{15}$$

$$I_F^1(1) = \begin{bmatrix} 05 & 04 \\ 04 & 03 \end{bmatrix}^{15} = \begin{bmatrix} 2{,}748{,}592{,}520{,}889{,}300 & 2{,}146{,}036{,}191{,}130{,}000 \\ 2{,}146{,}036{,}191{,}130{,}000 & 1{,}675{,}574{,}425{,}324{,}300 \end{bmatrix}$$

$$= \begin{bmatrix} 2.7486 & 2.1460 \\ 2.1460 & 1.6756 \end{bmatrix} \times 10^{13}$$

When additional units are not ordered, z=0

$$D_F^0(1) = \begin{bmatrix} 06 & 07 \\ 05 & 03 \end{bmatrix}^{15} = \begin{bmatrix} 1{,}499{,}718{,}770{,}286{,}141 & 1{,}380{,}724{,}426{,}661{,}006 \\ 9{,}862{,}317{,}333{,}292{,}900 & 9{,}079{,}797{,}302{,}885{,}670 \end{bmatrix}$$

$$= \begin{bmatrix} 1.4997 & 1.3807 \\ 0.9862 & 0.9080 \end{bmatrix} \times 10^{15}$$

$$I_F^0(1) = \begin{bmatrix} 08 & 05 \\ 07 & 05 \end{bmatrix}^{15} = \begin{bmatrix} 2,002,905,062,800,333 & 1,317,132,632,407,562 \\ 1,843,985,685,370,587 & 1,212,625,483,355,796 \end{bmatrix}$$

$$= \begin{bmatrix} 2.0029 & 1.3171 \\ 1.8440 & 1.2126 \end{bmatrix} \times 10^{16}$$

ITEM 1:
We assume the selling price per unit $P_s(1)$ is ₦1000, the cost of ordering $C_o(1)$ is ₦200, the cost of holding $C_h(1)$ is ₦100, the cost for shortage $C_s(1)$ is ₦50 and the cost price $C_p(1)$ is ₦500, then we have the following computations:

3.10 Computation of Model Parameters`

$$P_{i_n j_n}(1) = 1000 - 500 = 500$$

The matrices for the demand transition and profit are computed using equations (11) and (15). we have:

$$Q^1(1) = \begin{bmatrix} \dfrac{D_{FF}^1(1)}{D_{FF}^1(1) + D_{FU}^1(1)} & \dfrac{D_{FU}^1(1)}{D_{FF}^1(1) + D_{FU}^1(1)} \\ \dfrac{D_{UF}^1(1)}{D_{UF}^1(1) + D_{UU}^1(1)} & \dfrac{D_{UU}^1(1)}{D_{UF}^1(1) + D_{UU}^1(1)} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{3,363,213,869,197,363}{6,174,126,370,537,381} & \dfrac{2,810,912,501,340,018}{6,174,126,370,537,381} \\ \dfrac{3,342,427,084,450,015}{5,300,184,702,977,369} & \dfrac{1,957,757,618,527,354}{5,300,184,702,977,369} \end{bmatrix}$$

$$\Rightarrow Q^1(1) = \begin{bmatrix} 0.545 & 0.455 \\ 0.631 & 0.369 \end{bmatrix}$$

When z=1, we have the profit matrix as follows:

$$P^1(1) = \left\{ P_{i_n,j_n}(1) D^1_{i_n,j_n}(1) - \{C_o(1) + C_h(1)\} I^1_{i_n,j_n}(1) - \{C_o(1) + C_s(1)\}[D^1_{i_n,j_n}(1) - I^1_{i_n,j_n}(1)] \right\}; D^1_{i_n,j_n} \geq I^1_{i_n,j_n}$$

$$\Rightarrow P^1(1) = 500\begin{bmatrix} 07 & 06 \\ 05 & 04 \end{bmatrix}^{15} - 300\begin{bmatrix} 05 & 04 \\ 04 & 03 \end{bmatrix}^{15} - 250\left(\begin{bmatrix} 07 & 06 \\ 05 & 04 \end{bmatrix}^{15} - \begin{bmatrix} 05 & 04 \\ 04 & 03 \end{bmatrix}^{15}\right)$$

