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A Perusal of Transaction Details from Silk Road 2.0 and its Cogency using the Riemann Elucidation of Integrals

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ABSTRACT

The 21st century witnessed a huge boom in technology, improvement in living standards, or 'just' terms, people felt the thrills of modernization. But, as we know, all great deeds come at a pleasant cost. Alongside the numerous positive aspects of modernization, we got to experience several negative aspects of it. One such is the growth of the internet black market. In the past decade, the collaboration of several international intelligence agencies unveiled the sheer extent of these operations. Normally operated on the unregulated Darknet, these markets grew unbeknownst to the masses surfing the World Wide Web. In this paper, we will be studying the transaction details of one particular Darknet marketplace, namely SilkRoad 2, and will analyze its minute details with statistical essence. Now, unlike the Surface Web, the information from the darknet neither has the verified tags clinging to them nor has been stored in secure vaults. So, there's a huge chance of the transaction details being sandwiched with fallacies. To cope with this dilemma, one needs some strong pillar of evidence supporting it. So, do we. In this literature, we have proposed a new hypothesis, namely Riemann elucidation of Integrals, and have validated the transaction details, using some pre-existing principles and have compared its outcome to that of the outcome through our Hypothesis which has served as a witness to justify the effectiveness of our proposition.

Keywords: Crypto-markets, Silk Road 2.0, Cryptocurrencies, Zipf's Law, Dark Web, Riemann Integration

1 INTRODUCTION

Indexing the World Wide Web dates back to the early 1990s and by 2000, several major search engines like Google and Yahoo! rose to prominence. Modern search engines use a software called Web crawler (also known by other names such as "Spider") which "crawls" across the World Wide Web, indexing the pages along the way [1]. While the size of indexed pages rose to billions, tens of billions more cannot be indexed and hence are not retrievable using a search engine. These pages are often behind a paywall or are password-protected, so an unauthorized Web crawler cannot index them. Such pages collectively constitute the "hidden" Web or Deep Web, while those indexed constitute the Surface Web. The data and number of documents on the Deep Web massively outnumber that on the Surface Web. As early as 2001, the figures estimated were nearly 400:1 and 550:1 [2].

The material of the World Wide Web that is only accessible with specific software, configurations, or authorization is known as the "dark web,"[3][4] which refers to overlay networks that utilise the Internet. Private computer networks can interact and conduct transactions anonymously on the dark web sans disclosing personal information like a user's location. Although the phrase "deep web" is occasionally misused to refer particularly to the dark web, the dark web only makes up a small portion of the deep web, the portion of the Web that search engines do not index [5].

Small peer-to-peer networks between friends are included in the darknets that make up the dark web, along with huge, well-known networks run by individuals and public entities like Tor, Freenet, I2P, and Riffle. Now, while the Deep Web is unindexed, a vast majority of it is supposed to be under some regulation and can be viewed and accessed through a regular Web browser. However, a portion of the Deep Web, known as the Dark Web cannot be accessed by usual means. If the Deep Web is considered to be the set of all pages that cannot be indexed [6], then Dark Web is a subset of Deep Web where the pages are not only unindexed but the pages cannot be viewed in a standard Web browser or accessed without the use of specialized software or network configuration. They also use encryption to provide anonymity and privacy to the hosts and visitors. The overlay network of such pages is called darknet. Pages hosted on darknets largely avoid state and other forms of surveillance [7], and hence ideal for illegal activities while keeping the perpetrator anonymous.

Now, the merit of the paper includes the proposition, Riemann elucidation of Integrals, which is a methodology acquainted with data validation. The antecedent behind the nomenclature is quite promising, it’s the definition of integrals, which is the combination of infinitesimally small data to get to the result.

2 CRYPTO-MARKETS

According to predictions, the market capitalization of cryptocurrencies could reach $1–2 trillion in 2018. With peak trade volumes of almost $3 billion per day, the market capitalization of Bitcoin hit $70 billion.

