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A Heuristic Technique For Finding A Solution Of Job Sequencing Problem

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ABSTRACT

A finite set of sequential jobs performed through a setup machines assignment within a time-bound shift-wise manner or processing with a minimum time delay and using effectiveness process of machine sequencing order within the limited resource is called job sequencing problem. This paper proposed a heuristic technique known as a time deviation for an appropriate solution to the job sequencing problem to minimize the total minimum elapsed time and idle time in detail. We have written a process n number of sequential jobs through the m machines. Initially, we discussed the job sequencing problem for n jobs processed through two and three machines separately. We have also expanded the mentioned variables in n jobs for m machines. In conclusion, we have found the sequence of the job specification with the assigned machines. If the elapsed time for the n number of jobs process through m number of machines is known, for a more significant value of n, the final optimal assignment sequential jobs determined a listed solution by the MATLAB programming is discussed in this paper. If the total elapsed time of machines' jobs is concerned, a relationship of pre-assigned matrix elements.

1 INTRODUCTION

The role of sequencing and scheduling plan a sequence and completion a task. Rapidly increase the deterministic theory of sequencing and scheduling in past few decades such as scheduling of single machine, scheduling of parallel machines, identical, uniform, flow-shop, open-shop and scheduling of job-shop (JSS) [1].The specially ordered machines focused on overall optimizing the waiting time of machines when it formed in n jobs (processing stations) introduced by [2]. One of the combinatorial problems that have received the more focus in the literature is the scheduling of job-shop problem (JSSP).The standard JSSP when each operation to be performed by multiple resources is defined as the problem of flexible job-shop (FJSP). The problem of machine assignment and the problem of operation sequencing are the two challenges covered by the FJSP problems. Two objective function make-span and bi-criteria objective function reduce the time investigated by the flexible job shop problem (FJSP) of scheduling and setup dependent time sequence [3]. The classical job shop scheduling problem (SFJSP) has a novel problem of scheduling that considers flexibility of sequence under consideration. The improved genetic algorithm (IGA) uses to solve sequence flexibility job shop (SFJSP) explains by [4]. Gupta also explained the algorithm for reduce the rental-cost of machines under the individual rental-policy

of criteria of block job and interval break-down policy [5]. An optimal scheduling sequence is obtained for (FSSP) with one, two, three machines relating time of transportation, time of break-down, a new algorithm and weighted jobs are proposed by several works[6 - 10]. The mini-max and maxi-min sequencing problems solving methods and results compared to existing Johnson methods is proposed by [11]. In the production the schedule process in studies of decision-making optimization, the sequencing process entails creating an appropriate time-table for jobs scheduled, apparatus, public, materials, operational time, and services. The ultimate focus on problem of sequencing is to minimize the total process time between the first job and last job in a specific sequence. However, the job sequencing or order can be easily carried out by a computer application using an appropriate language. Job sequencing time optimization computational procedure does not have any programming code in any language in any literature [12 - 14]. A heuristic algorithm for a job sequencing problem proposed by [15]. A computer programs solving job sequencing problems of n number of jobs and m number of machines. Graphical procedure is well-known an extension for solving n number of jobs two machines in jobs sequencing problems. It's similar to find the shortest route between two vertices in a limited network. To the investigated value of giving solution is an optimal much better than feasible solution.

In the first part of this paper, we will discuss the process to solve the job sequencing problem for n number of jobs and m number of machines known as Johnson methods and describe it by solving an example. The second section explains this by the time deviation algorithm. We can solve the same problem by a time deviation algorithm and obtain a result that minimum total process time and free time of all machines. In the final section, we draw conclusions about the sequencing problem's outcome by comparing it with time deviation approaches.

2   PROBLEM WITH n JOBS AND m MACHINES

Assume n number of jobs is selected to be process on two, three or m number of machines through the job sequencing. The problem is that a sequential order for a job. The common theory in n number of jobs selection through the process of some or all k distinct machines satisfying the restrictions of order where each job to be processing through the machines and the job sequence performing must optimize the time measure effectiveness. The required processing time to complete a job through processing on a particular machine. Total minimum elapsed time is time interval involving time completion of jobs first to last including the idle time by the set of machines in a particular order would be minimum. The Idle time is that time in which the machine remains free have no job process.

In the outline, we will search a solution for n number of jobs through m number of machines in sequencing problem initially select $m = 2$ and $n = 3$ in particular cases of this article the solutions of the following cases are discussed:

a) n Jobs each of which is to be processed through and two machines $M1$ and $M2$, in the order $M1M2$.

b) n Jobs each of which is to be processed through three machines $M1, M2$ and $M3$, all jobs processed in the order $M1M2M3$

c)   Problems with n jobs each of which is to be processed through m machines $M1, M2, ... , Mm$ processed in the order $M1M2M3 ... Mm$.