$$\Rightarrow \begin{bmatrix} 8{,}394{,}291{,}710{,}388{,}964 & 7{,}016{,}551{,}072{,}394{,}395 \\ 5{,}845{,}337{,}530{,}169{,}038 & 4{,}886{,}016{,}617{,}419{,}176 \end{bmatrix}$$

$$\Rightarrow P^1(1) = \begin{bmatrix} 8.3943 & 7.0166 \\ 5.8453 & 4.8860 \end{bmatrix} \times 10^{17}$$

For z=0, m=1 we have:

$$Q^0(1) = \begin{bmatrix} \dfrac{D^0_{FF}(1)}{D^0_{FF}(1) + D^0_{FU}(1)} & \dfrac{D^0_{FU}(1)}{D^0_{FF}(1) + D^0_{FU}(1)} \\ \dfrac{D^0_{UF}(1)}{D^0_{UF}(1) + D^0_{UU}(1)} & \dfrac{D^0_{UU}(1)}{D^0_{UF}(1) + D^0_{UU}(1)} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1{,}499{,}718{,}770{,}286{,}141}{2{,}880{,}443{,}196{,}947{,}147} & \dfrac{1{,}380{,}724{,}426{,}661{,}006}{2{,}880{,}443{,}196{,}947{,}147} \\ \dfrac{9{,}862{,}317{,}333{,}292{,}900}{18{,}942{,}114{,}636{,}178{,}570} & \dfrac{9{,}079{,}797{,}302{,}885{,}670}{18{,}942{,}114{,}636{,}178{,}570} \end{bmatrix}$$

$$\Rightarrow Q^0(1) = \begin{bmatrix} 0.521 & 0.479 \\ 0.521 & 0.479 \end{bmatrix}$$

For z=0, m=1 we have:

$$P^0(1) = P_{i_n,j_n}(1) D^0_{i_n,j_n}(1) - \{C_o(1) + C_h(1)\} D^0_{i_n,j_n}(1); \quad \mathrm{D}^0_{i_n,j_n}(1) \leq I^0_{i_n,j_n}(1)$$

$$P^0(1) = 500\begin{bmatrix} 06 & 07 \\ 05 & 03 \end{bmatrix}^{15} - 300\begin{bmatrix} 06 & 07 \\ 05 & 03 \end{bmatrix}^{15} = \begin{bmatrix} 2{,}999{,}437{,}540{,}572{,}282 & 2{,}761{,}448{,}853{,}322{,}012 \\ 1{,}972{,}463{,}466{,}658{,}580 & 1{,}815{,}959{,}460{,}577{,}134 \end{bmatrix}$$

$$\Rightarrow P^0(1) = \begin{bmatrix} 2.9994 & 2.7614 \\ 1.9725 & 1.8160 \end{bmatrix} \times 10^{17}$$

3.11 Computation of expected total profit for first planning horizon for item 1:

For z=1, the matrices $Q^1(1)$ and $P^1(1)$ yield profit as follows:

$$a_1^1(1) = Q^1(1)[P^1(1)]^T = \begin{bmatrix} 0.545 & 0.455 \\ 0.631 & 0.369 \end{bmatrix} \begin{bmatrix} 8,394,291,710,388,964 & 7,016,551,072,394,395 \\ 5,845,337,530,169,038 & 4,886,016,617,419,176 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0.545 & 0.455 \\ 0.631 & 0.369 \end{bmatrix} \begin{bmatrix} 8,394,291,710,388,964 & 5,845,337,530,169,038 \\ 7,016,551,072,394,395 & 4,886,016,617,419,176 \end{bmatrix}$$

$$\Rightarrow a_{1,f}^1(1) = 8,394,291,710,388,964(0.545) + 7,016,551,072,394,395(0.455)$$