The 27 ways that blockchain might fundamentally alter operations in fields as broad as banking, cybersecurity, voting, and academia have been revealed by technology principal analyst CB Insights.

According to the World Economic Forum, cryptographic protocols will be used to hold 10% of the world's GDP by 2027. China is home to the majority of mining pools, which account for more than 70% of all Bitcoin mining. The majority of cryptocurrency mining hardware is produced in China, which also benefits from competitive electricity costs [9][10].

Illicit marketing on the internet is not new. The first reported one can be dated back to 1972 [11] and since then the sophistication of technology and the advent of anonymous internet services led to a perfect platform for online black markets on the darknet [12]. These marketplaces are collectively referred to as darknet markets or crypto–markets. Items listed on darknet markets are usually restricted, like drugs and prescriptions being the majority, or unethical, like selling stolen personal information, hacking services, and distributing ransomware [13].

Like Surface Web, there are a lot of sites enlisted in Dark Web. The only difference between them is that the sites enlisted under Dark Web provide some illegal services. One such site was the Silk Road, which derived its name from the Eurasian trade route [14]. Silk Road is an online marketplace similar to Amazon or Flipkart, but with a huge stock of illegal items, like Drugs, Guns, etc. This site was developed in the year 2011 by Ross Ulbricht (pseudonym Dread Pirate Roberts). Silk Road sold goods and services to buyers all over the world, and during this time, the total revenue generated was somewhat around $183 million in total, though the transactions were carried out in Bitcoins [15][16]. In October 2013, the Federal Bureau of Investigation seized the website, but its mirror, Silk Road 2.0 was hosted by the former administrator(s) of Silk Road within a month.

3 CRYPTOCURRENCY

A cryptocurrency, also known as a cryptocurrency or crypto, is a type of digital currency that operates as a means of an exchange over an intranet and is not supported or maintained by any one central organization, such as a bank or government. Interfaces and data, a computerized database that uses strong encryption to secure public ledger [17][18], regulate the production of new coins, and confirm the transfer of currency ownership, is where individual coin ownership records are kept. Some coin maintenance schemes employ validators. A proof-of-stake model requires owners to pledge their tokens as security. In exchange, individuals receive control over the token in proportion to their investment. These token stakes typically acquire more shareholding over time through network fees, newly created tokens, or other similar compensation systems [19].

Despite claims to the contrary, cryptocurrencies are not thought of as currencies in the traditional sense. Although they have received a variety of classifications, including those of commodities, securities, and currencies, in reality, cryptocurrencies are typically seen as a separate asset class. Some cryptographic methods employ validators [20].

Even before Silk Road came into action, several illicit marketplaces served in the same way as Silk Road. In 2006 – 2012, there existed one such site namely, The Farmer's Market, which was an online vendor for illegal drugs, that used to accept payments via PayPal and Western Union [21]. These transactions were easily traceable by Law Enforcers, and that set a reason for not using usual forms of transactions for these operations [22].

An innovation came when cryptocurrencies became popular. Later on, such marketplaces dealing with illicit services started using cryptocurrencies as the main form of transaction. Bitcoin is one such widely used cryptocurrency in today's world. It is a type of currency that is not controlled by some centralized organization or bank. In a blockchain, bitcoins are registered at some specific bitcoin address and to create such an address requires nothing but selecting a random valid private key (hash value) and computing the corresponding address [23][24]. It doesn't have a single administrator, rather it is managed and maintained by several ledgers who are equally powered and are referred to as miners.

4 ZIPF'S LAW

Between the late 19'th Century and Early 20s, one observation that stroke many well-known linguists, stenographers, and physicists, was that some words appear more frequently than others in some well-managed sets of natural languages. Later, the American Linguist, G.K. Zipf, explained it quantitatively [25]. He said an inverse relationship exists between the rank and frequency of statistical data sets. In other words, we can interpret him as "The product of rank and frequency is always a constant".