Let the processing of n number of jobs through m number of machines, say $M1, M2, ... Mm$ in the order $M1M2M3 ... Mm$. Let t_{1j} is the time-span by the ith machine to complete the jth job.

The iterative procedure for determining the optimal sequence can be summarized as follows:

Step 1: Find (a)$min_j (t_{1j})$(b) $min_j (t_{nj})$ and $max_j (t_{2j}, t_{3j}t_{4ji}, t_{n-1j})$ for $all\ j = 1,2, \ldots \ldots, m$.

Step 2: Check the following inequality

$min_j(t_{1j}) \geq max_j(t_{ij})$ for $\quad i = 2,3 \ldots \ldots, n - 1$

Or $min_j(t_{nj}) \geq max_j (t_{ij})$ for $i = 2,3, \ldots \ldots, n - 1$

Step 3: If the inequality in Step 2 are not met then methods fails otherwise go to the next step.

Step 4: Converted m machines into two fictitious machines G and H, such that

$$t_{Gj} = \sum_{i=1}^{n-1} t_{ij} \text{ and } t_{Hj} = \sum_{i=2}^{n} t_{ij}.$$

Find an optimal job sequence for n number of jobs through two machines corresponding sequencing problem with the set order by using algorithm of the optimal sequence.

Step 5: In adding up the given condition in Step 4, if $t = \sum_{i=2}^{n} t_{ij} = C$ is a fixed constant(positive)for all $i = 1, 2, 3, \ldots m$ then determine the optimal sequence of n jobs and two machines $M1$ and Mn in the order $M1Mn$ by using the optimal sequence algorithm.

Problem 1:

Find optimal job sequence to minimize total elapsed time if passing is not allowed. Also, find idle time for each machines given processing time (hours)are as follows.

Table 1: The processing time of machines

Jobs	Machines					
	A	B	C	D	E	F
J1	18	8	7	2	10	25
J2	17	6	9	56	8	19
J3	11	5	8	5	7	15
J4	20	4	3	4	8	12

Solution

Here Minimum $Ai = 11$, Minimum $Fi = 12$

Maximum$(Bi, Ci, Di) = 8,9,6,10$; respectively

Since Minimum Fi = Maximum(Bi, Di, Ei) and Minimum Ai = Maximum Ci satisfied therefore the problem can be converted into four jobs and two fictitious machines G and H as follows:

Table 2: Converted into four jobs and two fictitious machines

Job	Fictitious Machine	
	$G_i = A_i + B_i + C_i + D_i + E_i$	$H_i = B_i + C_i + D_i + E_i + F_i$
J1	45	52
J2	46	48
J3	36	40
J4	39	31

The above sequence will be: J3   J1   J2   J4

Table 3: Total Elapsed Time Corresponding to Optimal Sequence can be obtained as follows:

Jobs	Machines					
	A	B	C	D	E	F
J3	0-11	11-16	16-24	24-29	29-36	36-51
J1	11-29	29-37	37-44	44-46	46-56	56-81
J2	29-46	46-52	52-61	61-67	67-75	81-100
J4	44-66	66-70	70-73	73-77	77-85	100-112

The minimum total elapsed time of the job sequence is 112 hours. Idle time for A is 46 hours, idle time for B is 89 hours, idle time for C is 85 hours, idle time for D is 100 hours, idle time for E is 79 hours, Idle time for F is 40 hours.

2.1 A Heuristic Method

We proposed a heuristic method known as time deviation in sequencing problem to determine an optimal sequence. The row deviation is the difference between times in the corresponding cell; the minimum time in that row is the value equal to the time of the cell by subtracting the minimum time of the corresponding row. Similarly, column deviation is the difference between the times in the corresponding cell, the minimum time in that column is the value equal to the time by subtracting the minimum time deviation of the corresponding column. Let a_i is the minimum time for ith row and b_i is the minimum time for the ith column. The row time deviation of the (i, j)th allocation denoting r_{ij} is defined as $r_{ij} = t_{ij} - a_i$. Similarly, the column time deviation denoting c_{ij} is defined as $c_{ij} = t_{ij} - b_i$. The sequencing problem for two machines in form of the time deviation shown as follows

Table 4:Time deviation table

Machines/Jobs	J1	J2	J3	……..,	Jn
M1	(r_{11}, c_{11})	(r_{12}, c_{12})	(r_{13}, c_{13})	………	(r_{1n}, c_{1n})
M2	(r_{21}, c_{21})	(r_{22}, c_{22})	(r_{23}, c_{23})	………	(r_{2n}, c_{2n})

The required job sequence is obtained from the processing time table, which value is based on the row and column deviation. The time deviation are shown as follows

Make the table of time deviation vector for the given problem of job sequencing.