$$= 7,767,419,720,101,435$$

$$a_{1,u}^1(1) = 5,845,337,530,169,038(0.631) + 4,886,016,617,419,176(0.369)$$

$$= 5,491,348,113,364,339$$

Also, for z=0, the matrices $Q^0(1)$ and $P^0(1)$ yield profit as follows:

$$a_1^0(1) = Q^0(1)\{P^0(1)\}^T = \begin{bmatrix} 0.521 & 0.479 \\ 0.521 & 0.479 \end{bmatrix} \begin{bmatrix} 2,999,437,540,572,282 & 2,761,448,853,322,012 \\ 1,972,463,466,658,580 & 1,815,959,460,577,134 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0.521 & 0.479 \\ 0.521 & 0.479 \end{bmatrix} \begin{bmatrix} 2,999,437,540,572,282 & 1,972,463,466,658,580 \\ 2,761,448,853,322,012 & 1,815,959,460,577,134 \end{bmatrix}$$

$$\Rightarrow a_{1,f}^0(1) = 2,999,437,540,572,282(0.521) + 2,761,448,853,322,012(0.479)$$
$$= 2,885,460,119,379,403$$

$$a_{1,u}^0(1) = 1,972,463,466,658,580(0.521) + 1,815,959,460,577,134(0.479)$$

$$= 1,897,498,047,745,567$$

This shows that z=1 is the optimal ordering policy for favorable state since ₦7,767,419,720,101,435 is greater than ₦2,885,460,119,379,403 with associated total profit of ₦7,767,419,720,101,435 and EOQ of 4units. Also, it shows that z=1 is the optimal ordering policy for unfavorable state since ₦5,491,348,113,364,339 is greater than ₦1,897,498,047,745,567 with associated profit of ₦5,491,348,113,364,339 and EOQ of 2units for the first planning horizon.

3.12 Computation of accumulated profit for second planning horizon for item 1:

The accumulated profit for favorable and unfavorable demands for the second planning horizon are computed as follows:

For favorable demand, we have:

$$a_{2,f}^{1}(1) = 7,767,419,720,101,435 + (0.545)7,767,419,720,101,435 + (0.455)5,491,348,113,364,339$$
$$= 14,499,226,859,137,491$$

$$a_{2,f}^{0}(1) = 2,885,460,119,379,403 + (0.521)7,767,419,720,101,435 + (0.479)5,491,348,113,364,339$$
$$= 9,562,641,539,853,769$$

This shows that z=1 is the optimal ordering policy for favorable demand since ₦14,499,226,859,137,491 is greater than ₦9,562,641,539,853,769 with accumulated profit of ₦14,499,226,859,137,491 and EOQ of 4 units.

On the other hand, the accumulated profit for unfavorable state is as follows:

$$a_{2,u}^{1}(1) = 5,491,348,113,364,339 + (0.631)7,767,419,720,101,435 + (0.369)5,491,348,113,364,339$$

$$= 12,418,897,410,579,786$$

$$a_{2,u}^{0}(1) = 1,897,498,047,745,567 + (0.521)7,767,419,720,101,435 + (0.479)5,491,348,113,364,339$$
$$= 8,574,679,468,219,933$$

From the above computations, z=1 is the optimal ordering policy for the unfavorable state with associated profit ₦12,418,897,252,410,579,786 which is greater than ₦8,574,679,468,219,933 and EOQ of 2units.

3.13 Computation of accumulated total profit for third planning horizon for item 1:

The accumulated profits for favorable and unfavorable states of the third planning horizon are computed as follows:

For favorable demand, we have:

$$a_{3,f}^{1}(1) = 14,499,226,859,137,491 + (0.545)14,499,226,859,137,491 + (0.455)12,418,897,410,579,786$$
$$= 28,051,903,819,181,226$$

$$a_{3,f}^{0}(1) = 9,562,641,539,853,769 + (0.521)14,499,226,859,137,491 + (0.479)12,418,897,410,579,786$$
$$= 23,065,390,593,132,119$$

Which shows that z=1 is the optimal ordering policy for favorable demand since ₦28,051,903,819,181,226 is greater than ₦23,065,390,593,132,119 with accumulated profit of ₦28,051,903,819,181,226 and EOQ of 4units.