Mathematically,

According to Zipf's Law,

$f(r) \times r = c^2$

where,

$f(r)$ is the frequency corresponding to rank r.

c^2 being a positive constant.

Zipf's Law finds its usage nowadays in a lot of domains, like linguistics, informatics, and many more.

5 RIEMANN ELUCIDATION OF INTEGRALS HYPOTHESIS

In this Section, we have introduced a new methodology for validating a chunk of data.

If we consider, the rank to be acquainted along the x – axis, and the frequency along the y – axis, their 2 – D plot on the $x - y$ coordinate system will be a Rectangular Hyperbola with the x, and y axes as their asymptotes.

So,

$\varphi(x) \times x = c^2$.

The set of Discrete point(s) within domain corresponding to $\varphi(x)$ will be:

$\{(1, \varphi(1)), (2, \varphi(2)), (3, \varphi(3)), (4, \varphi(4)), (5, \varphi(5)), (6, \varphi(6)), \ldots, (9, \varphi(9))\}$.

On a general note, for an abscissa of μ, the projection on the axes of the ordinate will be $\varphi(\mu)\ \forall\ 1 \leq \mu \leq 9$.

Graphically,

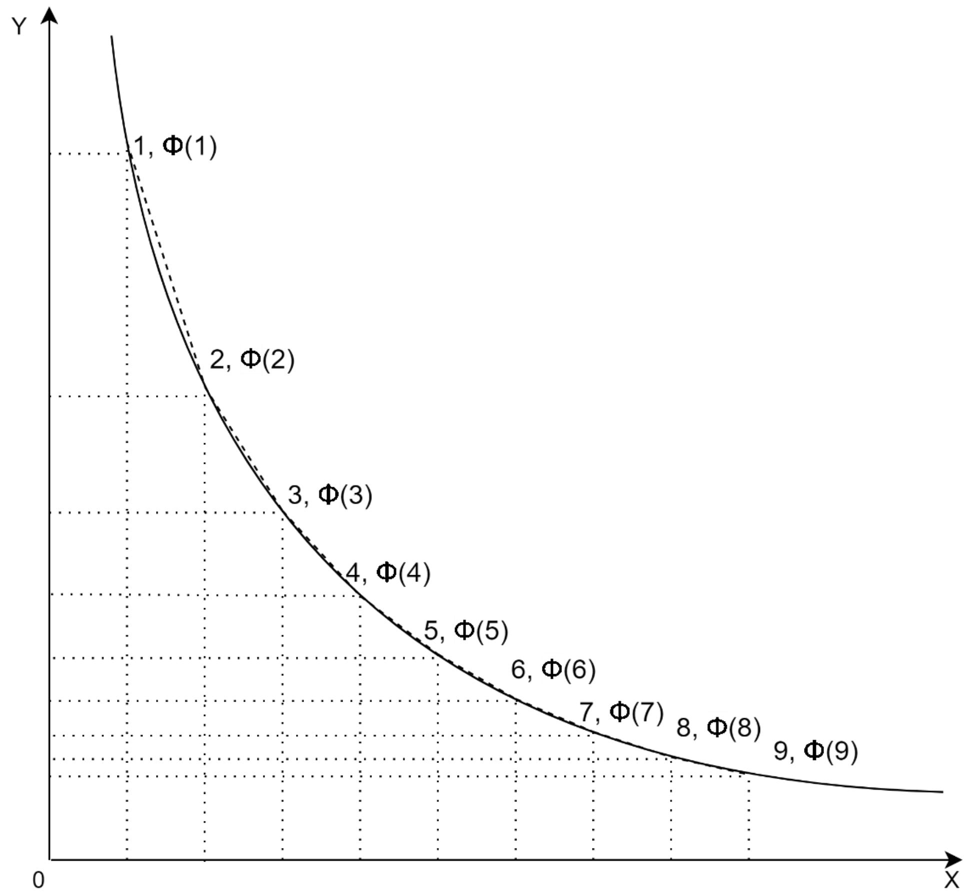

Figure 1 : Graphical abstraction of the Proposed Methodology.