1) Find the allocation of time in which both are zero for M1 machine and execute first the corresponding jobs.

2) Assume that if deviation vector equal to zero in multiple allocation, then compute the sum of time deviations of the parallel columns. Execute the job is the largest sum deviation first, then the job with the subsequently largest sum deviation, and so on.

3) Finally, appear for the allocation with time deviation vectors that are both zero for M2 machine.

4) Assume that multiple allocation such as both deviation vectors equal to zero, and then compute the sum of time deviations of the parallel columns. Execute the job with the largest sum deviation last, followed by the job with the subsequently largest sum divergence proceeding to last, and so on.

5) Stop the process if we obtain the sequence concerning all jobs for the job sequencing problem. Otherwise, proceed to the next step.

6) Make a table for reduced time duration with only unassigned jobs.

7) Repeat steps (1) through (6) above for the reduced time duration table. However, for the allocation where both deviation vectors are zero, perform the jobs next to the previously assigned jobs for M1 machine, and for the cells where both time deviation vectors are zero, perform the jobs prior to the last assigned jobs for M2 machine.

8) Determine the total time required to process all jobs in a sequence, as well as the time for which machines are available i.e. idle time.

Problem 2:

Find optimal job sequence to minimize total elapsed time if passing is not allowed. Also, find idle time for each machine given processing time (hours) as follows.

Table 5: The processing time of machines

Job	Machine				
	A	B	C	D	E
J1	7	5	2	3	9
J2	6	6	4	5	10
J3	5	4	5	6	8
J4	8	3	3	2	6

Solution:

Table 6: The problem converted into four jobs, two fictitious machines

Job/Machines	G	H
J1	17	19
J2	21	25
J3	20	23
J4	16	14

Table 7:G and H converted the table of time deviation as follows.

Job/Machines	G	H
J1	(1,0)	(5,2)
J2	(5,0)	(11,4)
J3	(4,0)	(9,3)
J4	(0,2)	(0,0)

The time deviation for job sequencing is shown below

	J4

Table 8: The remaining job sequence's reduced time deviation table is provided below

Job/Machines	G	H
J1	17	19
J2	21	25
J3	20	23

Table 9: The following is the time deviation table for the jobs on G and H's reduced time deviation table.

Job/Machines	G	H
J1	(0,0)	(0,2)
J2	(4,0)	(6,4)
J3	(3,0)	(4,3)

The time deviation for the job sequencing is shown below

J1	J4

Table 10:The remaining job sequence's reduced Time duration is in the table below.

Job/Machines	G	H
J2	21	25
J3	20	23

Table 11: the following is the time deviation table for the jobs on G and H's reduced time-duration table.

Job/Machines	G	H
J2	(1,0)	(0,0)
J3	(2,4)	(0,3)

The required steps for described problem of the job sequencing are follows

J1	J3	J2	J4

Table 12: The time required to perform the tasks in this sequence

Job	Machines				
	A	B	C	D	E
J1	0-7	7-12	12-14	14-17	17-26
J3	7-12	12-16	16-21	21-27	27-35
J2	12-18	18-24	24-28	28-33	35-45
J4	18-26	26-29	29-32	33-35	45-51

The total minimum elapsed time for job sequence is 51(hours). For machine A the idle time is 25 hours, for machine B the idle time is 23 hours, for machine C the idle time is 27 hours, for machine D the idle time is 35 hours and for machine E the idle time is 18 hours.

3   RESULTS AND DISCUSSION

In this paper the we obtained an optimum job sequence, total minimum elapsed time, and idle time for each machine in this algorithm we obtained the outcome from the heuristic method which are much better than Johnson's algorithm. These results are shown in the above. The heuristic algorithms is major improvement of using although number of possible sequence is very high we obtained optimal sequence within a fraction of second by computing on a normal system. The minimum total elapsed time and idle times not improved than a traditional algorithm but a different ways to solve such type problem in easier ways.

The computational time complexity of the MATLAB program which can be obtained by comparing the matrix rows and columns transformation is of order $O\ (n^2)$.M1 and M2 are machines, used in the job sequence, and each job has its processing path (M1, M2, and M1). The worst performance of ratio of 3/2 is increased time heuristic best previously available that creates a schedule that is at most 4/3 times as long as an optimal schedule. The space complexity is observed from the replacement of rows and columns if number of jobs and number of machines are very large may be greater than ten.

4   CONCLUSION

In this paper a new concept of *n* number of jobs processed through the *m* number of machines is introduced. The heuristic methods known as time deviation methods is a frame for finding optimal sequence and total elapse time of *n* number of jobs and *m* number of machines. Here we proposed at least build on this basic understanding in approaching more complicated situations.

In future we extend new heuristic technique for solving job sequencing problem to obtained optimal sequence without existing conditions.

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