On the other hand, the accumulated profit for unfavorable demand is as follows:

$$a_{3,u}^{1}(1) = 12,418,897,410,579,786 + (0.631)14,499,226,859,137,491 + (0.369)12,418,897,410,579,786$$
$$= 26,150,482,703,199,484$$

$$a_{3,u}^{0}(1) = 8,574,679,468,219,933 + (0.521)14,499,226,859,137,491 + (0.479)12,418,897,410,579,786$$
$$= 22,077,428,521,498,283$$

Which shows clearly that z=1 is the optimal ordering policy since ₦26,150,482,703,199,484 is greater than ₦22,077,428,521,498,283 with accumulated profit of ₦26,150,482,703,199,484 and EOQ of 2units.

3.14 Computation of accumulated total profit for fourth planning horizon for item 1:

The accumulated profit for favorable and unfavorable demands for the fourth planning are computed as follows:

For favorable demand, we have:

$$a_{4,f}^{1}(1) = 28,051,903,819,181,226 + (0.545)28,051,903,819,181,226 + (0.455)26,150,482,703,199,484$$
$$= 55,238,661,030,590,759$$

$$a_{4,f}^{0}(1) = 23,065,390,593,132,119 + (0.521)28,051,903,819,181,226 + (0.479)26,150,482,703,199,484$$
$$= 50,206,513,697,758,091$$

It is clear that z=1 is the optimal ordering policy for favorable demand since ₦55,238,661,030,590,759 is greater than ₦50,206,513,697,758,091 with accumulated profit of ₦55,238,661,030,590,759 and EOQ of 4 units.

On the other hand, the accumulated profit for unfavorable demand is as follows:

$$a_{4,u}^{1}(1) = 26,150,482,703,199,484 + (0.631)28,051,903,819,181,226 + (0.369)26,150,482,703,199,484$$
$$= 53,500,762,130,583,447$$

$$a_{4,u}^{0}(1) = 22,077,428,521,498,283 + (0.521)28,051,903,819,181,226 + (0.479)26,150,482,703,199,484$$
$$= 49,218,551,626,124,255$$

Which shows clearly that z=1 is the optimal ordering policy since ₦53,500,762,130,583,447 is greater than ₦49,218,551,626,124,255 with accumulated profit of ₦53,500,762,130,583,447 and EOQ of 2units.

3.15 Computing the accumulated total profit for fifth planning horizon for item 1:

The accumulated profit for favorable and unfavorable demands for the fifth planning are computed as follows:

For favorable demand, we have:

$$a_{5,f}^{1}(1) = 55,238,661,030,590,759 + (0.545)55,238,661,030,590,759 + (0.455)53,500,762,130,583,447$$
$$= 109,686,578,061,678,191$$

$$a_{5,f}^{0}(1) = 50,206,513,697,758,091 + (0.521)55,238,661,030,590,759 + (0.479)53,500,762,130,583,447$$
$$= 104,612,721,155,245,348$$

It shows that z=1 is the optimal ordering policy for favorable demand since ₦109,686,578,061,678,191 is greater than ₦104,612,721,155,245,348 with accumulated profit of ₦109,686,578,061,678,191 and EOQ of 4 units.

On the other hand, the accumulated profit for unfavorable demand is as follows:

$$a_{5,\mathrm{u}}^{1}(1) = 53,500,762,130,583,447 + (0.631)55,238,661,030,590,759 + (0.369)53,500,762,130,583,447$$
$$= 108,098,138,467,071,508$$

$$a_{5,\mathrm{u}}^{0}(1) = 49,218,551,626,124,255 + (0.521)55,238,661,030,590,759 + (0.479)53,500,762,130,583,447$$
$$= 103,624,759,083,611,512$$

Similarly, it is clear that z=1 is the optimal ordering policy since ₦108, 098,138,467,071,508 is greater than ₦103,624,759,083,611,512 with accumulated profit of ₦108, 098,138,467,071,508 and EOQ of 2units.