Graphical Representation of $\varphi(x) \times x = c^2$ along with the discrete points.

Considering the discrete points, total area, $\mathcal{A}$ will be

$$\frac{1}{2}\sum_{i=1}^{8}\big(\varphi(i) + \varphi(i+1)\big)$$

We know, the total area, $\mathcal{A}'$ under $y = \varphi(x)$ will be

$$\int_{\alpha}^{\beta} \varphi(x)\, dx$$

where α, and β are the extremum.

Here, $\varphi(x) = \frac{c^2}{x}$. So,

$$\mathcal{A}' = \int_1^9 \left(\frac{c^2}{x}\right) dx = c^2\, log_e(9)$$

Equating, we get,

$$\mathcal{A} + \varepsilon = \mathcal{A}'$$

So,

$$2\left(\sum_{i=1}^{9} \varphi(i)\right) - \big(\varphi(1) + \varphi(9)\big) + \varepsilon = 2c^2\, log_e(9)$$

Now, in accordance to Zipf's Law,

$$i \times \varphi(i) = c^2\ \forall\ i \in \mathbb{Z}^+\ \&\ 9 \geq\ i \geq 1$$

So,

$$\sum_{i=1}^{9} \varphi(i) + \varepsilon = c^2\left(log_e(9) + \left(\frac{5}{9}\right)\right)$$

where, $(\frac{\Delta\varepsilon}{\varepsilon})$ = Relative difference between the two areas.

Thus, we have :-

$$(\frac{\Delta\varepsilon}{\varepsilon}) = \left| c^2\left(log_e(9) + \left(\frac{5}{9}\right)\right) - \sum_{i=1}^{9} \varphi(i) \right| = \ c^2 \times \left| log_e(9) + \left(\frac{5}{9}\right) - \sum_{i=1}^{9} \frac{1}{i} \right| = \ c^2 \times 0.0761$$

Here, we have, $c^2 =\ \varphi(1)$.

Relative Error $(\frac{\Delta\varepsilon}{\varepsilon}) =\ \frac{c^2 \times 0.0761}{c^2 \times 2.8289} = 0.02690$

% Relative error in Area $(\frac{\Delta\varepsilon}{\varepsilon} \times 100) = 2.690$

6 MATHEMATICAL ANALYSIS OF THE DARKNET DATASET

6.1 Using Riemann elucidation of Integrals Hypothesis

Here, in this subsection, we will apply the Riemann elucidation of Integrals Hypothesis on the famous Darknet Dataset by Gwern Branwen. Originally the dataset was sized somewhat around 1.6 TB, which was released publicly with a size of 50 GB. Dealing with such a big dataset quantitatively is too difficult to manage, so we will work qualitatively on this dataset and will try to justify our views. Given below is a glimpse of the Dataset.

Table 1. DarkNet Dataset by Gwern Branwen.

Title	*Seller ID*	*Price USD*	*Price BTC*
Ray Ban Tech RB3460 001 Aviator/Flip Out/ Sunglasses Replica	FoxyGirl	61.16154	0.098232
Ray Ban RB3025 Aviator Classic Sunglasses Replica	FoxyGirl	37.96259	0.060972
Rolex - Watch Box (AAA Grade Replica)	RepAAA	100.4298	0.161301
Ray Ban RB3016 - W0365 | Clubmaster Sunglasses Replica	FoxyGirl	45.34379	0.072827
Rolex - Submariner 2Tone YG/SS Black [Replica]	RepAAA	149.6351	0.24033

We have considered both the price in USD, and price in BTC for our analysis.

For our analysis, we have considered the frequency of the first digits for the Data, which turned out to be like

Table 2. First Digit Frequency for data with prices in USD.