The Tables below shows the EOQ's and profits for all the 5-planning horizons for the five items.

Table 11: Table showing the EOQ and profits for the 5-planning horizons for item 1(m=1)

FIRST PLANNING	Z=1	Z=0
$a_{1,f}^{z}(1)$	7,767,419,720,101,435	2,885,460,119,379,403
$a_{1,\mathrm{u}}^{z}(1)$	5,491,348,113,364,339	1,897,498,047,745,567
$O_{f}^{z}(1)$	4	0
$O_{u}^{z}(1)$	2	0
SECOND PLANNING	Z=1	Z=0

$a_{2,f}^{z}(1)$	14,499,226,859,137,491	9,562,641,539,853,769
$a_{2,\mathrm{u}}^{z}(1)$	12,418,897,410,579,786	8,574,679,468,219,933
THIRD PLANNING	Z=1	Z=0
$a_{3,f}^{z}(1)$	28,051,903,819,181,226	23,065,390,593,132,119
$a_{3,\mathrm{u}}^{z}(1)$	26,150,482,703,199,484	22,077,428,521,498,283
FOURTH PLANNING	Z=1	Z=0
$a_{4,f}^{z}(1)$	55,238,661,030,590,759	50,206,513,697,758,091
$a_{4,\mathrm{u}}^{z}(1)$	53,500,762,130,583,447	49,218,551,626,124,255
FIFTH PLANNING	Z=1	Z=0
$a_{5,f}^{z}(1)$	109,686,578,061,678,191	104,612,721,155,245,348
$a_{5,\mathrm{u}}^{z}(1)$	108,098,138,467,071,508	103,624,759,083,611,512

The same computations were carried out for items 2, 3, 4 and 5. The results obtained are shown in the following tables: The EOQ for the first planning horizon is the same all through the planning horizons.

ITEM 2:

We assume the unit selling price $P_s(2)$ is ₦1000, the ordering cost $C_o(2)$ is ₦300, the holding cost $C_h(2)$ is ₦180, the shortage cost $C_s(2)$ is ₦20 and the cost price $C_p(2)$ is ₦500, then we obtain the following results for the planning horizons:

Table 12: Table showing the profits and EOQ's for the 5-planning horizons for item 2 (m=2)

FIRST PLANNING	Z=1	Z=0
$a_{1,f}^{z}(2)$	1,146,768,439,159,696	1,321,099,016,817,400
$a_{1,\mathrm{u}}^{z}(2)$	4,336,972,904,889,318	1,692,032,451,268,560
$O_f^z(2)$	7	0
$O_u^z(2)$	3	0
SECOND PLANNING	Z=1	Z=0
$a_{2,f}^{z}(2)$	3,825,010, 705,609,459	4,337,119,158,731,218

$a_{2,u}^{z}(2)$	7,015,215,171,339,081	4,708,052,593,182,378
THIRD PLANNING	Z=1	Z=0
$a_{3,f}^{z}(2)$	9,367,273,070,014,215	10,179,328,276,528,055
$a_{3,u}^{z}(2)$	12,557,477,535,743,837	10,550,261,710,999,215
FOURTH PLANNING	Z=1	Z=0
$a_{4,f}^{z}(2)$	20,616,768,513,200,372	21,243,107,245,477,361
$a_{4,u}^{z}(2)$	23,806,972,978,929,994	22,066,109,871,215,299
FIFTH PLANNING	Z=1	Z=0
$a_{5,f}^{z}(2)$	43,013,615,338,731,418	43,927,107,033,155,102
$a_{5,u}^{z}(2)$	46,203,819,804,461,040	44,750,109,658,893,040

ITEM 3:

We assume the unit selling price $P_s(3)$ is ₦900, the ordering cost $C_o(3)$ is ₦200, the holding cost $C_h(3)$ is ₦80, the shortage cost $C_s(3)$ is ₦20 and the cost price $C_p(3)$ is ₦400, then we obtain the following results for the planning horizons:

Table 13: Table showing the profits and EOQ's for the 5-planning horizons for item 3 (m=3)

FIRST PLANNING	Z=1	Z=0
$a_{1,f}^{z}(3)$	4,671,195,960,608,100	3,638,758,406,874,681
$a_{1,u}^{z}(3)$	3,756,458,413,165,815	2,926,462,866,743,006
$O_{f}^{z}(3)$	5	0
$O_{u}^{z}(3)$	1	0
SECOND PLANNING	Z=1	Z=0
$a_{2,f}^{z}(3)$	8,983,814,802,618,824	7,932,167,760,389,117
$a_{2,u}^{z}(3)$	8,069,077,255,176,539	7,219,872,220,257,442

THIRD PLANNING	Z=1	Z=0
$a_{3,f}^{z}(3)$	17,464,332,486,640,272	16,485,365,955,914,277
$a_{3,u}^{z}(3)$	16,639,594,939,197,987	15,773,070,415,782,602
FOURTH PLANNING	Z=1	Z=0
$a_{4,f}^{z}(3)$	34,605,367,854,683,168	33,609,081,835,460,885
$a_{4,u}^{z}(3)$	33,780,630,307,240,883	32,896,786,295,329,210
FIFTH PLANNING	Z=1	Z=0
$a_{5,f}^{z}(3)$	68,887,438,590,768,960	67,873,833,083,050,389
$a_{5,u}^{z}(3)$	68,062,701,043,326,675	67,161,537,542,918,714

ITEM 4:

We assume the unit selling price $P_s(4)$ is ₦1000, the ordering cost $C_o(4)$ is ₦300, the holding cost $C_h(4)$ is ₦70, the shortage cost $C_s(4)$ is ₦30 and the cost price $C_p(4)$ is ₦400, then we obtain the following results for the planning horizons:

Table 14: Table showing the profits and EOQ's for the 5-planning horizons for item 4 (m=4)

FIRST PLANNING	Z=1	Z=0
$a_{1,f}^{z}(4)$	7,869,301,269,933,555	5,118,464,752,907,168
$a_{1,u}^{z}(4)$	5,990,837,524,008,164	5,118,464,752,907,168
$O_{f}^{z}(4)$	6	0
$O_{u}^{z}(4)$	3	0
SECOND PLANNING	Z=1	Z=0
$a_{2,f}^{z}(4)$	14,842,575,333,060,698	11,496,411,991,119,346
$a_{2,u}^{z}(4)$	12,964,111,587,135,307	11,496,411,991,119,346

THIRD PLANNING	Z=1	Z=0
$a^z_{3,f}(4)$	28,789,123,459,314,984	24,150,305,886,151,953
$a^z_{3,u}(4)$	26,910,659,713,389,593	24,150,305,886,151,953
FOURTH PLANNING	Z=1	Z=0
$a^z_{4,f}(4)$	56,682,219,711,823,556	49,356,093,094,813,417
$a^z_{4,u}(4)$	54,803,755,965,898,165	49,356,093,094,813,417
FIFTH PLANNING	Z=1	Z=0
$a^z_{5,f}(4)$	112,468,412,216,840,700	99,665,666,930,732,596
$a^z_{5,u}(4)$	110,589,948,470,915,309	99,665,666,930,732,596

ITEM 5:

We assume the unit selling price $P_s(5)$ is ₦1000, the ordering cost $C_o(5)$ is ₦200, the holding cost $C_h(5)$ is ₦100, the shortage cost $C_s(5)$ is ₦50 and the cost price $C_p(5)$ is ₦500, then we obtain the following results for the planning horizons:

Table 15: Table showing the profits and EOQ's for the 5-planning horizons for item 5 (m=5)