Digits	*Frequency*
1	128746
2	78512
3	64476
4	55140
5	44722
6	33584
7	25231
8	22564
9	21945

for the Prices in USD, and,

Table 3. First Digit Frequency for data with prices in BTC.

Digits	*Frequency*
1	159473
2	71076
3	51904
4	41649
5	36027
6	33019
7	29399

8	29126
9	23247

for the Prices in BTC.

Considering the Discrete Area for prices in USD

$$\mathcal{A}'|_{USD} = 2\left(\sum_{i=1}^{9} \varphi(i)\right) - \left(\varphi(1) + \varphi(9)\right) = 399574.5\ sq.units$$

Considering the Continuous Area for prices in USD

$$\mathcal{A}|_{USD} = c^2\ log_e(9) = 282883.8754\ sq.units$$

So, Relative error in Area $(\frac{\Delta\varepsilon}{\varepsilon})$ = $\left[\frac{max(\mathcal{A}',\mathcal{A}) - min(\mathcal{A}',\mathcal{A})}{max(\mathcal{A}',\mathcal{A})}\right] = 0.292037216$

Percentage Relative error in Area $(\frac{\Delta\varepsilon}{\varepsilon} \times 100)$ = $0.292037216 \times 100 = 29.2037216\% \approx 29\%$, which is in inacceptable ranges.

Again,

Considering the Discrete Area for prices in BTC

$$\mathcal{A}'|_{BTC} = 2\left(\sum_{i=1}^{9} \varphi(i)\right) - \left(\varphi(1) + \varphi(9)\right) = 383560\ sq.units$$

Considering the Continuous Area for prices in BTC

$$\mathcal{A}|_{BTC} = c^2\ log_e(9) = 350397.995\ sq.units$$

So, Relative error in Area $(\frac{\Delta\varepsilon}{\varepsilon})$ = $\left[\frac{max(\mathcal{A}',\mathcal{A}) - min(\mathcal{A}',\mathcal{A})}{max(\mathcal{A}',\mathcal{A})}\right] = 0.086458455$

Percentage Relative error in Area $(\frac{\Delta\varepsilon}{\varepsilon} \times 100)$ = $0.086458455 \times 100 = 8.6458455\% \approx 9\%$, which is in acceptable ranges

Now, from these analytics, we can conclude that the prices in BTC is a better mark than prices in USD as $\left(\frac{\Delta\varepsilon}{\varepsilon} \times 100\right)_{BTC} \ll \left(\frac{\Delta\varepsilon}{\varepsilon} \times 100\right)_{USD}$

One logical explanation for the surging height of $\left(\frac{\Delta\varepsilon}{\varepsilon} \times 100\right)_{USD}$ is the rise and fall of Government Controlled Currency, here USD. As we know, Bitcoins are not controlled by any foreign bodies, so, we can say that the price stability of BTC will be much higher than that of the USD.

6.2 Using Zipf's Law

To validate our result from REOI Hypothesis, we will make use of Zipf's Law that links the rank (here first digit) of the inputs with the corresponding frequencies. Basically, when observations (such as words) are ordered according to how frequently they occur, Zipf's law states that the frequency of a specific observation is inversely proportional to its rank, or Frequency is inversely proportional to Rank.

For Price in USD,

Table 4. Rank Frequency table for data with prices in USD.

Rank	*Frequency*	*Product*
1	128746	128746
2	78512	157024
3	64476	193428
4	55140	220560
5	44722	223610
6	33584	201504
7	25231	176617
8	22564	180512
9	21945	197505

Now coming to the question i.e., what is a log-log graph?

To display more detail in the extremities, a log-log graph is employed, where the log of the rank serves as the scale for the horizontal axis and the log of the frequency serves as the vertical axis.