FIRST PLANNING	Z=1	Z=0
$a^z_{1,f}(5)$	13,238,518,826,742	5,642,219,816
$a^z_{1,u}(5)$	7,529,095,786,927	3,761,479,877
$O^z_f(5)$	4	0
$O^z_u(5)$	2	0
SECOND PLANNING	Z=1	Z=0
$a^z_{2,f}(5)$	24,016,276,323,324	10,389,449,526,651
$a^z_{2,u}(5)$	18,306,853,283,509	10,387,568,786,712
THIRD PLANNING	Z=1	Z=0

$a_{3,f}^{z}(5)$	45,571,791,316,488	31,551,014,330,068
$a_{3,u}^{z}(5)$	39,862,368,276,673	31,549,133,590,129
FOURTH PLANNING	Z=1	Z=0
$a_{4,f}^{z}(5)$	88,682,821,302,816	74,268,094,126,649
$a_{4,u}^{z}(5)$	82,973,398,263,001	74,266,213,386,710
FIFTH PLANNING	Z=1	Z=0
$a_{5,f}^{z}(5)$	174,904,881,275,472	160,096,203,909,558
$a_{5,u}^{z}(5)$	169,195,458,235,657	160,094,323,169,619

From the above computations, the EOQ's for the 5 items and the planning horizons are as follows:

$O_1 = 4units$

$O_2 = 7units$

$O_3 = 5units$

$O_4 = 6units$

$O_5 = 4units$

The above unconstrained order quantities do not satisfy the requirements of the company as such, the LP method will be applied to obtain the optimal order quantities as shown below:

The Objective Function:

Maximize

$$500O_1 + 500O_2 + 500O_3 + 600O_4 + 500O_5$$

Constraints:

Capital Constraint:

$$500O_1 + 500O_2 + 400O_3 + 400O_4 + 500O_5 \leq 30,000$$

Order Quantity Constraints:

$$O_1 \geq 6,\ O_2 \geq 8,\ O_3 \geq 6,\ O_4 \geq 7,\ O_5 \geq 5$$

Warehouse Space Constraint:

$$20O_1 + 30O_2 + 35O_3 + 30O_4 + 40O_5 \leq 1000$$

Non-negativity and integer constraints:

$$O_1, O_2, O_3, O_4, O_5 \geq 0$$

Formulation: The complete LP problem can be formulated as follows:

Maximize

$$500O_1 + 500O_2 + 500O_3 + 600O_4 + 500O_5$$

Subject to:

$$500O_1 + 500O_3 + 400O_1 + 400O_3 + 500O_5 \leq 30,000$$

$$20O_1 + 30O_2 + 35O_3 + 30O_4 + 40O_5 \leq 1000$$

$$O_1 \geq 6,\ O_2 \geq 8,\ O_3 \geq 6, O_4 \geq 7,\ O_5 \geq 5$$

$$O_1, O_2, O_3, O_4, O_5 \geq 0$$

The above problem has been solved using the Matlab software. The optimal order quantities obtained are as follows:

$O_1 = 6 units$

$O_2 = 8 units$

$O_3 = 6 units$

$O_4 = 8 units$

$O_5 = 5 units$

To obtain the optimal profit, we substitute the values into the objective function as follows:

$500(6) + 500(8) + 500(6) + 600(7) + 500(5) = 16,700$. Therefore, the optimal profit is ₦16,700.

4 CONCLUSION

In this study, we developed an N-period dynamic programming model for constrained multi-item EOQ problem under stochastic demand. Linear programming technique is applied to deal with the constraints. We consider five items in inventory subject to the constraints of capital, order quantity and warehouse, with each item having different holding costs, ordering costs, shortage costs, cost prices and selling prices respectively. The demand for each item over the planning horizon is either in favorable state or in unfavorable state. We obtain the EOQ for the five items using dynamic programming method, we realized the result does not satisfy the constraints given by the management. Therefore, we apply the linear programming method to handle the constraints in order to obtain an optimal profit.

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