Plotting the logarithmic plots including rank and frequency, we get nearly a straight-line curve. This observation helps us in concluding that the Dataset is free from fallacies theoretically.,

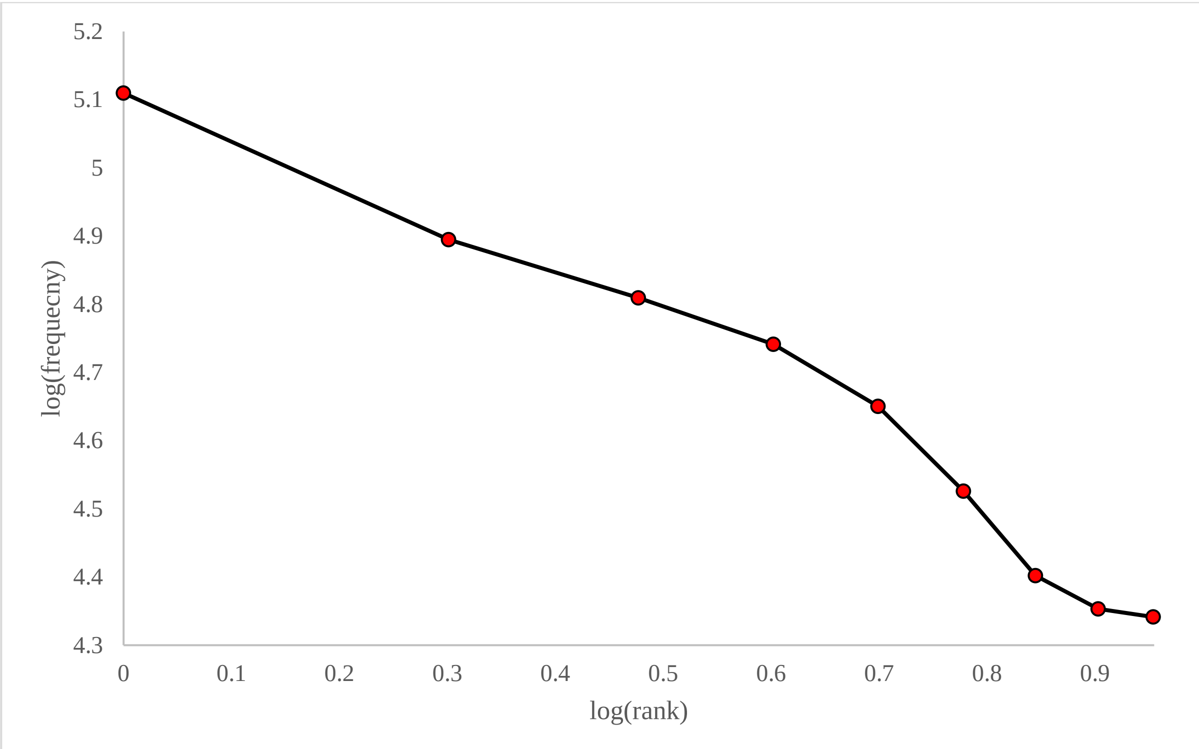

Figure 2: log – log plot for Frequency – Rank abstraction of the price in USD.

For Price in BTC,

Table 5. Rank Frequency table for data with prices in BTC.

Rank	*Frequency*	*Product*
1	159473	159473
2	71076	142152
3	51904	155712
4	41649	166596
5	36027	180135
6	33019	198114
7	29399	205793
8	29126	233008
9	23247	209223

Plotting the logarithmic plots including rank and frequency, we get nearly a straight-line curve. This observation helps us in concluding that the Dataset is free from fallacies theoretically.

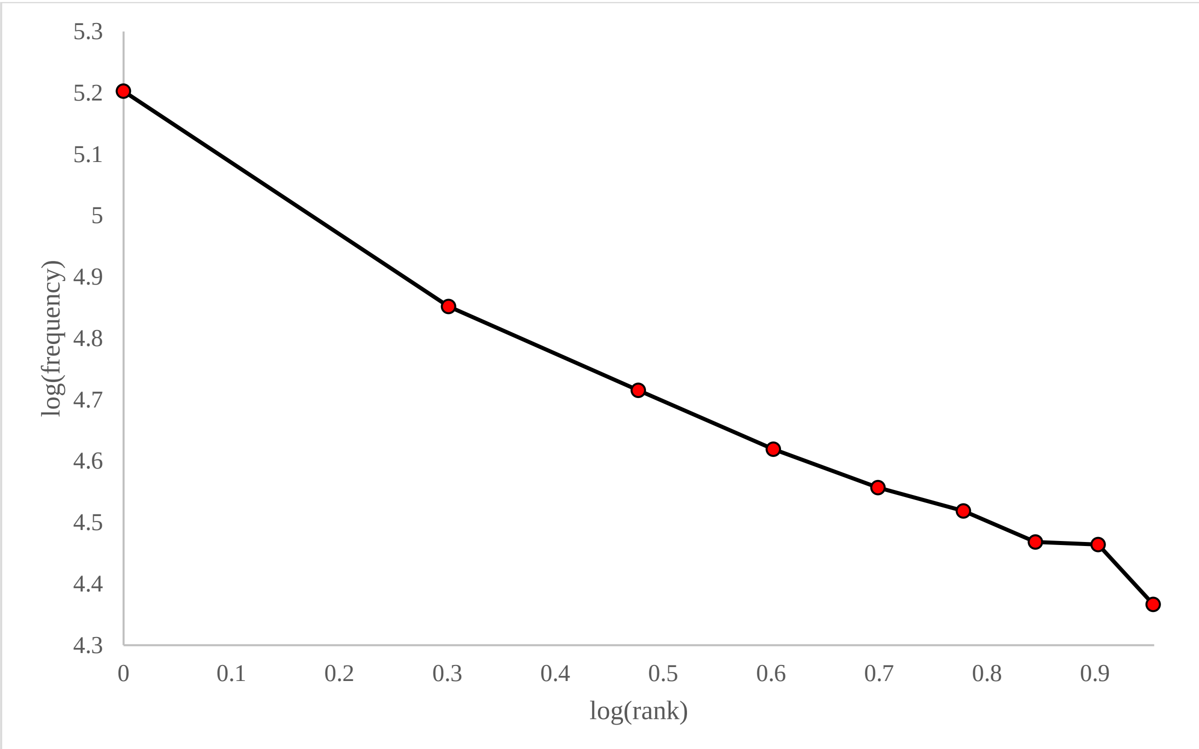

Figure 3 : log – log plot for Frequency – Rank abstraction of the price in BTC.

7 CONCLUSION

They serve largely as black marketplaces, facilitating trade in legal commodities as well as illegal ones including drugs, cyber-arms, firearms, fake money, credit card information, false documents, unlicensed medications, steroids, and other illegal things.

The formula highlighted and derived in this paper differs and takes over Zipf's Law in a few key ways i.e., some advantages:

- Zipf's Law does not state whether a dataset is valid or not because there is no estimate for the conformation range under which we may claim a dataset to be theoretically valid.

- The absence of error limitations in Zipf's law prevents us from determining the acceptable level of dataset departure from the ideal scenario.

Our formula satisfies the criteria for the Zipf's law shortcomings listed above.

- The absolute percentage error for both datasets is the same up to four decimal places, according to the validation described above.

- We can infer that the absolute percentage error is independent from the observation above.

- Since the dataset must be legitimate, we may infer from the aforementioned observation that the absolute percentage error is independent of the term c^2, meaning that the percentage error will be constrained by a particular conformity range.

Future developments that can be done in our formula:

This conjecture can be given a conformation range for absolute percentage error, and if the absolute percentage error of a particular dataset falls within this range, we can say that the conjecture is theoretically valid. It is not being purely established that the value of the absolute percentage error is independent of the term c^2, rather we have taken an approximation to make the expression independent of c^2. Future research on this hypothesis can be done if it is shown that the statement immediately adjacent is accurate.

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